Physical Optics 2018/ *Dr. Muwafaq Fadhil Al-Mishlab*

6th lecture [Michelson interferometer, Fabry-Perot interferometer]

1. Michelson Interferometer

The Michelson interferometer is an important example of amplitude division. Here the two beams obtained by amplitude division are sent in quite different directions against plane mirrors, whence they are brought together again to form interference fringes. The arrangement is shown schematically in Figure (1). The main optical parts consist of two highly polished plane mirrors M_1 and M_2 and two plane-parallel plates of glass Gland G_2 Sometimes the rear side of the plate G_1 is lightly silvered (shown by the heavy line in the figure) so that the light coming from the source S is divided into (1) a reflected and (2) a transmitted beam of equal intensity. The light reflected normally from mirror M_1 passes through G_1 a third time and reaches the eye as shown. The light reflected from the mirror M_2 passes back through G_2 for the second time, is reflected from the surface of G_1 and into the eye.

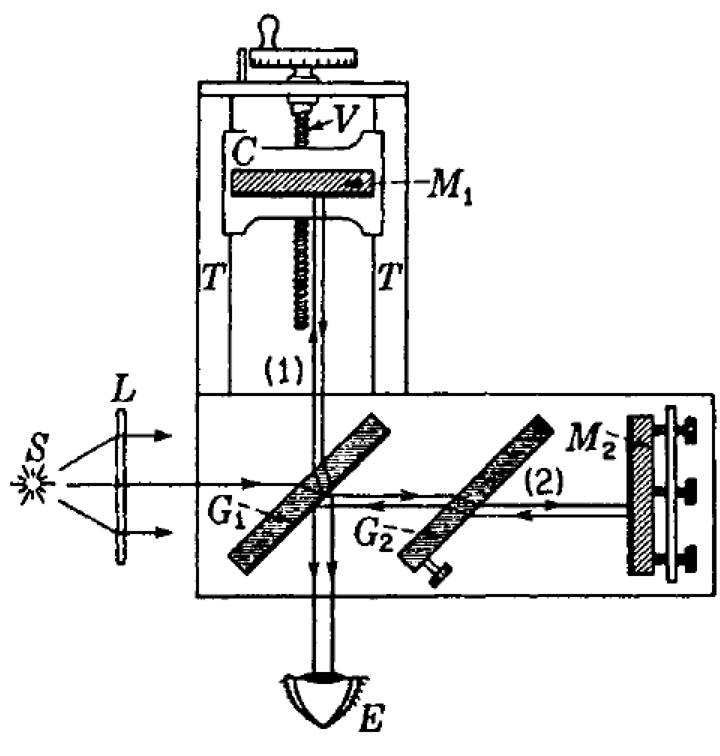

Fig. 1: Diagram of the Michelson interferometer.

The purpose of the plate G_2, called the compensating plate, is to render the path in glass of the two rays equal. This is not essential for producing fringes in monochromatic light, but it is indispensable when white light is used.

The mirror M_1 is mounted on a carriage C and can be moved along the well-machined ways or tracks T. This slow and accurately controlled motion is accomplished by means of the screw V, which is calibrated to show the exact distance the mirror has been moved. To obtain fringes, the mirrors M_1 and M_2 are made exactly perpendicular to each other by means of screws shown on mirror M_2.

Even when the above adjustments have been made, fringes will not be seen unless two important requirements are fulfilled. **First**, the light must originate from an extended source (A sodium flame or a mercury arc). A point source or a slit source, as used in the methods previously described, will not produce the desired system of fringes in this case. The reason for this will appear when we consider the origin of the fringes. **Second**, the light must in general be monochromatic, or nearly so.

In order to obtain the fringes, the next step is to measure the distances of M_1 and M_2 to the back surface of G_1 roughly with a millimeter scale and to move M_1 until they are the same to within a few millimeters. The mirror M_2 is now adjusted to be perpendicular to M_1 by observing the images of a common pin, or any sharp point, placed between the source and G_1. Two pairs of images will be seen, one coming from reflection at the front surface of G_1 and the other from reflection at its back surface. When the tilting screws on M_2 are turned until one pair of images falls exactly on the other, the interference fringes should appear. When they first appear, the fringes will not be clear unless the eye is focused on or near the back mirror M_1, so the observer should look constantly at this mirror while searching for the fringes.

