Definition: LASER is an acronym of:

<u>L</u>ight <u>A</u>mplification by the <u>S</u>timulated <u>E</u>mission of <u>R</u>adiation

The electromagnetic spectrum

Light, radio waves , x-rays , γ-rays are all E.M radiation,

The only difference between them is their frequency.

E.M radiation is propagated through space as a *<u>transverse wave</u>*; the speed of propagation (c) is related to frequency by:

$$c = \lambda * f \qquad \lambda = \text{wavelength}$$

One of the most important parameters of a wave is its wavelength.

Wavelength (λ) (Lamda) is the distance between two adjacent points on the wave, which have the same phase. As an example (see figure 1.1 below) the distance between two adjacent peaks of the wave.

In a parallel way it is possible to define a wave by its frequency. Frequency (ν) is defined by the number of times that the wave oscillates per second (The number of periods of oscillations per second).Between these two parameters the relation is: $c = \lambda \nu$

From the physics point of view, all electromagnetic waves are equal (have the same properties) except for their wavelength (or frequency).
As an example: the speed of light is the same for visible light, radio waves, or x-rays.

<u>Wave Description:</u> A wave can be described in two standard forms:

1. Displacement as a function of space when time is held constant.
2. Displacement as a function of time at a specific place in space.

1. Displacement as a function of space, when time is "frozen" (held constant), as described in figure 1.1. In this description, the minimum distance between two adjacent points with the same phase is wavelength (ν). Note that the horizontal (x) axis is space coordinate.

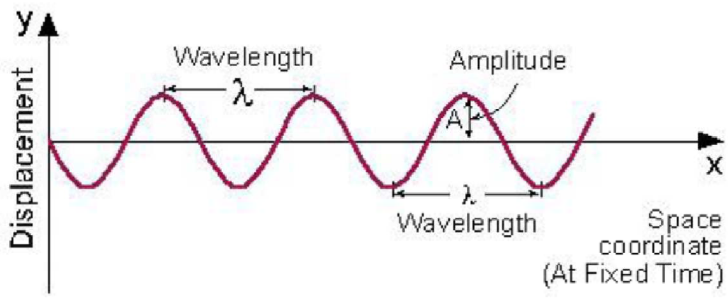


Fig 1.1: Displacement as a function of space coordinates (at fixed time)
A = Amplitude = Maximum displacement from equilibrium.

2. Displacement as a function of time: in a specific place in space, as described in figure 1.2. In this description, the minimum distance between two adjacent points with the same phase is period (T). Note that the horizontal (x) axis is time coordinate!

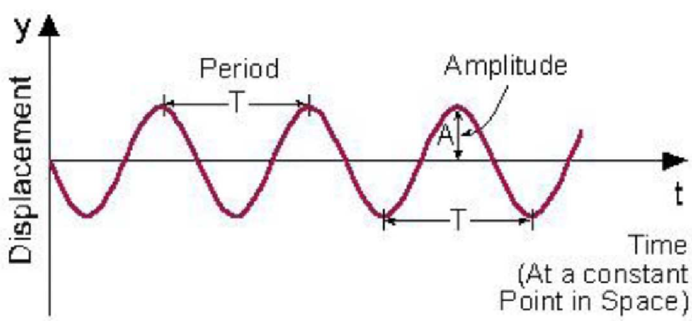


Figure 1.2: Displacement as a function of time (at a fixed point in space)

Wavelengths Comparison

Figure 1.3 describes how two different waves (with different wavelengths) look at a specific moment in time. Each of these waves can be uniquely described by its wavelength.

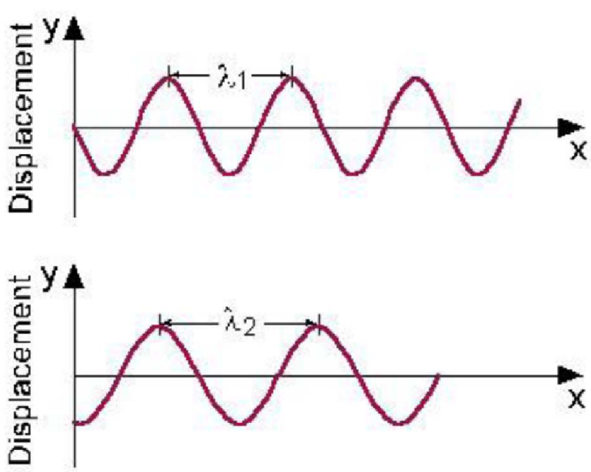


Figure 1.3: Short wavelength (λ_1) compared to longer wavelength (λ_2)

In vacuum the speed of light for any E.M radiation is equal to :

$c = 3*10^8\ m.s^{-1}$

In a transparent medium the velocity (c') is less than (c).

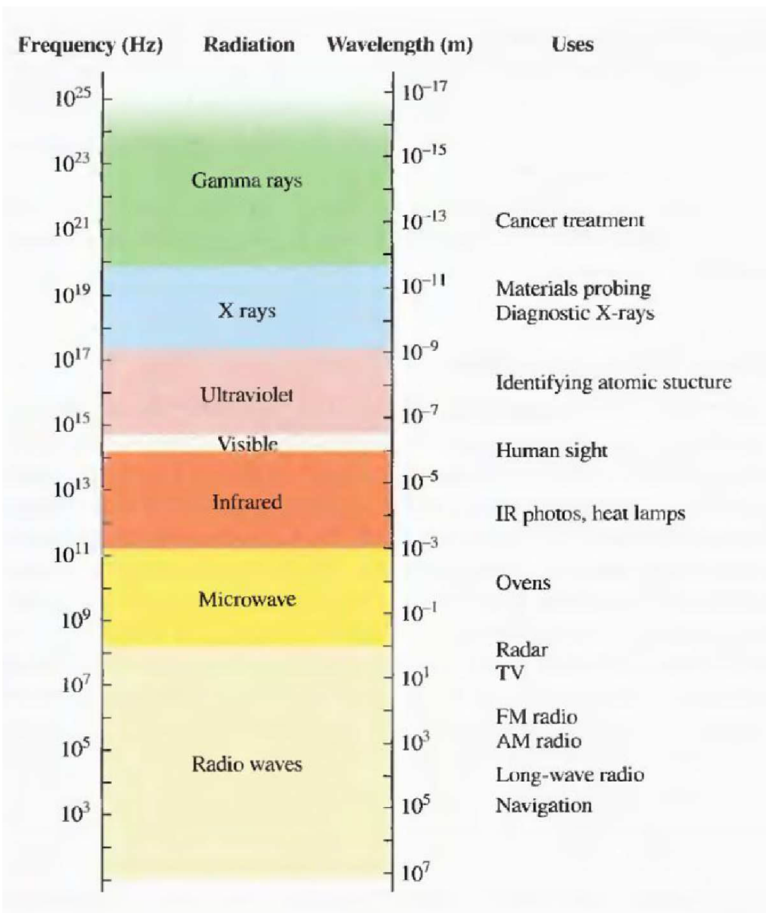
This reduction is related to the refractive index of medium by:

Refractive index of the medium (n) = (velocity in vacuum / velocity in medium) = c/c'

so : $c' = c/n$

As the radiation enters a region of higher ref. index, the wavelength is reduced, the frequency remains constant.

Ref. index of air is ~ 1.0028 for visible light, the effect on λ due to air may be ignored except for high accuracy work.



Photon energy	Wavelength (m)	Frequency (HZ)	Region
100 kev	10^{-11}	10^{20}	γ-ray
10 kev	10^{-10}	10^{19}	
1 kev	10^{-9}	10^{18}	x-ray
100 ev	10^{-8}	10^{17}	
10 ev	10^{-7}	10^{16}	v.uv
1 ev	10^{-6}	10^{15}	u.v
0.1 ev	10^{-5}	10^{14}	visible
10^{-2} ev	10^{-4}	10^{13}	I.R
10^{-3} ev	10^{-3}	10^{12}	F.I.R
10^{-4} ev	10^{-2}	10^{11}	
10^{-5} ev	10^{-1}	10^{10}	microwave
10^{-6} ev	0	10^{9}	
10^{-7} ev	10	10^{8}	radio freq
.	10^{2}	10^{7}	
.	10^{3}	10^{6}	
10^{-10} ev	10^{4}	10^{5}	

The E.M. spectrum

The speed of electromagnetic waves

$$v = \sqrt{\frac{1}{\mu\varepsilon}} = \frac{c}{n}.$$

The quantity n is the **index of refraction** of a given medium.

The energy of a quantum of light depends on its frequency.

