1. Optical Path

To derive one of the most fundamental principles in geometric optics, it is appropriate to define a quantity called the optical path. The path d of a ray of light in any medium is given by the product velocity times time:

$d = vt$	1

Since n = c/v

$d = \frac{c}{n}\, t \;\; or\, nd = c\, t$	2

The product *nd* is called the *optical path* Δ= *nd*

The optical path represents the distance light travels in a vacuum in the same time it travels a distance *d* in the medium. If a light ray travels through a series of optical media of thickness *d, d', d''*, ... and refractive indices *n*, *n'*, *n''*, ... , the total optical path is just the sum of the separate values:

$\Delta \; = \; nd \; + \; n'd' \; + \; n''d'' + \ldots$	3

A diagram illustrating the meaning of optical path is shown in Fig. 1.

Three media of length *d, d'*, and *d''*, with refractive indices *n, n'*, and *n''*, respectively, are shown touching each other. Line *AB* shows the length of the actual light path through these media, while the line *CD* shows the distance A, the distance light would travel in a vacuum in the same amount of time *t.*

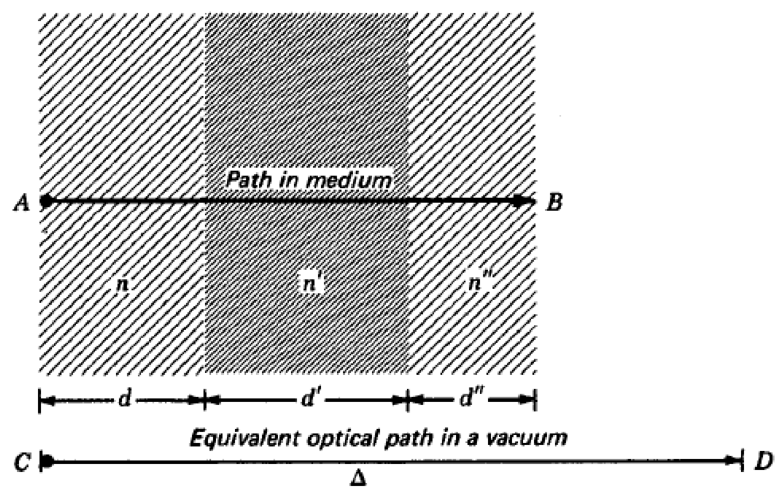


Fig. I: The optical path through a series of optical media.

2. Fermat's Principle

The first way of thinking that made the law about the behavior of light evident was discovered by Fermat in about 1650, and it is called *the principle of least time*, or *Fermat's principle*. His idea is this: ***that out of all possible paths that it might take to get from one point to another, light takes the path which requires the shortest time.***

Consider a ray of light that must pass through a point *Q* and then, after reflection from a plane surface, pass through a second point *Q"* (see Fig. 2). To find the real path, we first drop a perpendicular to *GH* and extend it an equal distance on the other side to *Q'*. The straight-line *Q' Q"* is drawn in, and from its intersection *B* the line *QB is* drawn. The real light path is therefore *QBQ"*, and, as can be seen from the symmetry relations in the diagram, it obeys the law of reflection.

Consider now adjacent paths to points like *A* and C on the mirror surface close to *B*. Since a straight line is the shortest path between two points, both the paths *Q' A Q"* and *Q' CQ"* are greater than *Q' BQ"*. By the above construction and equivalent triangles, *QA* = *Q'A*, and *QC* = *Q'C*, so that *QAQ"* > *QBQ"* and *QCQ"* > *QBQ"*. Therefore, the real path *QBQ"* is a minimum. A graph of hypothetical paths closes to the real path *QBQ"*, as shown in the lower right of the diagram, indicates the meaning of a minimum, and the flatness of the curve between *A* and C illustrates that to a first approximation adjacent paths are equal to the real optical path.

