Physical Optics 2018/ *Dr. Muwafaq Fadhil Al-Mishlab*

9th lecture [Diffraction Grating]

1. Introduction

A periodic, multiple-slit device designed to take advantages of the sensitivity of its diffraction pattern to the wavelength of the incident light is called a *diffraction grating*. In figure (1), the net path difference for waves from successive slits is

$$\Delta = \Delta_1 + \Delta_2 = a\sin\theta_i + a\sin\theta_m \quad \text{-------------------- (1)}$$

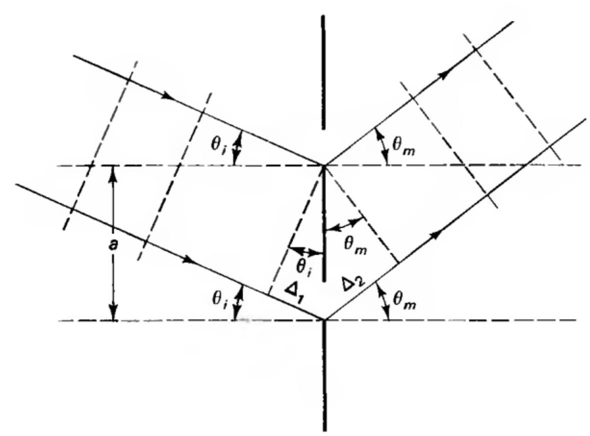

Fig. 1: Neighboring grating slits illuminated by light incident at angle θ with the grating normal.

The two sine terms in the path difference may add or subtract, depending on the direction θ_m of the diffracted light. When the incident and diffracted rays are on the same side of the normal, as they are in Fig. 1, θ_m is considered positive. When the diffracted rays are on the side of the normal opposite to that of the incident rays, θ_m is considered negative. In the latter case, the net path difference for waves from successive slits is the difference $\Delta_1 - \Delta_2$. In either case, when $\Delta = m\lambda$, all diffracted waves are in phase and the grating equation becomes:

$$a\,(\sin\theta_i + \sin\theta_m) = m\lambda, \qquad m = 0, \pm1, \pm2, \ldots.. \quad \text{---------------- (2)}$$

When it is not necessary to distinguish angles, the subscript on the angle of diffraction θ_m is dropped. For each value of m, monochromatic radiation of wavelength λ is enhanced by the diffractive properties of the grating. By Eq. 2, the zeroth order m = 0, occurs at $\theta_m = -\theta_i$, the direction of the incident light, for all λ. Thus, light of all wavelengths appears in the central or zeroth order peak of the diffraction pattern. Higher orders – both plus and minus- produce *spectral lines* appearing on either side of the zeroth order. For a fixed direction of incidence given by θ_i, the direction θ_m of each principle maximum varies with wavelength. As a dispersing element, the grating is superior to a prism in several ways. Figure (2a) illustrates the formation of the spectral orders of diffraction for monochromatic light. Figure (2b) shows the angular spread of the continuous spectrum of visible light for a particular grating. Not that second and third orders in this case partially overlap. Unlike the prism, a grating produces greater deviation from the zeroth-order point for longer wavelengths.

