Physical Optics 2018/ *Dr. Muwafaq Fadhil Al-Mishlab*

7th lecture [Applications of Michelson interferometer]

1. Applications of Michelson interferometer

The Michelson interferometer is easily adaptable to the measurement of thin film. It is also easily adaptable to the determination of the index of refraction of a gas. An evacuable cell with plane, parallel windows is interposed in the path of beam 3, figure (1) and is filled with a gas at a pressure and temperature for which its index of refraction is desired. The fringe system established under these conditions is monitored as the gas is gradually pumped out of the cell.

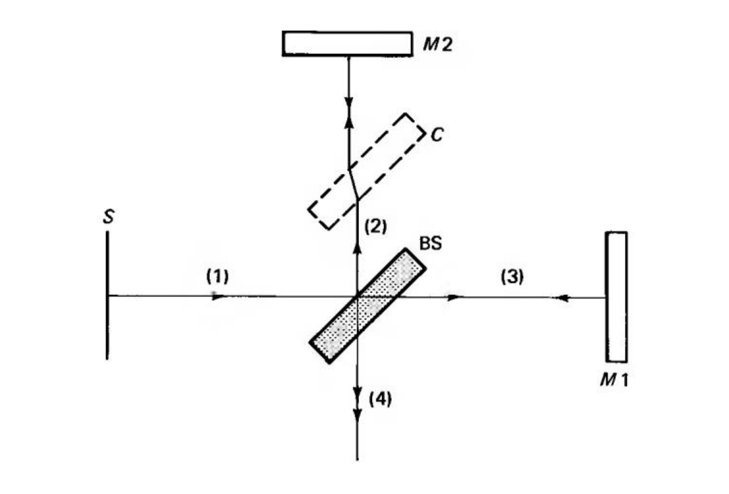

Fig. 1: Michelson interferometer.

A count Δm of the net fringe shift is related to the change in optical path during the replacement of the gas by vacuum. If the actual length of the cell is accurately known to be L, the change in optical path is given by:

$\Delta d = nL - L = L\,(n - 1)$ -------------------- (1)

And using following Eq.

$\Delta m = \frac{2\,\Delta d}{\lambda}$ ----------------- (2)

$n - 1 = \left(\frac{\lambda}{2L}\right)\Delta m$ ------------------------ (3)

Consider another direct application of the Michelson interferometer, the determination of wavelength difference between two closely spaced components of a spectral line λ and $\lambda^\backslash$. each wavelength forms its own system of circular fringes according to Eq. (4)

$2\ d \cos\theta\ = m\lambda$ -------------- (4)

Suppose we view the circular system near their center, so that cos θ=1. Then for given path difference *d* of the interferometer, the product $m\lambda$ is fixed, that is, $m\lambda = m^\backslash \lambda^\backslash$. when the fringe system coincide, the pattern appears sharp, whereas when the fringes of one system in the region of observation lie midway between the fringes of the second system, the pattern appears rather uniform in brightness. The mirror movement Δd required between consecutive coincidences is related to the wavelength difference $\Delta\lambda$ as follows. At one coincidence, when fringes are (in step) the orders of the two systems corresponding to λ and $\lambda^\backslash$ must be related by

$m = m^\backslash + N$

Where N is an integer. If the optical path difference at this time is d_1 then from Eq. 4

$\frac{2d_1}{\lambda} = \frac{2\ d_1}{\lambda^\backslash} + N$ ----------------- (5)

Let the optical path difference be increased to d_2, when the next coincidence is found. Then

$m = m^\backslash + (N+1)$ or

$\frac{2d_1}{\lambda} = \frac{2\ d_1}{\lambda^\backslash} + N + 1$ --------------- (6)

By subtracting Eq. 5 from Eq. 6 and by writing mirror movement $\Delta d = d_2 - d_1$, we find

$\lambda^\backslash - \lambda = \frac{\lambda\lambda^\backslash}{2\ \Delta d}$ --------------------------------- (7)

Now since λ and $\lambda^\backslash$ are very closely, the wavelength of the two unresolved components can be approximated by

$\Delta\lambda = \frac{\lambda^2}{2\ \Delta d}$ ------------------------ (8)

This technique is employed in an optics lab to measure the wavelength difference of 6 A between two components of the yellow of sodium. Finally, there are many ways in which a beam of light may be split into two parts and reunited after traversing divers path, such as Twyman and Green interferometer and Mach-Zehnder interferometer.