$E = h*f = h * \upsilon$

Where: h= planck's constant = $6.63*10^{-34}$ J.s

E in joules & f in $Hz = s^{-1}$

E (joule) = hc/ λ = $(6.63*10^{-34} * 3*10^{8}) / \lambda$ (joules)

But: 1 ev = $1.6*10^{-19}$ J

So : E in (ev) = $(6.63*10^{-34} * 3*10^{8}) / (\lambda * 1.6*10^{-19})$ ev

$= (1.243*10^{-6}) / \lambda$ ev

Units

Multiplication factor	name	symbol
10^{12}	Tera	T
10^{9}	Giga	G
10^{6}	Mega	M
10^{3}	Kilo	K
10^{-2}	Centi	c
10^{-3}	Milli	m
10^{-6}	Micro	µ
10^{-9}	Nano	n
10^{-12}	Pico	p
10^{-15}	Femto	f
10^{-18}	atto	a

Velocity of light $c = 3*10^{8}\ ms^{-1}$
Frequency = cycles per sec (cps) or Hz

Since frequency used lie in the range (10^{10} -- 10^{20}) Hz
Frequency usually expressed as *wave number*
wave number (cm^{-1}) = 1/ λ (cm) = f (Hz)/c(=$3x10^{10}$ cm/s)

freq & wave number units have two advantages over wavelength units

1) **They are both independent of the medium.**
2) **They are both directly proportional to the photon energy**

Historical preview:

The idea of the amplification of the electro-magnetic radiation by the stimulated emission was started after the 2nd world war. The first use of stimulated emission was in the building of the first **MASER**:

Microwave **A**mplification by the **S**timulated **E**mission of **R**adiation) in 1954 by TOWNES and his group in the USA (the system works in the radiation wavelength of 1.25 cm which is used in RADAR), the active medium was the ammonia gas.

The population inversion was achieved by splitting the high energy molecules from those in the lower level.

Later was the achievement of *population inversion* by pumping (optical pumping) in 1955 then was the design of solid state masers (Ruby) in 1957.

There was a great hope to extend the application of the *stimulated emission* to the visible part of the electro-magnetic spectrum (later called the Optical maser).
Townes and Shawlow worked together in a study in 1958 to solve the problems evolved from the manufacturing of ***optical maser***, which may be;

1. The differences in energy levels to produce visible radiation are large comparing to (KT) at room temperature, while for MASER those differences are small. This means searching for different population schemes to stimulate the energy level greater than that for MASER.

2. The ***spontaneous emission*** is greater than the ***stimulated emission*** (will be discussed later), they also suggest interference system (Fabry –Perot interferometer to work as a ***Resonator*** (two opposed mirrors and the active medium is inserted between them).

The first optical maser was built by Theodore Maiman in 1960 in the USA, using Ruby as an active medium producing 694.3 nm. This later called the **LASER.**

After that, other solid lasers were discovered (using Uranium ions and rare earth metal ions) such as Nd: YAG laser using flash lamps for pumping (before the end of 1960).

Ali Javan at the end of 1960 operated the First gas laser ever, the He-Ne laser. He successfully pumped it by electrical discharge producing 632.8nm wavelength.

Semiconductor laser technology started in 1962 by exciting a semiconducting material using electric field to produce optical emission at the junction region (p-n). This construction is called the DIODE. Gallium Arsenide (GaAs) used as the first semiconductor to produce a 810 nm laser, this technology depends on:

a. Impurity concentration in the active medium
b. Temperature
c. The current passing through the diode.

Later on the discovery of liquid (Dye) laser and chemical lasers and many other lasers was about 1963.

Lasing Process

To understand lasing processes, we have **three parts**:

1. Basic elements of the structure of matter **- the atom.**

2. Wave theory - especially **electromagnetic waves.**

3. The **interaction of electromagnetic radiation with matter.**

2.1 Bohr model of the atom.

Lasing action is a process that occurs in matter. Since matter is composed of atoms, we need to understand (a little) about the structure of the atom, and its energy states. We shall start with Bohr model of the atom.

According to this model, every atom is composed of a very massive nucleus with a positive electric charge (Ze); around it electrons are moving in specific paths.
Z = Number of protons in the nucleus,
e = Elementary charge of the electrons:
$e = 1.6*10^{-19}$ Coulomb
Figure 2.1 illustrates a simple, but adequate, picture of the atom, the Bohr model:

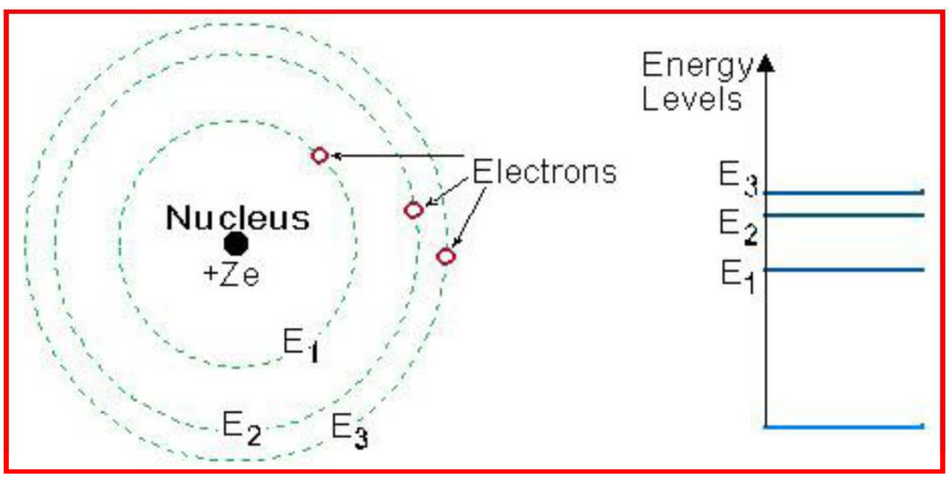


Fig 2-1: Bohr picture of the Atom •

Every "orbit" of the electron around the nucleus, is connected to a specific energy level. The energy level is higher as the distance of the "orbit" from the nucleus increases.

Since for each atom there are only certain "allowed orbits", only certain discrete energy levels exist, and are named: E_1, E_2, E_3, etc.

Energy States (Levels)

Every atom or molecule in nature has a specific structure for its energy levels.

The lowest energy level is called the ground state,

which is the naturally preferred energy state. As long as no energy is added to the atom, the electron will remain in the ground state.

When the atom receives energy (electrical energy, optical energy, or any form of energy), this energy is transferred to the electron, and raises it to a higher energy level (in our model further away from the nucleus).The atom is then considered to be in an ***excited state.***

The electron can **"jump"** from one energy level to another, while receiving or emitting specific amounts of energy.

These specific amounts of energy are equal to the difference between energy levels within the atom.

Energy transfer to and from the atom

Energy transfer to and from the atom can be performed in two different ways:

1. **Collisions with other atoms, and the transfer of kinetic energy as a result of the collision. This kinetic energy is transferred into internal energy of the atom.**
2. **Absorption and emission of electromagnetic radiation.**

Since we are now interested in the lasing process, we shall concentrate on the second mechanism of energy transfer to and from the atom (The first excitation mechanism is used in certain lasers, like Helium-Neon, as a way to put energy into the laser.

2.2 Photons and the energy diagrams

Electromagnetic radiation has, in addition to its wave nature, some aspects of "particle like behavior". **These are called "Photons"**.

The relation between the amount of energy (E) carried by the photon and its frequency (ν), is determined by the formula (first given by Einstein):

$$E = h\nu$$

The proportionality constant in this formula is Planck's constant (h):

$h = 6.626 * 10^{-34}$ Joule-sec

This formula can be expressed in different form, by using the relation between the frequency (ν) and the wavelength: $c = \lambda \nu$ to get:

$$E = h * c/\lambda.$$

This formula shows that the energy of each photon is inversely proportional to its wavelength.

This means that each photon of shorter wavelength (such as **violet** light) carries more energy than a photon of longer wavelength (such as **red** light).

- When this energy is absorbed or emitted in a form of electromagnetic radiation, the energy difference between these two energy levels (E_2-E_1) determines uniquely the frequency (ν) of the electromagnetic radiation: $(\Delta E) = E_2 - E_1 = h\nu$

Example 2.1: Visible Spectrum

The visible spectrum wavelength range is: 0.4 - 0.7 μm (400-700 nm).
The wavelength of the violet light is the shortest, and the wavelength of the red light is the longest.
Calculate:
a) What is the frequency range of the visible spectrum?
b) What is the amount of the photon's energy associated with the violet light, compared to the photon energy of the red light?

Solution to example 2.1:

The frequency of violet light:

$$\nu_1 = \frac{c}{\lambda_1} = \frac{3 \cdot 10^8 \cdot \frac{m}{sec}}{0.4 \cdot 10^{-6} \cdot m} = 7.5 \cdot 10^{14} \cdot \frac{1}{sec}$$

The frequency of red light:

$$\nu_2 = \frac{c}{\lambda_2} = \frac{3 \cdot 10^8 \cdot \frac{m}{sec}}{0.7 \cdot 10^{-6} \cdot m} = 4.3 \cdot 10^{14} \cdot \frac{1}{sec}$$

The difference in frequencies:

$$\Delta\nu = \nu_1 - \nu_2 = 7.5 \cdot 10^{14} - 4.3 \cdot 10^{14} = 3.2 \cdot 10^{14} \cdot \frac{1}{sec}$$

The energy of a violet photon:

$$E_1 = h \cdot \nu_1 = (6.626 \cdot 10^{-34} \cdot J \cdot sec) \cdot \left(7.5 \cdot 10^{14} \cdot \frac{1}{sec}\right)$$

$$E_1 = 5 \cdot 10^{-19} \cdot Joule$$

The energy of a red photon:

$$E_2 = h \cdot \nu_2 = (6.626 \cdot 10^{-34} \cdot J \cdot sec) \cdot \left(4.3 \cdot 10^{14} \cdot \frac{1}{sec}\right)$$

$$E_2 = 2.85 \cdot 10^{-19} \cdot Joule$$

The difference in energies between the violet photon and the red photon is: ***$2.15*10^{-19}$ J.***

This example shows how much more energy the violet photon have compared to the red photon.