Circular fringes are produced with monochromatic light when the mirrors are in exact adjustment and are the ones used in most kinds of measurement with the interferometer. Their origin can be understood by reference to the diagram of Figure (2). Here the real mirror M_2 has been replaced by its virtual image $M_2^{\setminus}$ formed by reflection in G_1. $M_2^{\setminus}$ is then parallel to M_1. Owing to the several reflections in the real interferometer, we may now think of the extended source as being at L, behind the observer, and as forming two virtual images L_1 and L_2 in M_1 and $M_2^{\setminus}$;. These virtual sources are coherent in that the phases of corresponding points in the two are exactly the same at all instants. If d is the separation M_1M, the virtual sources will be separated by 2d. When d is exactly an integral number of half wavelengths, i.e., the path difference 2d equal to an integral number of whole wavelengths, all rays of light reflected normal to the mirrors will be in phase. Rays of light reflected at an angle, however, will in general not be in phase. The path difference between the two rays coming to the eye from corresponding points P' and P'' is 2d cos e, as shown in the figure. The angle θ is necessarily the same for the two rays when M_1 is parallel to $M_2^{\setminus}$ so that the rays are parallel. Hence when the eye is focused to receive parallel rays (a small telescope is more satisfactory here, especially for large values of d) the rays will support each other to produce maxima for those angles θ satisfying the relation:

$2\,d\cos\theta = m\,\lambda$ -------------------------- (1) m = 0,1, 2, …   (Dark fringes)

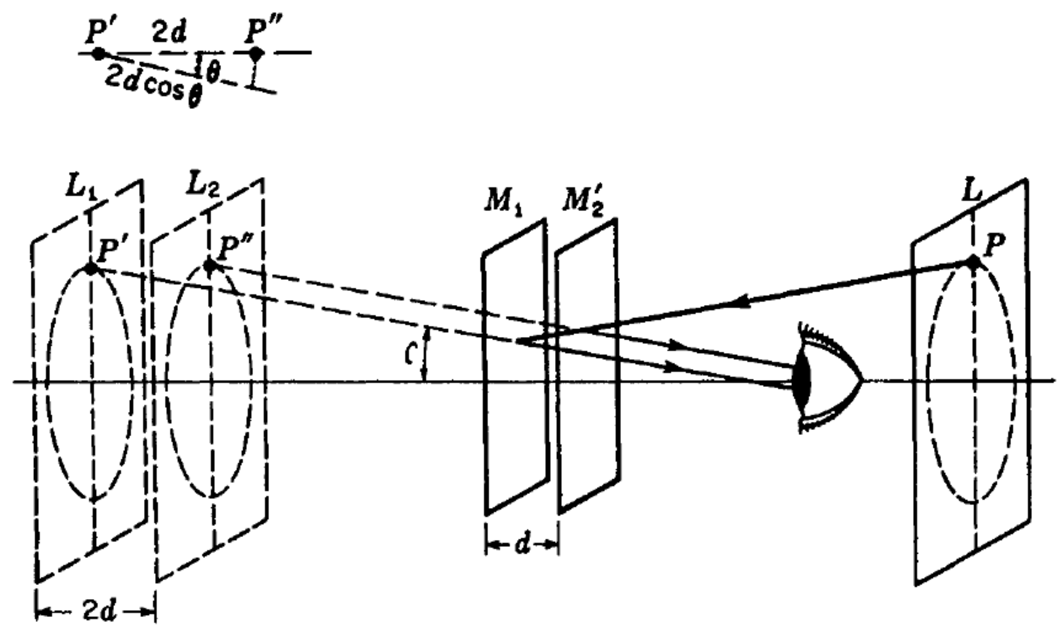

Fig. 2: Formation of circular fringes in the Michelson interferometer.

