Physical Optics 2018

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5th lecture [Interference in dielectric films., Newton ring Exp.]

1. Interference in Dielectric films

The familiar appearance colors on the surface of oily water and soap films are associated with the interference of light in single or multiple thin surface layers of transparent material.

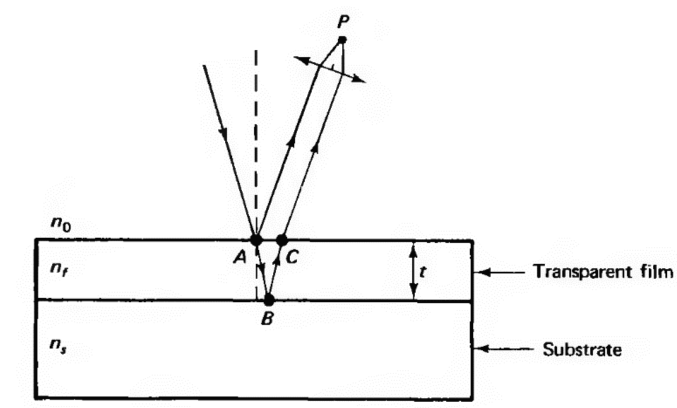


Fig. 1 Double beam interference of thin film.

Consider the case of a film of transparent material bounded by parallel planes, such as might be formed by an oil slick, a metal oxide layer or an evaporated coating on a flat glass substrate (figure (1)). A beam of light incident on the film surface at ***A*** divides into reflected and refracted portions. This separation of the original light into two parts, and interference is usually referred to as *Amplitude division.* The refracted beam reflects again at the film interface ***B*** and leaves the film at ***C,*** in the same direction as the beam reflected at ***A.*** part of the beam may reflect internally again at ***C*** and continue to experience multiple reflections within the film until it has lost its intensity. Unless the reflectance of the film is large, a good approximation is to consider only the first two emerging beams. The two parallel beams leaving the film at ***A*** and ***C*** can be brought together by a converging lens. The two beams intersecting at ***P*** superpose and interfere. Since the two beams travel different paths from point ***A*** onward, a relative phase difference developed that can produce constructive or destructive interference at ***P.*** the optical path difference Δ, in the case of normal incidence, is the additional path length ***ABC*** traveled by the refracted ray times the refractive index of the film. Thus

$\Delta = n(AB + BC) = n(2t)$ --------------------- (1)

Where t is the thickness of the film.

For example, if $2nt = \lambda_0$, the two interfering beams would be in phase and produce constructive interference. Suppose that $n_f > n_0$ *and* $n_f > n_s$. In fact $n_0 = n_s$ because the media bounding the film are identical, as in the case of a water film (soap bubble) in air. Then the reflection at ***A*** occurs with light going from lower index n_0 toward a higher index n_f , a condition usually called *external reflection.* The reflection at ***B*** occurs for light going from a higher index n_f toward lower index n_s, the condition of internal reflection. A relative phase shift of π occurs between the external and internal reflected beams, so that an additional path difference of λ/2 is introduced between the two beams. The net optical path difference between the beams is then λ + λ/2, which puts them precisely out of phase, and destructive interference results at ***P.*** if, instead, both reflections are external ($n_0 < n_f < n_s$) or both reflections are internal ($n_0 > n_f > n_s$), no relative phase difference due to reflection needs to be taken into account. In that case, constructive interference occurs at ***P.***

A frequent use of such single layer films is in the production of *antireflecting coating* on optical surfaces. In most cases, the light enters the film from air, so that $n_0 = 1$. Furthermore, if $n_s > n_f$, no relative phase shift between the two reflected beams occurs, and the optical path difference alone determines the type of interference. If the film thickness is $\lambda_f / 4$, where λ_f is the wavelength of light in the film, then $2t = \lambda_f / 2$ and the optical path difference $2n_f t = \lambda_0 / 2$, since $\lambda_0 = n_f \lambda_f$. destructive interference occurs at this wavelength. In general, all one can say is that for constructive interference the two amplitudes add (being in phase), and for destructive interference the amplitudes subtract (out of phase). for the difference to be zero, the amplitudes must be equal. In the case of normal incident, the reflectance coefficient is given by:

$$r = \frac{1-n}{1+n} \text{ --------------------------- (2)}$$

Where the relative index $n = n_2 / n_1$. The amplitudes of the electric field reflected internally and externally from the film in figure (1) are then equal, assuming a non - absorbance film, if the relative indices are equivalent for these cases, that is

$$\frac{n_f}{n_0} = \frac{n_s}{n_f} \; or \; n_f = \sqrt{n_0 \, n_s} \text{ ----------------- (3)}$$

Consider a multilayer stack of alternating high-low index dielectric films (figure (2)). If the film has thickness of $\lambda_f / 4$, a little analysis shows that in this case all emerging beams are in phase. multiple reflections in the region of λ_0 increase the total reflected intensity and quarter-wave stack performs as an efficient mirror.

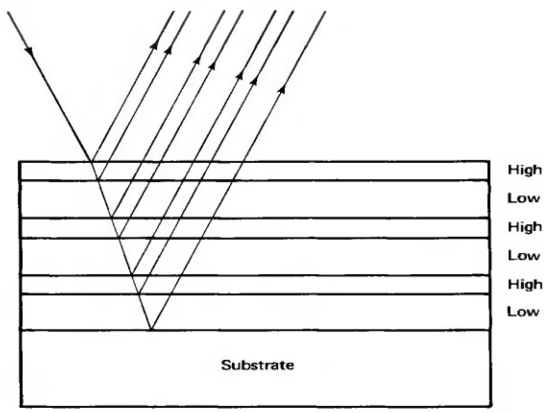


Fig. 2 Multilayer dielectric mirror of alternating high-low index.