Question 2.1:

Is it allowed to calculate first the wavelength difference (λ_2-λ_1), and then use the relation between frequency and wavelength (ν_2-ν_1)= c/(λ_2-λ_1) ?

Question 2.2:

Calculate in units of Nanometer, the wavelength of light emitted by the transition from energy level E_3 to energy level E_2 in a 3 level system in which:

E_1 = 0 eV
E_2 = 1.1 eV
E_3 = 3.5 eV

2.3 Absorption of electromagnetic Radiation

We saw that the process of photon absorption by the atom is a process of raising the atom (electron) from a lower energy level into a higher energy level (excited state), by an amount of energy which is equivalent to the energy of the absorbed photon.

When electromagnetic radiation passes through matter, part of it is transmitted, and part is absorbed by the atoms.
The intensity (I) of the transmitted radiation through a thickness (x) of homogeneous material is described by the experimental equation of exponential absorption (Lambert Law):

$$I=I_0 \exp(-\alpha x)$$

I_0 = Intensity of incoming radiation.
α = Absorption coefficient of the material.

The thicker the material (bigger x), the lower the intensity after the material (the transmitted beam).

It is common to use units of centimeter (10^{-2} m), to measure the width of the material (x), so the units of the absorption coefficient (α) are:
cm^{-1} = 1/cm.

Example 2.2: Absorption Coefficient (α)

Calculate the absorption coefficient (α) of materials which transmit 50% of the intensity of the incident radiation on a 10 mm width, to the other side. Solution to example 2.2:

Using the exponential absorption law:

$$\alpha = -1/x * \ln(I/I_0) = -1/1 * \ln(0.5/1) = 0.69\ cm^{-1}$$

2.4 Spontaneous emission of electromagnetic Radiation

One of the basic physical principles (which is the basis of a subject in physics called Thermodynamics) is that:
Every system in nature "prefers" to be in the lowest energy state.

This state is called the Ground state. As an example, we mentioned this principle in the Bohr model of the atom. When energy is applied to a system, the atoms in the material are excited, and raised to a higher energy level. (The terms "excited atoms", "excited states", and "excited electrons" are used here with no distinction).

These electrons will remain in the excited state for a certain period of time, and then will return to lower energy states while emitting energy in the exact amount of the **difference between the energy levels (delta E).**

If this package of energy is transmitted as electromagnetic energy, it is called photon. The emission of the individual photon is random, being done individually by each excited atom, with no relation to photons emitted by other atoms. When photons are randomly emitted from different atoms at different times, the process is called Spontaneous Emission.

2.5 Boltzmann Distribution Equation

From thermodynamics we know that a collection of atoms, at a temperature T [^{0}K], in thermodynamic equilibrium with its surrounding, is distributed according to Boltzmann equation which determines the relation between the population number of a specific energy level and the temperature:

$$N_i = \text{Constant} * \exp(-E_i/kT)$$

N_i = Population Number = number of atoms per unit volume at certain energy level E_i.

k = Boltzmann constant: $k = 1.38*10^{23}$ [Joule/^{0}K].

E_i = Energy of level i. We assume that $E_i > E_{i-1}$ (ex: $E_2 > E_1$).

Const = proportionality constant.

It is not important when we consider population of one level compared to the population of another level as we shall see shortly. T=Temperature in degrees Kelvin [^{0}K] (Absolute Temperature).

The Boltzmann equation shows the dependence of the population number (N_i) on the energy level (E_i) at a temperature T. From this equation we see that:

1. The higher the temperature, the higher the population number.

2. The higher the energy level, the lower the population number.

Relative Population (N_2/N_1):

The relative population (N_2/N_1) of two energy levels E_2 compared to E_1 is:

$$N_2/N_1 = const^* \exp(-E_2/kT)/ const^* \exp(-E_1/kT)$$

$$N_2/N_1 = \exp(-(E_2-E_1)/kT).$$

The proportionality constant (const) is canceled by division of the two population numbers.

Population at Thermodynamic Equilibrium

Figure 2.2 shows the population of each energy level at thermodynamic equilibrium.

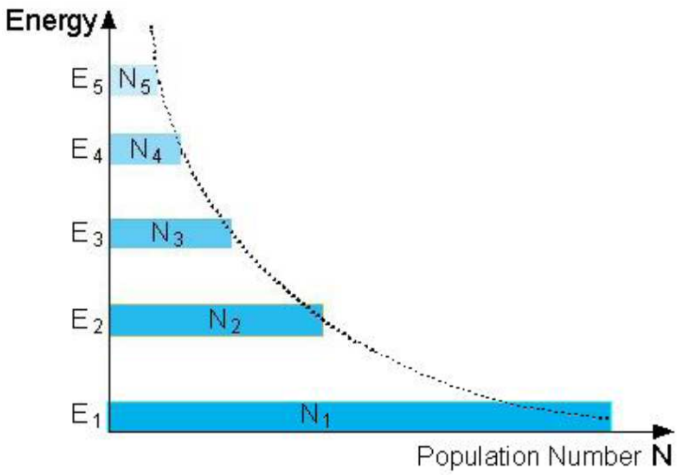


Figure 2.2: Population Numbers at "Normal Population"

Question 2.3: The difference between population numbers

Prove that the difference in population numbers (N_1, N_2) between two energy levels E_2 and E_1 is given by:

$$N_1-N_2 = N_1^*(1-\exp(-h\nu/kT).$$

$\nu = \nu_2 - \nu_1$ is the frequency which corresponds to the energy difference between the two levels E_2 and E_1.

In the equation in question 2.2, the second term inside the parenthesis is always less than 1. So, the parenthesis is always less than 1. Thus the very important conclusions:

1. In a thermodynamic equilibrium, the population number of higher energy level is always less than the population number of a lower energy level.

2. The lower the energy difference between the energy levels, the less is the difference between the population numbers of these two levels.

Physically, the electrons inside the atom prefer to be at the lowest energy level possible.

Example 2.3:

Calculate the ratio of the population inversion (N_2 / N_1) for the two energy levels E_2 and E_1 when the material is at room temperature (300^0K), and the difference between the energy levels is 0.5 eV. What is the wavelength (λ) of a photon which will be emitted in the transition from E_2 to E_1?

Solution to example 2.3:

When substituting the numbers in the equation, we get:

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{k_B \cdot T}\right) = \exp\left[-\frac{(0.5 \cdot eV) \cdot \left(1.6 \cdot 10^{-19} \cdot \frac{J}{eV}\right)}{\left(1.38 \cdot 10^{-23} \cdot \frac{J}{K}\right) \cdot (300K)}\right]$$

$= 4 * 10^{-9}$

This means that at room temperature, for every 1,000,000,000 atoms at the ground level (E_1), there are 4 atoms in the excited state (E_2).

To calculate the wavelength:

$$\lambda = \frac{h \cdot c}{\Delta E} = \frac{(6.626 \cdot 10^{-34} \cdot J \cdot sec) \cdot \left(3.10^8 \cdot \frac{m}{sec}\right)}{(0.5 \cdot eV) \cdot \left(1.6 \cdot 10^{-19} \cdot \frac{J}{eV}\right)} = 2.48 \cdot \mu m$$

This wavelength is in the Near Infra-Red (NIR) spectrum.

Question 2.4:

A material is in thermodynamic equilibrium at room temperature (300^0K).The wavelength of the photon emitted in the transition between two levels is 0.5 μm (visible radiation). Calculate the ratio of the population numbers for these energy levels.

2.6 Population Inversion

When a photon of the same energy between two levels is incidented on the sample, TWO possibilities may realized:

1. It is absorbed by an atom in the lower state; moving upward.
2. Stimulating an atom in the upper state; moving downward.

BOTH ARE POSSIBLE.

If $N_2 > N_1$ 2nd possibility (stimulated emission).

If $N_1 > N_2$ Absorption

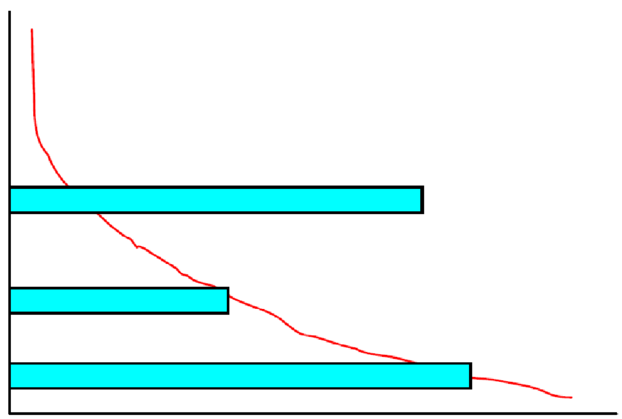
But We saw that in a thermodynamic equilibrium Boltzmann equation shows us that:

$\mathbf{N_1 > N_2 > N_3}$

Thus, the population numbers of higher energy levels are smaller than the population numbers of lower ones. This situation is called **"Normal Population"** (as described in Figure 2.3a).

By putting energy into a system of atoms, ***we can achieve a situation of "Population Inversion".***

In population inversion, at least one of the higher energy levels has more atoms than a lower energy level.