Since for a given m, λ, and d the angle θ is constant, the maxima will lie in the form of circles about the foot of the perpendicular from the eye to the mirrors. By expanding the cosine, it can be shown from Eq. (l) that the radii of the rings are proportional to the square roots of integers, as in the case of Newton's rings. The intensity distribution across the rings follows Eq. $I = A^2 = 4\, a^2\, cos^2\, \frac{\delta}{2}$, in which the phase difference is given by

$$\delta = \frac{2\pi}{\lambda}\, 2\, d \cos\theta$$

Fringes of this kind, where parallel beams are brought to interference with a phase difference determined by the angle of inclination e, are often referred to as *fringes of equal inclination*.

The upper part of Figure (3) shows how the circular fringes look under different conditions. Starting with M_1 a few centimeters beyond M_2, the fringe system will have the general appearance shown in (a) with the rings very closely spaced. If M_1 is now moved slowly toward M_2 so that d is decreased, Eq. (1) shows that a given ring, characterized by a given value of the order m, must decrease its radius because the product 2d cos θ must remain constant. The rings therefore shrink and vanish at the center, a ring disappearing each time 2d decreases by λ, or d by λ/2. This follows from the fact that at the center cos θ = 1, so that Eq. (1) becomes

$$2\, d = m\, \lambda$$

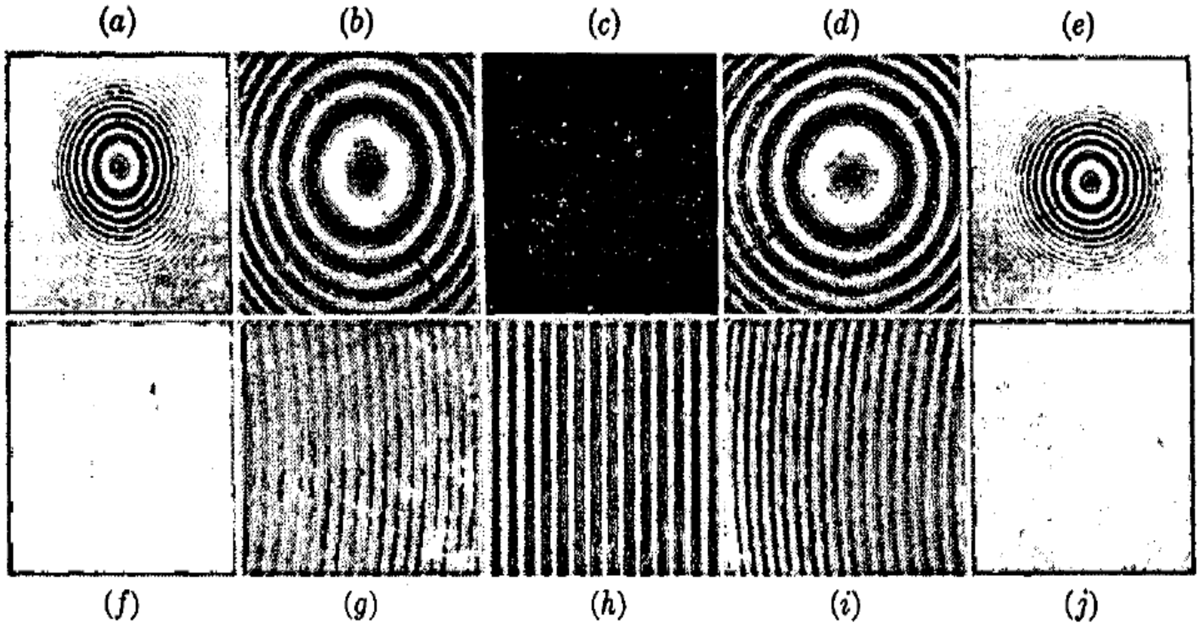

Fig. 3: Appearance of the various types of fringes observed in the Michelson interferometer. Upper row, circular fringes. Lower row, localized fringes.