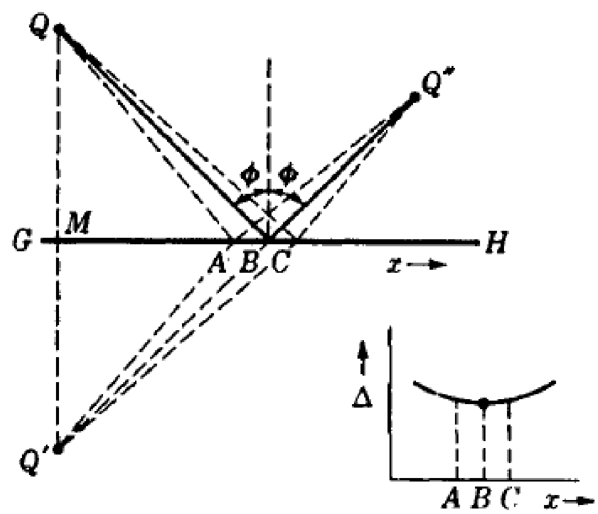
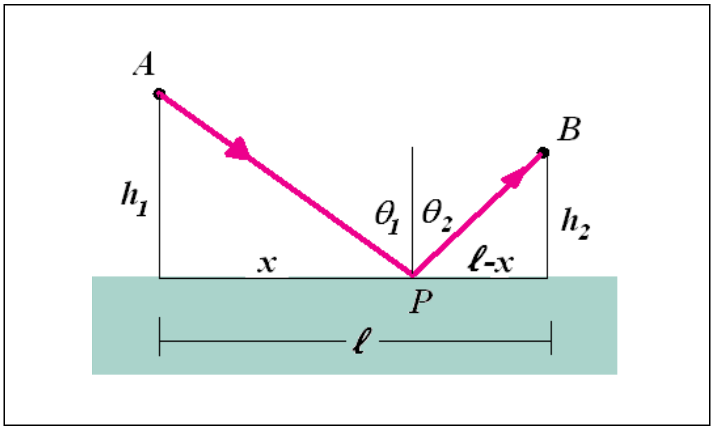


Fig. 2: Fermat's principle applied to reflection at a plane surface.

3. Derivation of The Laws of Reflection and Refraction

(a) Consider the light ray shown in the figure. A ray of light starting at point **A** reflects off the surface at point **P** before arriving at point **B**, a horizontal distance **L** from point **A**. We calculate the length of each path and divide the length by the speed of light to determine the time required for the light to travel between the two points.

$$t = \frac{\sqrt{x^2 + h_1^2}}{c} + \frac{\sqrt{(1-x)^2 + h_2^2}}{c} \qquad (4)$$



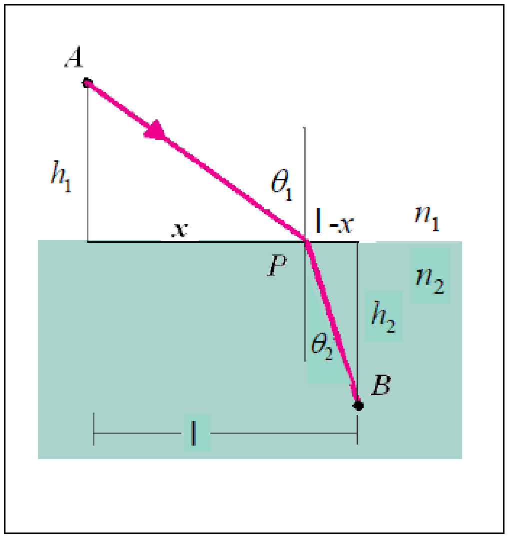
To minimize the time we set the derivative of the time with respect to equal to zero. We also use the definition of the sine as opposite side over hypotenuse to relate the lengths to the angles of incidence and reflection.

$$0 = \frac{dt}{dx} = \frac{x}{c\sqrt{x^2 + h_1^2}} + \frac{-(1-x)}{c\sqrt{(1-x)^2 + h_2^2}} \tag{5}$$

$$\frac{x}{\sqrt{x^2 + h_1^2}} = \frac{(1-x)}{\sqrt{(1-x)^2 + h_2^2}} \;, \sin\theta_1 = \sin\theta_2 \;, \theta_1 = \theta_2 \tag{6}$$

(b) Now we consider a light ray traveling from point A to point B in media with different indices of refraction, as shown in the figure. The time to travel between the two points is the distance in each medium divided by the speed of light in that medium.

$$t = \frac{\sqrt{x^2 + h_1^2}}{c/n_1} + \frac{\sqrt{(1-x)^2 + h_2^2}}{c/n_2} \tag{7}$$



To minimize the time we set the derivative of the time with respect to equal to zero. We also use the definition of the sine as opposite side over hypotenuse to relate the lengths to the angles of incidence and reflection.