An example is described in Figure 2.3b. In this situation there are more atoms (N_3) in a higher energy level (E_3), than the number of atoms (N_2) in a lower energy level (E_2).

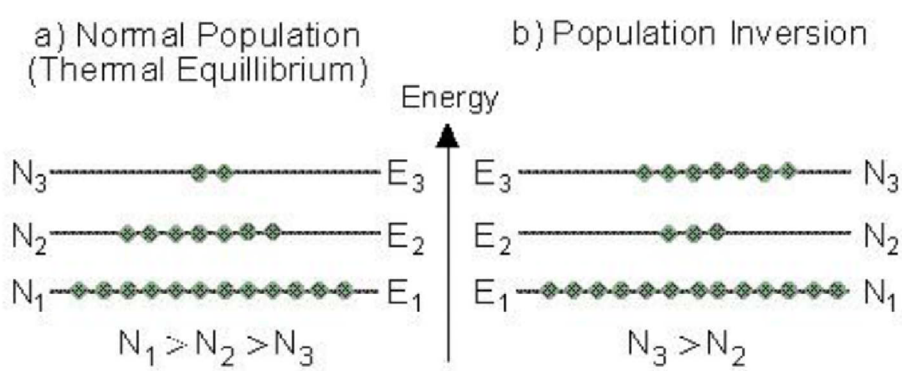


Figure 2.3: "Normal Population" compared to "Population Inversion".

This is one of the necessary conditions for lasing.
The process of raising the number of excited atoms is called **"Pumping".**
If this process is done by optical excitation (electromagnetic beam), it is called
"Optical Pumping".

2.7 Stimulated Emission

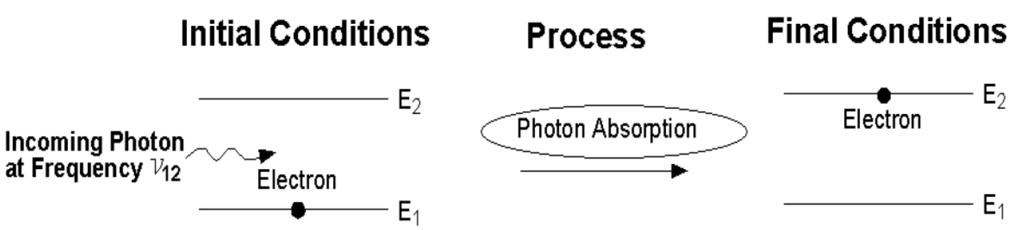
Atoms stay in an excited level only for a short time (about 10^{-8} [sec]), and then they return to a lower energy level by spontaneous emission.

If a population inversion exists between two energy levels, the probability is high that an incoming photon will stimulate an excited atom to return to a lower state, while emitting another photon of light.

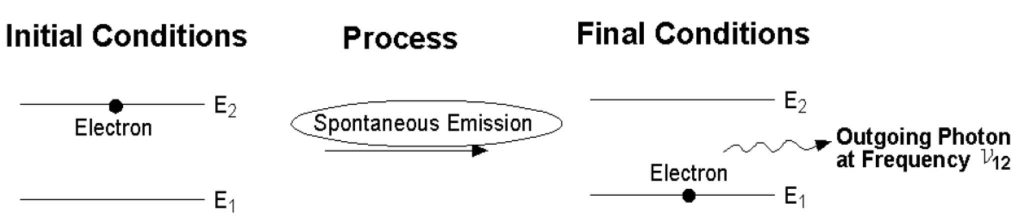
Possible Processes between Photons and Atoms

Figure 2.4 summarizes the three possible processes between photons and atoms: absorption, spontaneous emission, and stimulated emission.

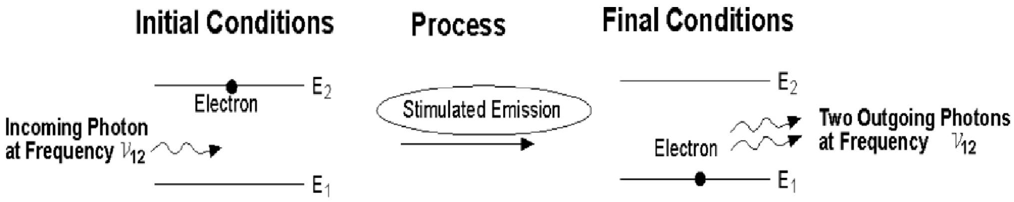
Photon Absorption: A photon with frequency ν_{12} hits an atom at rest (left), and excites it to higher energy level (E_2) while the photon is absorbed.



Spontaneous emission of a photon: An atom in an excited state (left) emits a photon with frequency ν_{12} and goes to a lower energy level (E_1).



Stimulated emission of a photon: A photon with frequency ν_{12} hit an excited atom (left), and cause emission of two photons with frequency ν_{12} while the atom goes to a lower energy level (E_1).



Now, $E_2 - E_1 = h\upsilon_{21} >>> kT$,

Hence the ratio of N_2 / N_1 is very small, i.e <u>only few atoms are in the upper level</u>.

So:

To measure kT at room temperature (T=300 K),

so at $kT=h\upsilon$,

Hence $\upsilon = 6x10^{12}$ Hz

So $\lambda = 50$ µm this is I.R

But for transition frequency in the visible and N.I.R (optical region):

So: $N_2 <<< N_1$

<u>Example:</u>

For a ruby crystal the energy gap (energy difference between transition levels) is corresponding to $\lambda = 0.69$ µm, (or $\upsilon = c/\lambda = 3x10^8/0.69x10^{-6}$)

So: $E2 - E1 = h\,\upsilon = 6.63x\ 10^{-34}\ x\ 3x10^8/0.69x10^{-6} = 2.88\ x\ 10^{-19}$ Joule

$KT= 1.38x10^{-23}$ J/k x 300 = $4.14x10^{-21}$

So: $N_2/N_1 = \exp(-\Delta E/kT)$

$= \sim 10^{-31}$

i.e For each 1 atom in the upper state there is 10^{31} atoms in the ground state !!!

Almost all atoms are in the ground state at thermal equilibrium.

So to get laser action in the sample we need a **<u>POPULATION INVERSION</u>**

Einstein Coefficients

In a two – level system, the no. of atoms in the two-level system is constant, hence:

$$N_{total} = N_1 + N_2$$

Where : N_1 and N_2 are the no. of atoms in each of the two levels.

It is either of the processes:
Absorption from E_1 to E_2
OR:
Emission from E_2 to E_1.

As we mentioned before, there are three types of interactions between the atomic system & the electromagnetic radiation:

1. **Absorption**

   The rate of depletion from ground (E_1) is proportional to the radiative density $\rho(\upsilon)$ and N_1.
   So:

   $$dN_1/dt = -B_{12}\,\rho(\upsilon)\,N_1 \quad \ldots\ldots\ldots\ldots\ldots(1)$$

   B_{12} = Proportionality constant (Einstein coefficient)

   $B_{12}\,\rho(\upsilon)$ = Probability per unit frequency that transition occur due to the field effect.

2. **Spontaneous emission**

   After absorption, the population of the upper level will be decayed spontaneously in a rate proportional to the upper level.

   $$dN_2/dt = -A_{21}\,N_2 \quad \ldots\ldots\ldots\ldots\ldots(2)$$

   A_{21} = probability of spontaneous emission.

   The solution of the differential equation (2) is :

   $$N_2(t) = N_2(0)\exp(-t/\tau_{21})$$

   Where:

   $$1/A_{21} = \text{lifetime} = \tau_{21}$$

   Lifetime = 1/transition probability

3. Stimulated emission

Due to the stimulation of the appropriate E.M radiation, the atom gives an energy to the radiative field as follows:

$$dN_2/dt = - B_{21}\, \rho(\upsilon)\, N_2 \quad \ldots\ldots\ldots\ldots\ldots..(3)$$

The essential factor for lasing action is $\mathbf{B_{21}}$**,** while $\mathbf{A_{21}}$ represents the amount of losses.

Adding together the three phenomena, get:

Since $N_1 + N_2 = N_{total}$

$dN_1/dt + dN_2/dt = 0$ (in thermal equilibrium)

$dN_1/dt = - dN_2/dt$

$0 = B_{21}\, \rho(\upsilon)\, N_2 - B_{12}\, \rho(\upsilon)\, N_1 + A_{21}\, N_2$

$A_{21}\, N_2 + B_{21}\, \rho(\upsilon)\, N_2 = B_{12}\, \rho(\upsilon)\, N_1$

Spon.em. Stim.em. Absorption

Using Boltzmann relation for $\mathbf{N_2/N_1}$**,** get:

$$\rho(\upsilon) = \frac{A_{21}/B_{21}}{B_{12}/B_{21}\ \exp(h\upsilon/kT) - 1} \quad \ldots\ldots\ldots\ldots..(4)$$

Comparing with the Black body radiation relation, which is:

The energy density(energy per unit volume per unit frequency):

$$\rho(\upsilon) = 8\pi h\, \upsilon^3/c^3\ \frac{1}{\exp(h\upsilon/kT) - 1} \quad \mathrm{J.s/m^3} \quad \ldots\ldots\ldots\ldots..(5)$$

Comparing with equation (4) & (5), get:

$$A_{21}/B_{21} = 8\pi h\, \upsilon^3/c^3 \quad \ldots\ldots\ldots\ldots.(6)$$

And

$$B_{12} = B_{21} \quad \ldots\ldots\ldots\ldots\ldots\ldots.(7)$$

Conclusion:

From eq(7), Einstein coeff. For stimulating emission and absorption are EQUAL i.e Probability of absorption is equivalent to probability of stimulated emission.