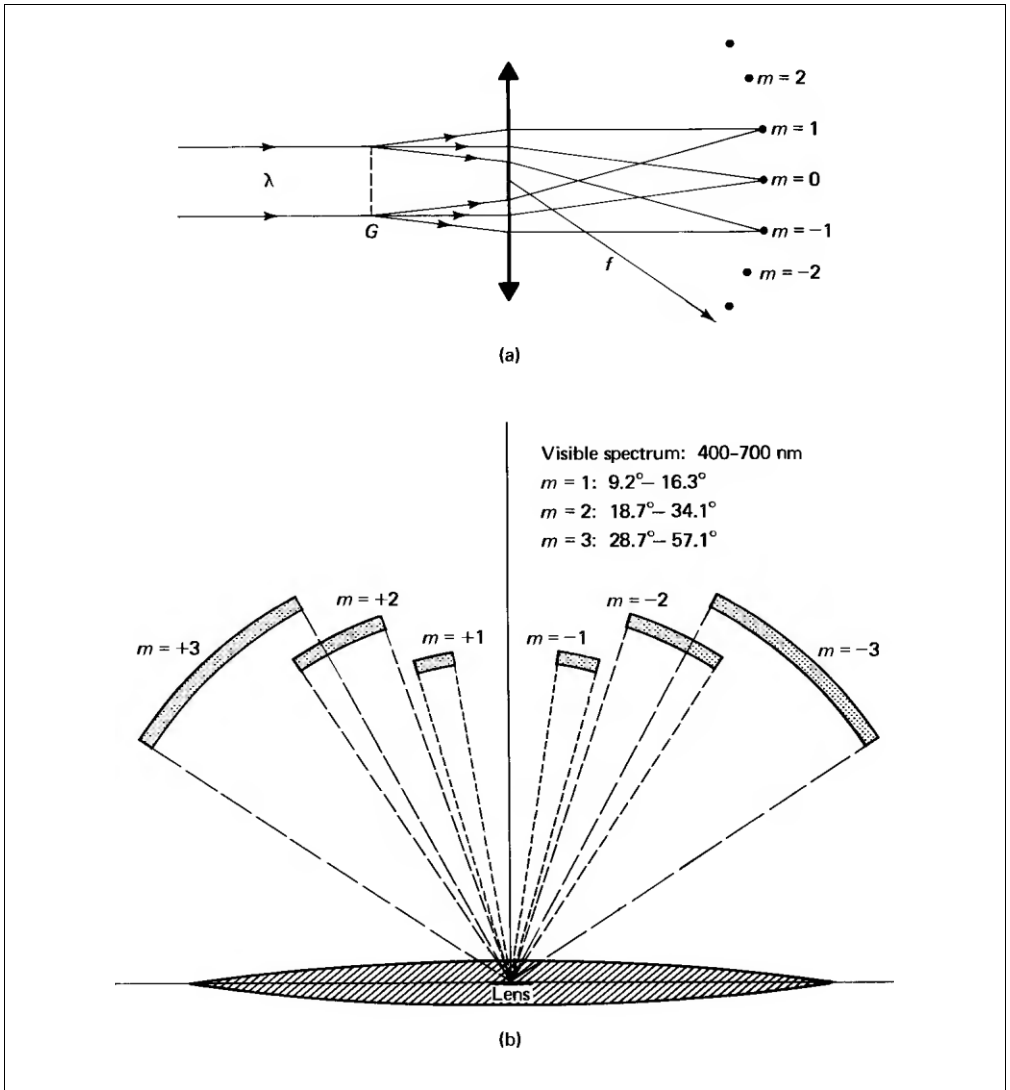

Fig. 1: (a) Formation of the orders of principle maxima for monochromatic light incident normally on grating G. (b) Angular spread of the first three orders.

2. Free Spectral Range of a Grating

The nonoverlapping wavelength range in a particular order is called the *free spectral range, F.* overlapping occurs because in the grating equation, the product (a sin θ) may be equal to several possible combinations of $m\lambda$ for the light actually incident and processed by the optical system. Thus, at the position corresponding to λ in the first order, we may also find a spectral line corresponding to λ/2 in the second order, λ/3 in the third order, and so on. The free spectral range in order *m* may be determined by the following argument.

If λ_1 is the shortest detectable wavelength in the incident light, then the longest nonoverlapping wavelength λ_2 in order *m* is coincident with the beginning of the spectrum again in the next higher order, or

$$m\lambda_2 = (m+1)\,\lambda_1$$

The free spectral range for order *m* is then given by

$$F = \lambda_2 - \lambda_1 = \frac{\lambda_1}{m} \text{ ------------------- (3)}$$

Example

The shortest wavelength of light present in a given source is 400 nm. Determine the free spectral range in the first three orders of grating diffraction.

Solution

$F = \frac{\lambda_1}{m}$ thus

$$F_1 = \frac{400}{1} = 400\ nm\ (from\ 400\ to\ 800\ nm\ in\ first\ order)$$

$$F_2 = \frac{400}{2} = 200\ nm\ (from\ 400\ to\ 600\ nm\ in\ second\ order)$$

$$F_3 = \frac{400}{3} = 133\ nm\ (from\ 400\ to\ 533\ nm\ in\ third\ order)$$

3. Dispersion of a Grating

Figure (2b) shows clearly that wavelengths are better separated as their order increases. This property is precisely described by the *angular dispersion* $D = \frac{d\theta_m}{d\lambda} = \frac{m}{a\cos\theta_m}$ ------------------- (4)

The equation shows in the first place that for a given small wavelength difference $d\lambda$, the angular separation $d\theta_m$ is directly proportional to the order *m*. Hence the second. order spectrum is twice as wide as the first order, the third three times as wide as the first, etc. In the second place, $d\theta_m$ is inversely proportional to the slit separation *a*, which is usually referred to as the *grating space*. The smaller the grating space, the more widely spread the spectra will be.