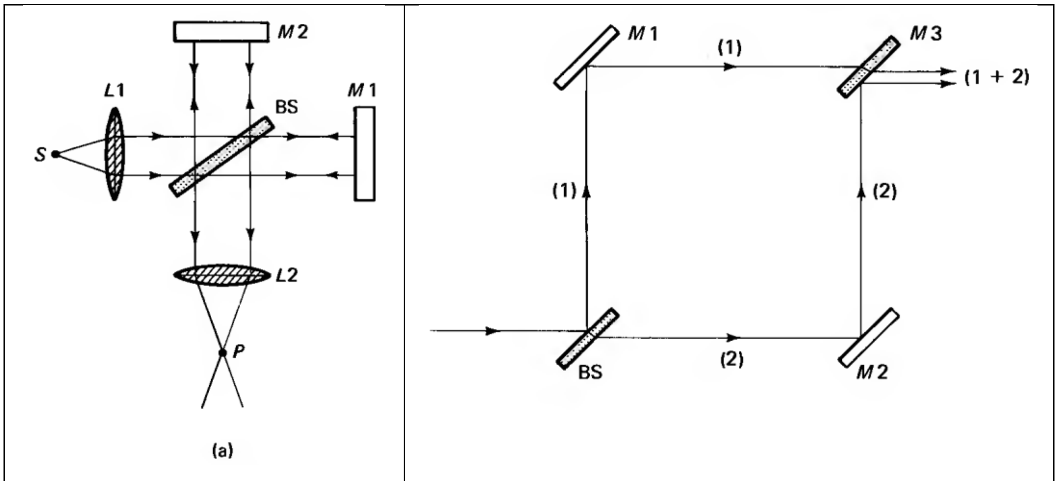

Fig. 2: a) Twyman and Green interferometer b) Mach-Zehnder interferometer.

2. Stokes Relations

We begin with an argument due to Sir George Stokes, which yield information concerning the amplitudes of reflected and transmitted portion of a plane wavefront incident on a plane refracting surface, as in figure (3a). Let E_1 represent the amplitude of the incident light. We define reflection and transmission coefficients by

$$r = \frac{E_r}{E_t} \ , \ t = \frac{E_t}{E_i} \quad \text{--------------------- (9)}$$

So that at the interface, E_i is divided into a reflected part, $Er = r\ E_i$, and a transmitted part, $Et = t\ E_i$ as shown. For ray incident from the second medium, we define similar quantities, which we distinguish with prime notation, r\ and t\. according to the principle of ray reversibility, the situation shown in figure (3b) must also be valid. In general, two rays incident at the interface, as in Fig.3b each result in a reflected and a transmitted ray, all of which are shown, with appropriate amplitudes, in Fig. 3c, we conclude that the situations depicted in Fig. 3b and c must be physically equivalent, so that we can write

$$E_i = \left(r^2 + t^{\backslash}\, t\right)E_i \qquad \text{and} \qquad 0 = \left(r^{\backslash}\, t + tr\right)E_i$$

Or

$$tt^{\backslash} = 1 - \ r^2 \ \text{----------------- (10)}$$

$$r = \ -r^{\backslash} \ \text{-------------------- (11)}$$

Eqs. 10 and 11 are the *Stokes relations* between amplitude coefficients for angles of incidence related through Snell's law. Eq. 11 states that the amplitudes of reflected beams for rays incident from either directions are the same in magnitude but differ by a π phase shift. This becomes clearer if Eq. 11 is written in the equivalent form $r = \ e^{i\pi}\, r^{\backslash}$.

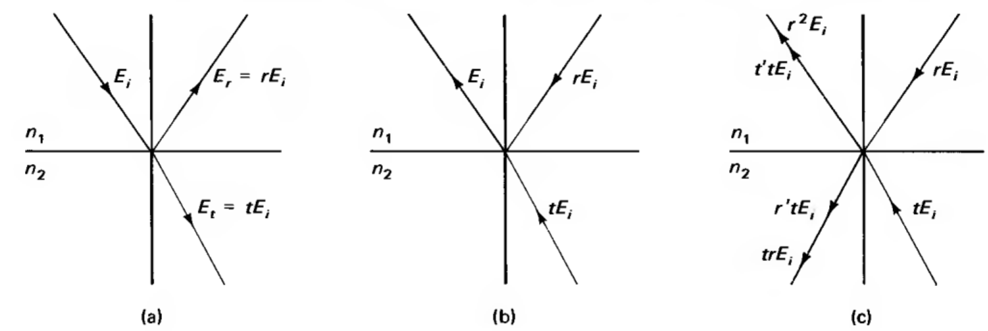

Fig. 4: figures used in deriving Stokes relations.