Now consider (R) ,as the **ratio of spontaneous to stimulated emission probabilities**, hence for a two level system of energy levels, we have:

$R = A_{21} N_2 / B_{21} \rho(\upsilon) N_2$

Substituting for A_{21}/B_{21} from eq(6) , and $\rho(\upsilon)$ from eq(5), get:

$$R = \exp(h\upsilon/kT) - 1 \quad \ldots\ldots\ldots.(8)$$

Example.1:

For a tungsten lamp that works at temp. ~ ***2000k*** and emission frequency $5x10^{14}$ Hz, Find R.

$R \sim 1.5x10^5$

At room temperature $R \sim 4x10^{34}$

Hence spontaneous emission is **PREDOMINENT.**

Example.2

Find R at 50µm and 2000k

R = 0.15

If $\lambda = 60$ µm & room temperature:

R = 1

Hence all examples are for systems in thermal equilibrium, stimulated emission needs population inversion (i.e To destroy the thermal equilibrium).

The Optical Resonator

In most cases the gain of a pumped medium is very small ($0.1 m^{-1}$), so the amplification of the optical beam passing once is minimal, but the amplification is increased by placing highly reflecting mirrors at each end of the medium, this is forming the RESONATOR

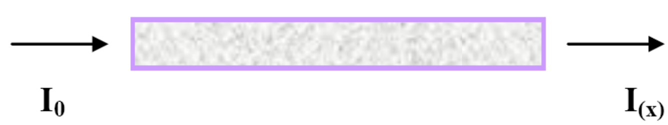
Resonator + A.M = Oscillator

The gain within the medium must be large enough to overcome the losses in the laser system.

The Gain coefficient

Assume a monochromatic light of intensity (I_0),passing through a transparent medium, so the intensity out of the medium is ($I_{(x)}$):

$$I_{(x)} = I_0 \, e^{-\alpha x}$$



α = Absorption coefficient
α – depends on the no. of atoms (N_1) in (E_1) and (N_2) in (E_2)

Now, in thermal equilibrium, $N_1 >>> N_2$, So $I_{(x)}$ decreases exponentially.

But if $N_2 > N_1$ (Population inversion) , so α is negative, hence – α is positive,
So the intensity will increase exponentially as follows:

$$I_{(x)} = I_0 \, e^{kx}$$

Where **k** is called the **Gain coefficient**.

A relation between k and the population inversion and some other laser medium parameters can be established:

$$k = (N_2 - N_1) \frac{nh\upsilon B_{21}}{c\Delta\upsilon}$$

<u>Thershold gain coefficient</u> (k_{th})

Losses in the laser system are due to:

1. **Transmission at mirrors.(out put).**
2. **Absorption and scattering by mirrors.**
3. **Diffraction around the mirror edges.**
4. **Absorption in laser medium (due to unwanted transitions).**
5. **Scattering at optical in homogeneities in laser medium.**

Losses are represented by (γ) the loss coefficient, which reduces the gain coefficient to be (k- γ).

The round trip gain (G) = 1 (This is threshold gain condition)

If G < 1 oscillation will decay

If G < 1 oscillation will grow

Assume active medium of length (L) between two mirrors of reflectivities (R_1 & R_2),

Hence the intensity will increase on passing from M_1 to M_2, where :

$I_1 = I_0\, e^{(k-\gamma)\, L}$

After reflecting from (R_2):

$I_1 = I_0\, R_2\, e^{(k-\gamma)\, L}$

After a round trip:

$I_1 = I_0\, R_1\, R_2\, e^{2(k-\gamma)\, L}$

So , the round trip gain (G):

G = Final intensity/ initial intensity

$= I_0\, R_1\, R_2\, e^{2(k-\gamma)\, L} / I_0$

$= R_1\, R_2\, e^{2(k-\gamma)\, L}$

Threshold condition for laser oscillation :

$R_1\, R_2\, e^{2(k-\gamma)\, L} = 1$

Hence:

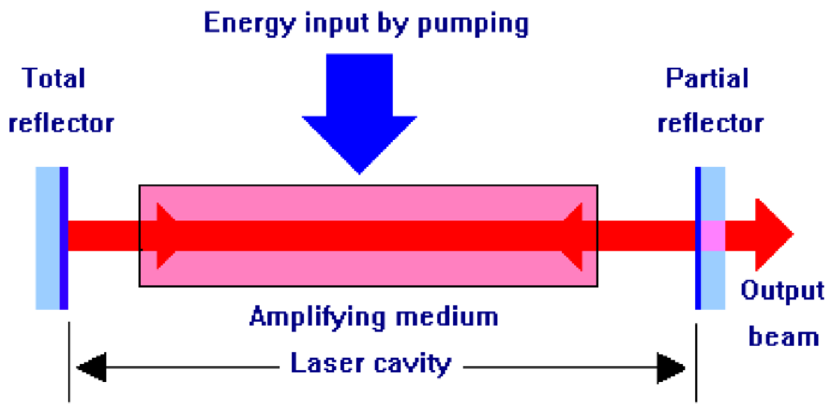
$$k_{th} = \gamma + (1/2L)\, \ln(1/R_1R_2)$$

BASIC COMPONENTS OF A LASER SYSTEM

1. Pumping : (To produce population inversion).

2. The Active medium :

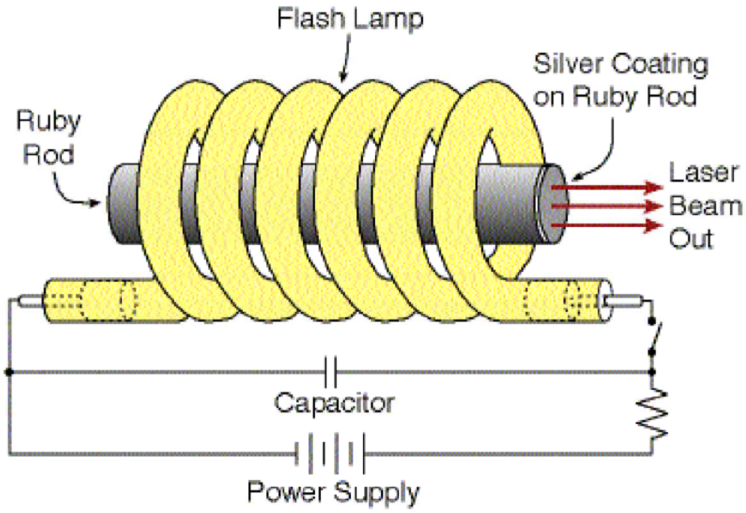
3. The Resonator : (For optical feedback).



Pumping process:

Types of pumping:

1. ***Optical pumping:*** High power light source to excite the active medium, where the A.M absorbes the energy that helps atoms, molecules or ions to move to the upper level, this is common in solid state lasers (Ruby, Nd:YAG,…….etc.) or Dye lasers.



Optical pumping can be implemented by :

a) Using Special lamps:

1. Pulesd laser – Xe-lamps
Or Kr-lamps (pressure value of 450 up to 1500 Torr).

2. C.W laser - Kr-lamps (pressure value of 4000 -8000 Torr).
Or Tungsten –Iodine lamp.

b) Another laser for pumping – as in Dye lasers, where Ar^+

2. Electrical pumping :

Used in :
Gas laser – electrical discharge
Semiconductor – potential difference.

Excitation in the laser gas:

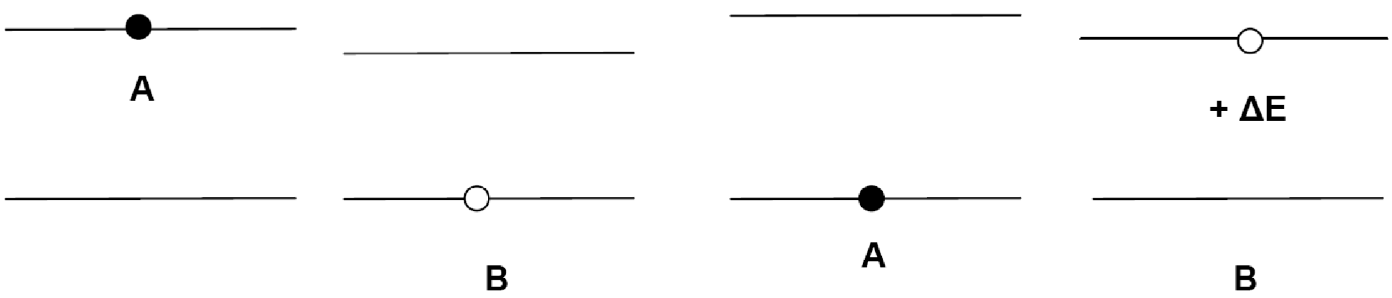
For gas laser in order to excite the gas, high voltage is needed in order to get a discharge in the gas medium, so generating ions and/or fast electrons gaining extra energy from theelectric field while colliding with gas atoms causing excitation.