If a photographic plate is used in the focal of the lens record the spectrum, it is convenient to describe the spread of wavelengths on the plate in terms of a *linear dispersion* $\frac{dy}{d\lambda}$, where *y* is measured along the plate. Since $dy = f\ d\theta$, the linear dispersion is given by

$$linear\ dispersion = \frac{dy}{d\lambda} = f\ \frac{d\theta_m}{d\lambda} = f\ D \text{ --------------- (5)}$$

Example

Light of wavelength 500 nm is incident normally on a grating with 5000 grooves/cm. determine its angular and linear dispersion in the first order when used with a lens of focal length 0.5 m.

Solution

The grating constant or groove separation *a* is

$$a = \frac{1}{5000\ cm^{-1}} = 2 \times 10^{-4}\ cm$$

For zeroth order, there is no dispersion. For first order, Eq. 4 requires the diffraction angle θ_1, which can be found the grating equation (2)

$$\sin\theta_1 = \frac{(1)\lambda}{a} = \frac{500 \times 10^{-7}}{2 \times 10^{-4}} = 0.25$$

Thus, $\theta_1 = 14.5^0$ and $\cos\theta_1 = 0.968$.

The angular dispersion in the wavelength region around 500 nm can now be calculated:

$$D = \frac{m}{a\cos\theta_m} = \frac{1}{(2 \times 10^{-4}\ cm)(0.968)} = 5164\frac{rad}{cm} = 5.164 \times 10^{-4}\ \frac{rad}{nm} \times \frac{180^0}{\pi\ rad} = 0.0296^0/nm$$

The linear dispersion is then found from

$$fD = (500mm)\left(5.164 \times 10^{-4}\ {}^{rad}/_{nm}\right) = 0.258\frac{mm}{nm}$$

4. Resolution of a Grating

By the resolution of grating, we mean its ability to produce distinct peaks for closely spaced wavelengths in a particular order. Recall that the *resolving power (R)* is define in general by

$$R = \frac{\lambda}{(\Delta\lambda)_{min}} \text{ -------------------- (6)}$$

Where $(\Delta\lambda)_{min}$ is the minimum wavelength interval of two spectral components that are just resolved by Rayleigh's criterion (it was decided by Rayleigh to arbitrarily fix the separation a: = 01 = Alb as the criterion for resolution of two diffraction patterns. This quite arbitrary choice is known as Rayleigh's criterion. The angle θ_1 is sometimes called the resolving power of the aperture *b*, although the ability to resolve increases as θ_1 becomes smaller. A more appropriate designation for θ_1 is the minimum angle of resolution. For normally incident light of wavelength $\lambda + \Delta\lambda$, and principle maximum of order m, we have by the grating Eq. 2,

$$a\sin\theta = m\,(\lambda + d\lambda) \text{ ------------------------- (7)}$$

To satisfy Rayleigh's criterion this peak must coincide (same θ) with the first minimum of neighboring wavelength's peak in the same order, or

$$a\sin\theta = \left(m + \frac{1}{N}\right)\lambda \text{ ----------------------- (8)}$$

From Eqs. 7 and 8, Hence we can equate the extreme path differences in the two cases and obtain

$mN\lambda + \lambda = mN\ (\lambda + \Delta\lambda)$

From which it immediately follows that

$\frac{\lambda}{\Delta\lambda} = mN$ -------------------- (9)

For a grating of N grooves, the resolving power is simply proportional to the order of the diffraction. In a given order of diffraction, the resolving power increases with the total number of grooves. If the grating has 5000 grooves/cm, and width of 8 cm, then N=40000 and the resolving power in first power is 40000. This means that in the region of λ = 500 nm, spectral components as close together as 0.0125 nm can be resolved. In the second order, this figure improves to 0.0063 nm, and so on. The best value for grating resolving power are in the range of $10^5\ to\ 10^6$.

Notice that the resolving power, like dispersion, is independent of groove spacing for a given diffraction angle. If we write (*N* = *W/a*) for a ruled grating width W and incorporate the grating equation for normal incidence, Eq. 9 becomes

$R = mN = \left(\frac{a \sin\theta_m}{\lambda}\right) \frac{W}{a}$

Or

$R = \frac{W \sin\theta_m}{\lambda}$ ----------------------- (10)

According to Eq. 10, the resolution of a grating at diffraction angle θ_m depends on the width of the grating rather than on the number of its grooves.

Actual gratings used in the study of spectra are made by ruling fine grooves with a diamond point either on a plane glass surface to produce a *transmission grating* or more often on a polished metal mirror to produce a *reflection grating*.

17-1. **What is the angular separation in second order between light of wavelengths 400 nm and 600 nm when diffracted by a grating of 5000 grooves/cm?**