To change m by unity, d must change by λ/2. Now as M_1 approaches M_2 the rings become more widely spaced, as indicated in Fig.3(b), until finally we reach a critical position where the central fringe has spread out to cover the whole field of view, as shown in (c). This happens when M_1 and $M_2^\backslash$ are exactly coincident, for it is clear that under these conditions the path difference is zero for all angles of incidence. If the mirror is moved still farther, it effectively passes through $M_2^\backslash$, and new widely spaced fringes appear, growing out from the center. These will gradually become more closely spaced as the path difference increases, as indicated in (d) and (e) of the figure.

2. Fabry-Perot Interferometer

This instrument utilizes the fringes produced in the transmitted light after multiple reflection in the air film between two plane plates thinly silvered on the inner surfaces (Figure. 4). Since the separation d between the reflecting surfaces is usually fairly large (from 0.1 to 10 cm) and observations are made near the normal direction, the fringes come under the class of fringes of equal inclination. To observe the fringes, the light from a broad source (S_1 S_2) of monochromatic light is allowed to traverse the interferometer plates E_1E_2 Since any ray incident on the first silvered surface is broken by reflection into a series of parallel transmitted rays, it is essential to use a lens L, which may be the lens of the eye, to bring these parallel rays together for interference.

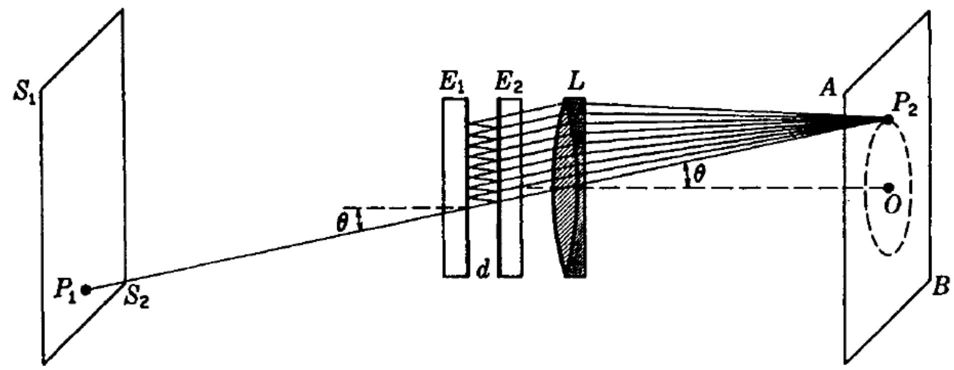

Fig. 3: Fabry-Perot interferometer E_1E_2 set up to show the formation of circular interference fringes from multiple reflections.

In Fig. 4 a ray from the point P_1 on the source is incident at the angle (θ), producing a series of parallel rays at the same angle, which are brought together at the point P_2 on the screen AB. It is to be noted that P_2 is not an image of P_1. The condition for reinforcement of the transmitted rays is given by Eq. ($2\ n\ d \cos\theta = m\ \lambda$) with n = 1 for air, so that

$2\ d \cos\theta = m\ \lambda$ (maxima)

This condition will be fulfilled by all points on a circle through P_2 with their center at 0, the intersection of the axis of the lens with the screen AB. When the angle (θ) is decreased, the cosine will increase until another maximum is reached for which *m* is greater by 1,2, ..., so that we have for the maxima a series of concentric rings on the screen with 0 as their center. the spacing of the rings is the same as for the circular fringes in that instrument and they will change in the same way with change in the distance *d.* In the actual interferometer one plate is fixed, while the other may be moved toward or away from it on a carriage riding on accurately machined ways by a slow-motion screw.

3. Chromatic Resolving Power

The great advantage of the Fabry-Perot interferometer over the Michelson instrument lies in the sharpness of the fringes. The difference in the appearance of the fringes for the two instruments is illustrated in Figure (4). where the circular fringes produced by a single spectral line are compared. If a second line were present, it would merely reduce the visibility in *(a)* but would show as a separate set of rings in *(b).* As will appear later, this fact also permits more exact inter-comparisons of wavelength.