Electrical pumping can be achieved by one of the following methods:

A) **e (fast) +X (atom in the ground state) = X^* (atom in the excited state) +e (Slow)**

B) A gas comprising of TWO species (A & B)

$$A^* + B = A + B^* \pm (\Delta E)$$



Before collision **After collision**

3. Chemical pumping

Does not need external source of energy, where the out put of the chemical reaction represents the active medium and the reaction generated energy can be used to excite the active medium and getting the population inversion.

Ex: Hydrogen fluoride laser

$$F + H_2 \longrightarrow HF^* + H$$

$$H + F_2 \longrightarrow HF^* + F$$

HF^* ↓ Active Medium

The Active medium (A.M)

a) Solid – Crystal ………… (Ex. Ruby, Nd:YAG, ….)

_ Glass ………….. (Ex. Nd:glass)

b) Dye _ Organic material dissolved in certain solvent.

c) Gas _ Atomic (He-Ne,…..)

_ Molecular (Co_2, N_2, …)

_ Ions (Ar^+)

OPTICAL RESONATORS

Formed usually by placing mirrors (plane or curved) to the axis along laser light propagation.

Resonator Configurations.

(i) Plane-Parallel resonator (Fabry-Perot):

- Two plane mirrors set parallel to one another.

- As approximation, the modes of this resonator can be thought as a superposition of two plane e.m works propagating in opposite directions along the cavity axis.

- Within this approximation, the resonant frequencies can be reading obtained by imposing the condition that the cavity length (L) must be an integral number of $\frac{1}{2}\lambda$, i.e

$$L = n\left(\frac{\lambda}{2}\right) \qquad \dots(1)$$

n≡ positive integer.

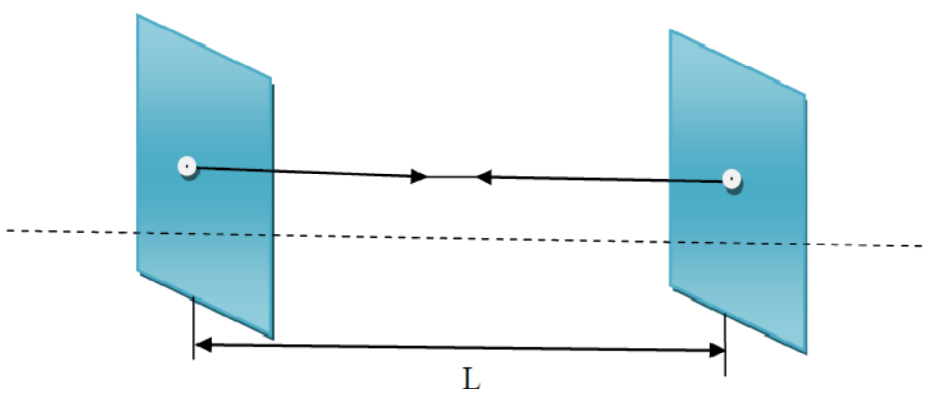


Figure (1): Resonator.

- This is a necessary condition for the electric field of an e.m standing wave to be zero on the two mirrors.

υ The resonant frequency:

$$\upsilon = \frac{nc}{2L} \qquad \dots(2)$$

Advantages:

- Optimal use of all the volume of the active medium. Thus, used in pulsed lasers which need the maximum energy.
- No focusing of the laser radiation inside the optical cavity. In high power lasers such

- Disadvantages:

- High diffraction losses.
- Very high sensitivity to misalignment. Thus, very difficult to operate.

(ii)Concentric (or Spherical) Resonator:

- Two spherical mirrors having the same radius (R) and separated by a distance (L) such that the mirror centre's of curvature C_1 & C_2 are coincident (i.e: L=2R).

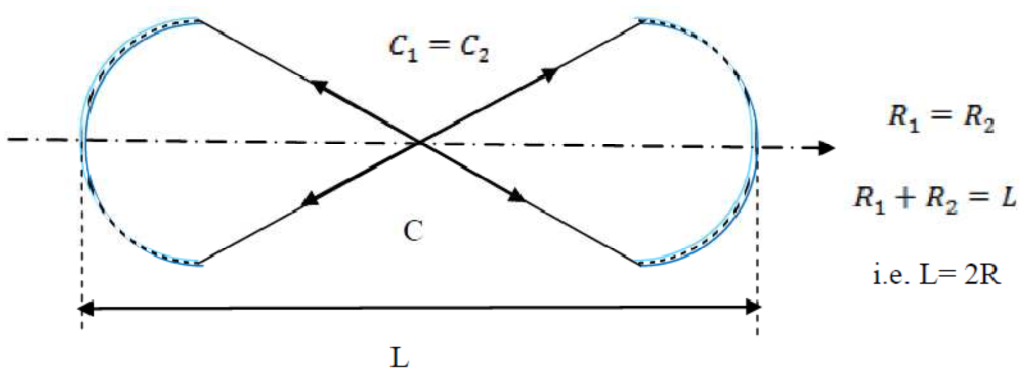


Figure (2): Concentric (spherical) resonator.

- The modes are approximated by a superposition of two oppositely spherical waves originating from point (C).

Advantages:

- Very low sensitivity to misalignment. Thus, very easy to align.
- Low diffraction losses.

Disadvantages:

- Limited use of the volume of the active medium. Used in optical pumping of continuous Dye lasers. In these lasers the liquid dye is flowing in the region of the beam focusing (The flow direction is perpendicular to the optical axis of the laser). Thus very high power density is used to pump the dye.
- Maximum focusing of the laser radiation inside the optical cavity. Such focusing can cause electric breakdown, or damage to the optical elements.

(iii) Confocal Resonator

- Two spherical mirrors of the same (R) and separated by (L) such that the foci F_1 & F_2 are coincident.

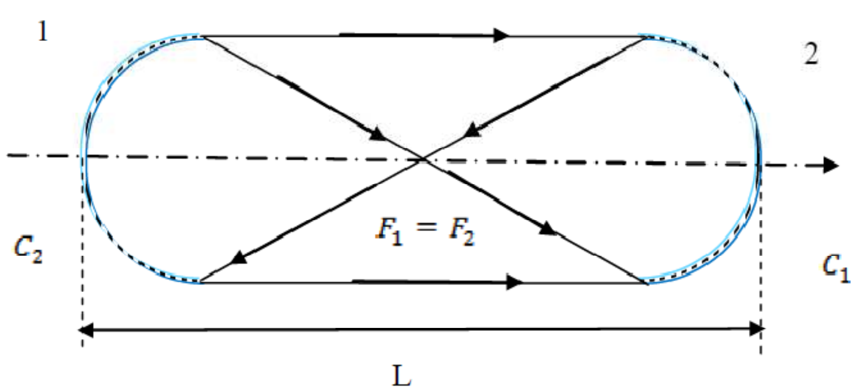


Figure (3): Confocal resonator.

- C of one mirror lies on the surface of the second mirror (L=R).

Advantages:

- Little sensitivity to misalignment. Thus, easy to align.
- Low diffraction losses.
- No high focusing inside the cavity.

The main difference between the Confocal cavity and the spherical cavity is that in the Confocal cavity the focal point of each mirror is at the center of the cavity, while in spherical cavity the center of curvature of the mirrors is in the center of the cavity.

Resonators using a combination of plane & spherical mirrors

- Called hemispherical resonators & hemiconfocal.

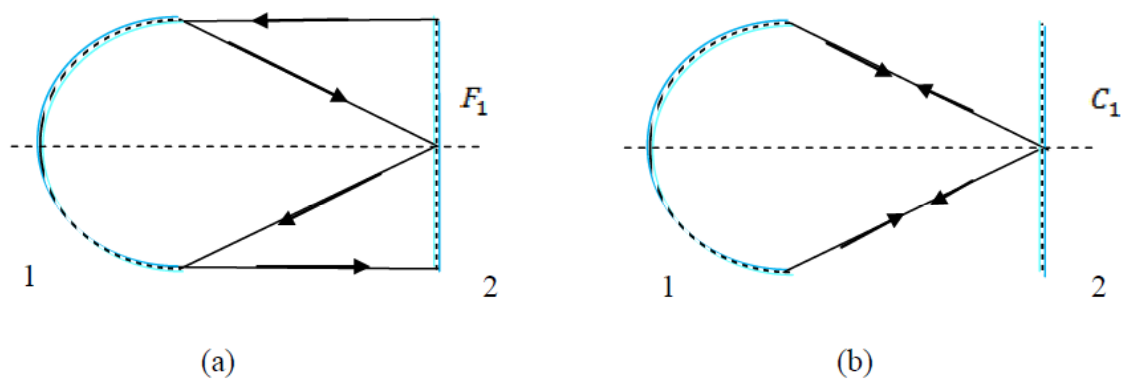


Figure (4): (a) Hemiconfocal resonator, (b) Hemispherical resonator.

- Also we can have spherical resonators of R < L < 2R (intermediate between confocal & concentric) & also L < R.

- All these resonators are particular examples of a general resonator of two spherical mirrors of different (R_1) & (R_2) (either +ve or –ve) and spaced by arbitrary (L).

Various resonators can be categorized as either:

1-Stable resonator:
Most lasers, in which the curvature of the mirrors keeps the light concentric near the resonator axis.