$$0 = \frac{dt}{dx} = \frac{n_1 x}{c\sqrt{x^2 + h_1^2}} + \frac{-n_2(1-x)}{c\sqrt{(1-x)^2 + h_2^2}} \tag{8}$$

$$\frac{n_1 x}{c\sqrt{x^2 + h_1^2}} = \frac{n_2(1-x)}{c\sqrt{(1-x)^2 + h_2^2}}, n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{9}$$

4. Critical Angle

The refractive index is not a constant but varies as a function of wavelength λ. The presence of this variation is called dispersion (dn/dλ) of the material. Thus, refraction causes a separation of white light into its component color, i.e. white light is spread out into a spectrum. The symbols F, D, and C are the names of the Fraunhofer wavelength in the solar spectrum (dark lines).

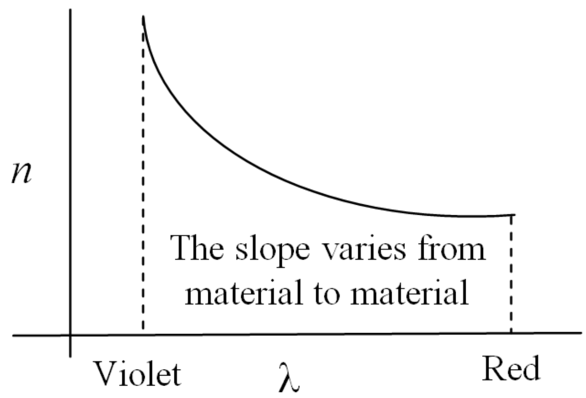


Fig. (3)

A number that indicates the amount of the dispersion (or dispersing properties) for a glass is the V-number, or Abbe' number, defined by:

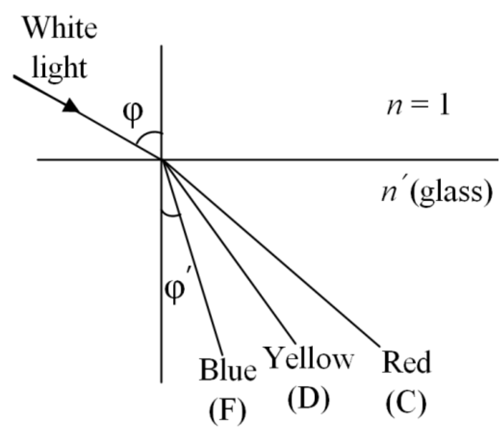


Fig. (4)

$$V = \frac{n_D - 1}{n_F - n_C} = dispersive\ index$$

$20 \leq V \leq 60$ for glass

The dispersive power = $\frac{1}{V} = \frac{n_F - n_C}{n_{D-1}}$

A large value of V indicates small dispersion. Because the dispersion (i.e. dispersive power) is greatest when ($n_F - n_C$) is largest, glasses with the strongest dispersion have the smallest V-number.

The angular dispersion ($\emptyset'_C - \emptyset'_F$) is proportional to ($n_F - n_C$). the deviation of the yellow (D) ray is ($\emptyset - \emptyset'_D$) which is proportional to ($n_D - 1$).

While many glasses have special names (crown glass K, flint glass F) all glasses can be labeled by their n_D and V. the glass number is a six digits number whose first three digits are the three most significant digits of $n_D - 1$ and whose second three digits are ten times (10 ×) the V-number. For example, the glass number for crown glass (n_D = 1.5320 and V = 58.7) is 532587.

Flint glass has a greater dispersion than crown since n of the flint glass › n of crown glass.

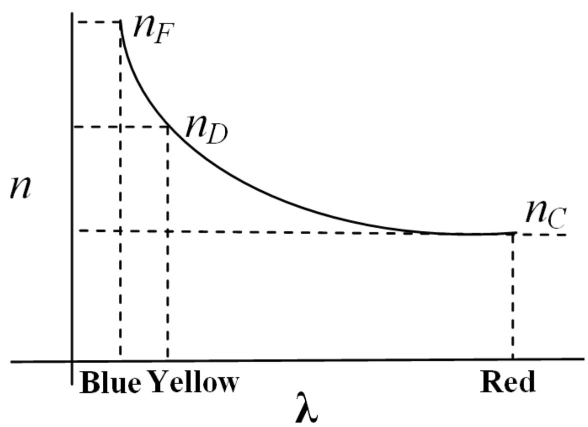


Fig. (5)