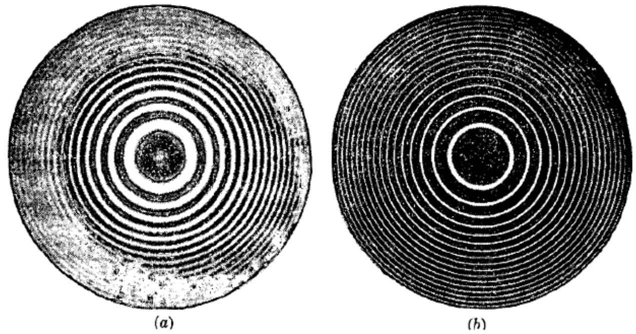

Fig. 4: Comparison of the types of fringes produced with *(a)* the Michelson interferometer and *(b)* the Fabry-Perot interferometer with surfaces of reflectance 0.8.

It is important to know how close together two wavelengths may be and still be distinguished as separate rings. The ability of any type of spectroscope to discriminate wavelengths is expressed as the ratio $\lambda / \Delta\lambda$, where λ is the mean wavelength of a barely resolved pair and $\Delta\lambda$ is the wavelength difference between the components. This ratio is called the *chromatic resolving power* of the instrument at that wavelength. In the present case, it is convenient to say that the fringes formed by λ and $\lambda + \Delta\lambda$ are just resolved when the intensity contours of the two in a particular order lie in the relative positions shown in Figure (5). If the separation $\Delta\theta$ is such as to make the curves cross at the half-intensity point, $I_T = 0.5 I_0$, there will be a central dip of 17 percent in the sum of the two, as shown in *(b)* of the figure. The eye can then easily recognize the presence of two lines.

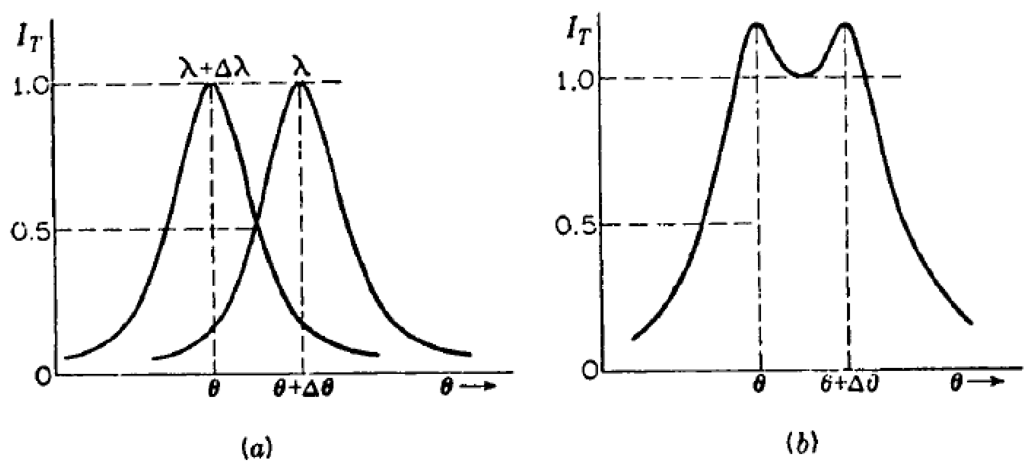

Fig. 5: Intensity contour of two Fabry-Perot fringes that are just resolved: *(a)* shown separately; *(b)* added, to give the observed effect.

In order to find the $\Delta\lambda$ corresponding to this separation, we note first that in going from the maximum to the halfway point the phase difference, this requires that:

$$sin^2 \frac{\delta}{2} = \frac{(1 - r^2)^2}{4\, r^2}$$

If the fringes are reasonably sharp, the change of $\frac{\delta}{2}$ from a multiple of π will be small. Then the sine may be set equal to the angle, and if we denote by $\Delta\delta$ the change in going from one maximum to the position of the other.

the chromatic resolving power

$$\frac{\lambda}{\Delta\lambda} = m\ \frac{\pi\, r}{1 - r^2}$$

It thus depends on two quantities, the order *m*, which may be taken as (2 d / λ) and the reflectance r^2 of the surfaces.