2-Unstable resonator:

Light ray keep on moving away from resonator axis.

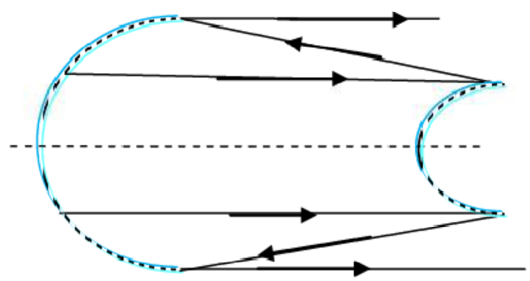
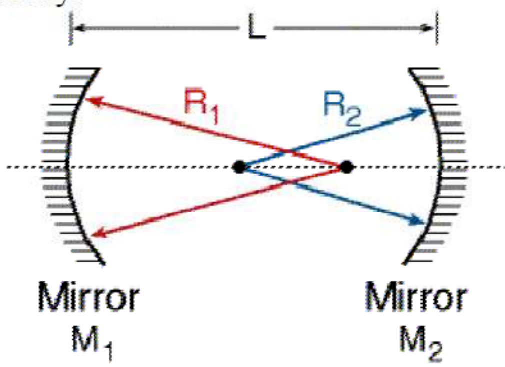


Figure (5): Un stable resonator.

- Unstable resonator have large losses, they can always be used with high gain media (i.e. Co_2).

The stability condition

- For an arbitrary mirror separation (L) and arbitrary curvatures (R_1) & (R_2), stability condition is satisfied by a low-loss resonator configuration, the g-parameters are defined by:



$$g_1 = 1 - \frac{L}{R_1} \qquad , g_2 = 1 - \frac{L}{R_2}$$

g_1 & g_2 : dimensionless parameters.

∴ Stability condition is:

$$0 < g_1 g_2 < 1 \qquad ...(3)$$

If $g_1 g_2 < 0$ *or* $g_1 g_2 > 1$ $\Rightarrow$ **unstable resonator**

When $g_1 g_2 = 0,1$ then the laser is on the boundary between stability & instability the called **marginally stable.**

Lines AB making an angle of 45° with g_1& g_2 axes corresponds to resonators having mirrors of the same $R_{s'}$ (symmetric resonator).

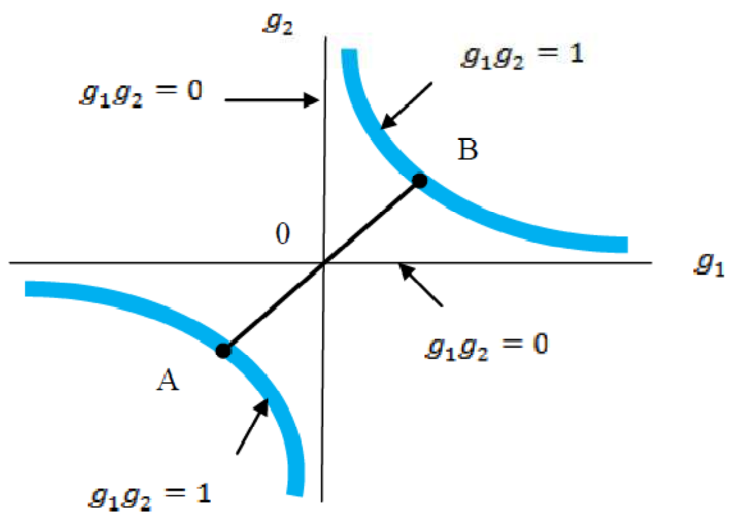


Figure (7): Symmetric resonators.

Example

A: concentric, o: confocal, B: plane resonator.
-The most commonly used laser resonator make use of either two concave mirrors of large radius of curvature or plane mirror & a concave of large radius.

Example 4.4: Unstable Resonator

The laser cavity length is 1 m. At one end a concave mirror with radius of curvature of 1.5 m. At the other end a convex mirror with radius of curvature of 10 cm. Find if this cavity is stable.

Solution to Example 4.4:

R_1 = 1.5 m.
As common in optics, a convex mirror is marked with minus sign:
R_2 = - 0.1 m
$g_1 = 1-L/R_1 = 1-1/1.5 = 0.333.$
$g_2 = 1-L/R_2 = 1+1/0.1 = 11$

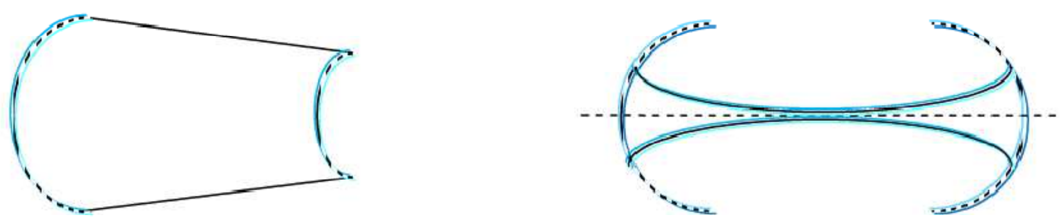
The product:

$g_1 * g_2 = 11*0.333 > 1$

The product is greater than 1, so the cavity is unstable.

Unstable resonators have several attractive features:

1- Highly efficient to utilization of A.M, even with short resonator.



(a) Confocal unstable resonator. (b) Large radius stable resonator.

Figure (8): Stable and unstable resonators

2- Their stability for adjustable output coupling, output coupler can be adjusted over a wide range of values.

Can be used only with A.M of high signal gain Co_2 *or* Co (since large losse

Properties of Laser Radiation

"Ordinary light" (from the sun or lamps) is composed of many different wavelengths, radiating in all directions, and there is ***no phase*** relation between the different waves out of the source.

Laser radiation is characterized by certain properties which are not present in other electromagnetic radiation:

1.2.1 Monochromaticity.

1.2.2 Directionality.

1.2.3 Coherence.

1.2.4 Brightness

1.2.1 Monochromaticity

Monochromaticity means **"One color"**. To understand this term, examine "white light" which is the color interpreted in the mind when we see all colors together. When "white light" is transmitted through a prism, it is divided into the different colors which are in it, as seen in figure.1:

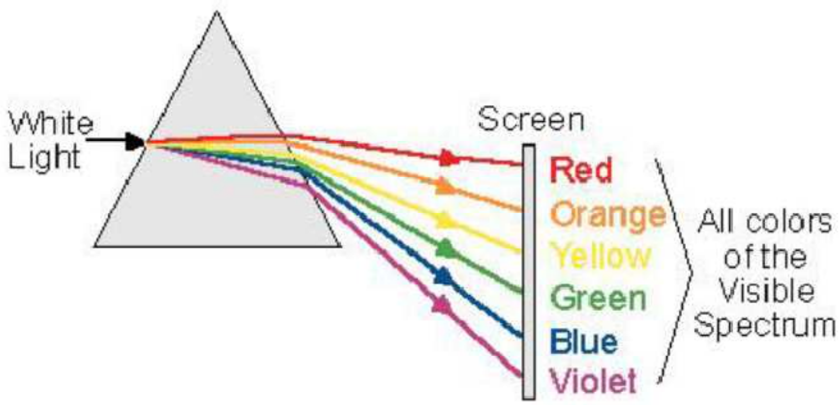


Figure 1 White light passing through a prism

The Meaning of "One Color"

In the theoretical sense "One Color", which is called "spectral line" means one wavelength (λ_0). A graph of light intensity versus wavelength for ideal "one color" is shown on the right side of figure.2. . In reality, every spectral line has a finite spectral width ($\Delta\lambda$) around its central wavelength (λ_0), as can be seen in the left side of figure .2 .

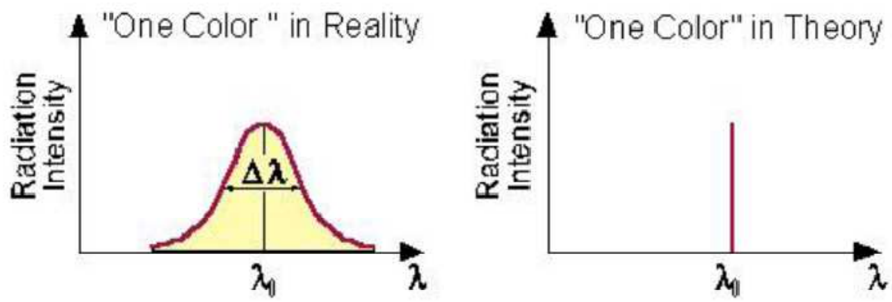


Figure .2: Bandwidth of laser radiation in Theory and in Reality

1.2.2. Directionality

Radiation comes out of the laser in a certain direction, and spreads at a defined divergence angle (θ) (see figure .3 and example 1.2).This angular spreading of a laser beam is very small compared to other sources of electromagnetic radiation, and described by a small divergence angle (of the order of ***milli-radians***).

In figure.3, a comparison is made between the radiation out of a laser, and the radiation out of a standard lamp.

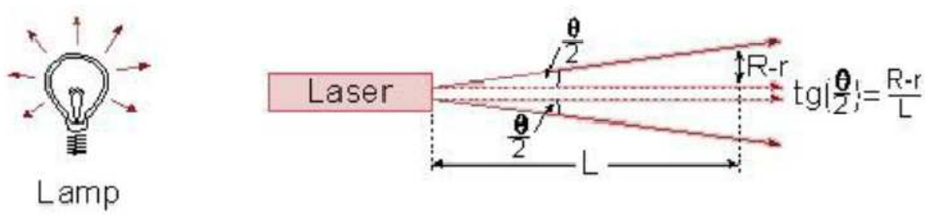


Figure .3: comparison between the light out of a laser, and the light out of an incandescent lamp

Divergence Angle

Divergence Angle is the full angle of opening of the beam. (Some books use half of this angle as divergence angle). The relation between radians and degrees is given by:

$360^0 = 2\pi$ Radians

1 Radian = 57.3^0,

1 milli-Radian = 1 mrad = 0.057^0.

Using the relation between minutes and degrees: 1^0=60', we get:

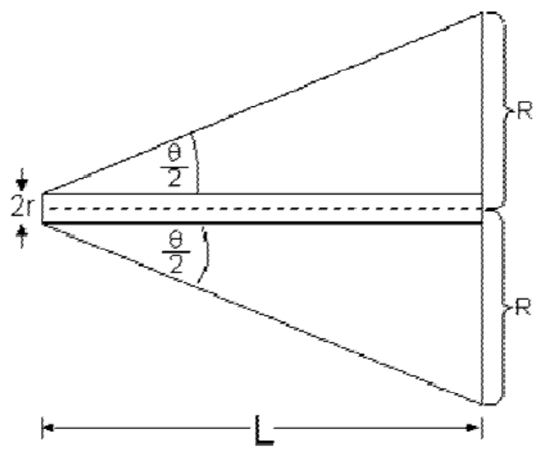
1 mrad = 0.057*60'≅ 3.5'

Since laser radiation divergence is of the order of milli-radians, the beam is **almost parallel**, and laser radiation can be send over long distances. We shall see later, how a laser beam was sent to the moon, and returned to Earth to measure the distance between Earth and the moon with accuracy of tens of centimeters.

Spot Size Measurement:

R = Radius of the illuminated spot at a distance L from the laser (see figure below).If the spot size measurement is done near the laser (where the spot is small), then the size of the beam at the output of the laser needs to be taken into account:

$$\tan\left(\frac{\Theta}{2}\right) = \frac{R-r}{L} \approx \frac{\Theta}{2}$$



Because the laser radiation has a very small divergence, the small angle approximation can be used.

Thus, we have set the tangent of the angle equal to the angle.

On a screen, the laser produces a spot. The diameter of this spot (2R) determines the spot size.

When the measurement is done very far from the laser, the spot size (2R) is big compared to the beam size at the laser output (2r), and it is accurate enough to measure the spot diameter and divide it by the distance, to find the beam divergence.

Example:

A laser with beam divergence of 1 milli-radian creates a spot of about 10 mm at a distance of 10 m. The laser power measured over a defined unit surface area is called Power Density. Looking at figure.3, it is clear that from a laser it is possible to achieve higher power density than from conventional sources (see example 1.2).

<u>This is the reason why a 5 [mW] laser radiation is considered dangerous, and the light out of a 100 W incandescent lamp is not!!!</u>

<u>Example 1.2: Numerical Calculation of Power Density</u>

Calculate the power density of radiation per unit area at a distance of 2 meters, from an incandescent lamp rated 100 [W], compared to a Helium-Neon laser of 1 mW. The laser beam diameter at the laser output is 2 mm, and its divergence is 1 mrad.

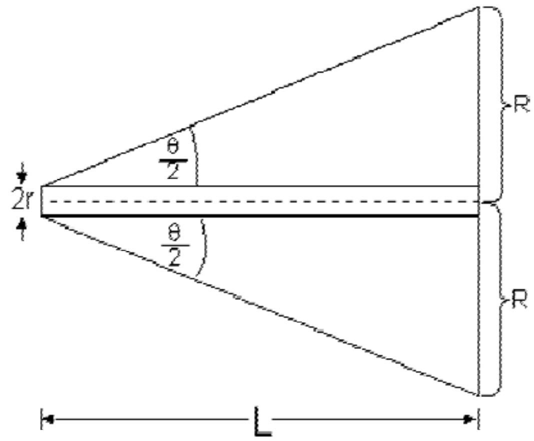
<u>Solution to example 1.2:</u>

Light from incandescent lamp is radiated to all directions, so it is distributed on a surface of a sphere with a radius of 2 m. The surface area is: πR^2, so the power density at a distance of 2 m is:

$$\frac{100 \cdot \mathbf{W}}{\mathbf{pi} \cdot 200^2 \cdot \mathbf{cm}^2} = 0.2 \cdot \frac{\mathbf{mW}}{\mathbf{cm}^2}$$

Compared to the incandescent lamp, the laser beam diameter at a distance of 2 [m] increased to 4 [mm] (see drawing below):

$$\tan\left(\frac{\Theta}{2}\right) = \frac{R - r}{L} \approx \frac{\Theta}{2}$$



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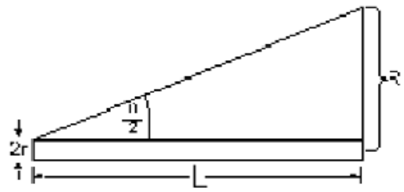

$$\tan\left(\frac{\Theta}{2}\right) = \frac{R - r}{L} = \frac{\Theta}{2} \quad \text{(in radians)}$$

$$R = r + L \cdot \tan\left(\frac{\Theta}{2}\right) = 1 \cdot \mathrm{mm} + 2000 \cdot \mathrm{tg}(0.5 \cdot \mathrm{mrad})$$

R = 2.1 mm = 0.2 cm

The power density of the laser radiation is:

$$\frac{1 \cdot \mathbf{mW}}{\mathbf{pi} \cdot 0.2^2 \cdot \mathbf{cm}^2} = 8 \cdot \frac{\mathbf{mW}}{\mathbf{cm}^2}$$

When calculating radiation power in the visible spectrum (used for illumination) the low efficiency of the incandescent lamp must be considered (A 100 W lamp emits only 1-3 W of visible radiation and all the rest is in the infrared spectrum).

At a distance of 2 m from the radiation source, the power density of the laser radiation is 4 times higher than from the lamp, although the power from the lamp is 5 times the original power of the laser.

1.2.3 Coherence

Since electromagnetic radiation is a wave phenomenon, every electromagnetic wave can be described as a sum (superposition) of sine waves as a function of time.

From wave theory we know that every wave is described by a wave function:

$y = A\cos(\omega t+\varphi)$
A=□ Amplitude.
$\omega = 2\pi\nu$ □ =□ Angular Frequency.
φ = Initial Phase of the wave (Describe the starting point in time of the oscillation).
$(\omega t+\varphi)$ = Phase of the wave.

Superposition of Waves

Coherent waves are waves that maintain the relative phase between them. Figure.4 describes, using the same time base, 3 waves marked y_1, y_2, y_3, and their superposition. In figure.4a, the waves are coherent, like the waves out of a laser.

In figure.4b, the waves have the same wavelength, but are not coherent with each other. Light from an incandescent lamp is composed of waves at many wavelengths, and each wave appears randomly with no systematic relation between its phase and that of the other wave.

Laser radiation is composed of waves at the same wavelength, which start at the same time and keep their relative phase as they advance. By adding (superposition) the wave amplitudes of the different waves, higher peaks are measured for laser radiation.

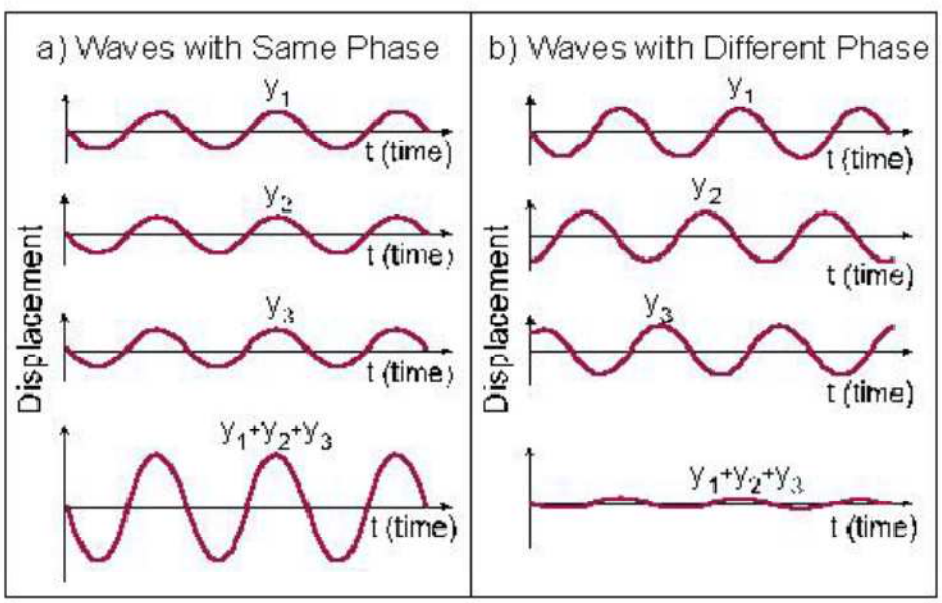

Figure.4: Superposition of waves