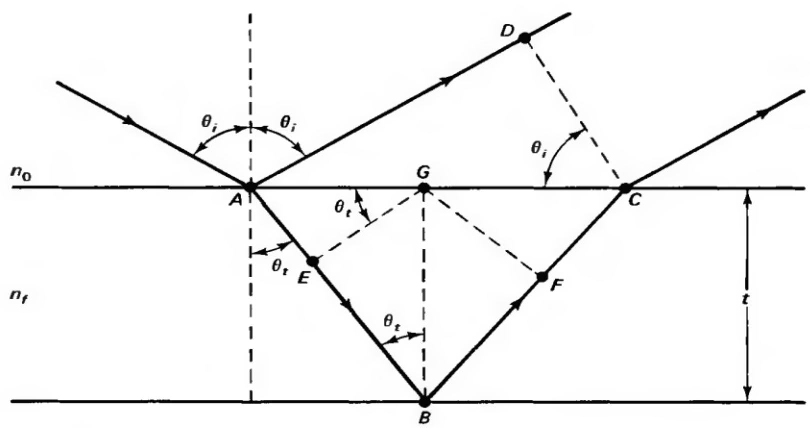


Fig. 3 single layer film interference with light incident at arbitrary angle θ_i.

Returning now to the single-layer film, we want first to generalize the conditions for constructive and destructive interference by calculating the optical path difference in the case incident rays are not normal. Figure (3) illustrates a ray incident on film at an angle θ_i. the phase difference at points C and D between emerging beams is due to the optical path difference between paths AD and ABC. After points C and D are reached, the respective beams are parallel and in the same medium, so that no further phase difference occurs. Point G is shown the midway between A and C at the foot of the altitude BG in the isosceles triangle ABC. Points E and F are determined by constructing the perpendiculars GE and GF to the ray paths AB and BC. The optical path difference between emerging beams is then:

$\Delta = n_f\,(AB + BC) - n_0\,(AD)$ ---------------------- (4)

Where n_f and n_0 are the refractive indices of film and external medium.

$\Delta = \left[n_f\ (AE + FC) - \ n_0\ AD\right] + \ n_f(EB + BF)$ ----------------------- (5)

$n_0\ \sin\theta_i = \ n_f \sin\theta_i$ ------------------- (6)

By inspection,

$AE = AG \sin\theta_t = \left(\frac{AC}{2}\right)\sin\theta_t$ ---------------------- (7) , and

$AD = AC \sin\theta_i$ ---------------------- (8)

From Eq. (7) and incorporating Eqs. (8) and (6)

$2AE = AC \sin\theta_t = AD\ \left(\frac{\sin\theta_t}{\sin\theta_i}\right) = AD\ \left(\frac{n_0}{n_f}\right)$, So that

$n_0\ AD = 2n_f\ AE = \ n_f\ (AE + FC)$ ----------------------- (9)

From Eq. 5

$\Delta = \ n_f\ (EB + BF) = 2n_f\ EB$ ------------------------------ (10)

The length EB is related to the film thickness *t* by EB = t cos θ_t, so we have

$\Delta = 2\ n_f t \cos\theta_t$ -------------------- (11)

The optical path difference Δ is in terms of the angle of refraction. For normal incidence, $\theta_i=\theta_t=0$ and $\Delta=2n_f$ t. the corresponding phase difference is $\delta = \ k\Delta = (2\pi/\lambda_0)\ \Delta$. If we call Δ_p the optical path difference given by Eq. 11 and Δ_r the equivalent path difference arising from phase change on reflection, we can state that:

Constructive interference: $\Delta_p + \ \Delta_r = m\lambda$ ------------------------- (12)

Destructive interference: $\Delta_p + \ \Delta_r = \left(m + \frac{1}{2}\right)\lambda$ ------------------- (13) where m=0,1,2,3,……

2. Fringes of Equal Thickness

If the film is of varying thickness t, the optical path difference $\Delta = 2\ n_f t \cos\theta_t$, varies even without variation in the angle of incidence. Thus, if the direction of the incident light is fixed, say at normal incidence, a bright fringe will be associated with a particular thickness for which Δ satisfies the condition for constructive or destructive interference. For this reason, fringes produced by a variable-thickness film are called *fringes of equal thickness*. In figure (4), an extended source is used in conjunction with a beam splitter set at an angle of 45^0 to the incident light. The beam splitter in this position enables light to strike the film at normal incidence, while at

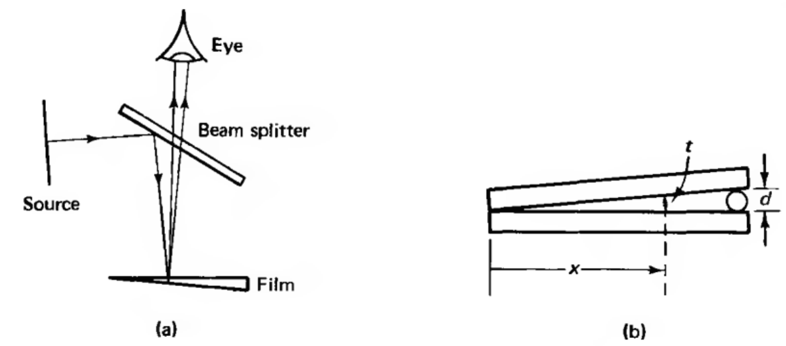


Fig. 4 interference from a wedge-shaped film, producing localized fringes of equal thickness. A) viewing assembly b) air wedge formed with 2 microscope slides

the same time providing for the transmission of part of the reflected light into the detector (eye). Fringes, often called *Fizeau fringes,* are seen localized at the film, from which the interfering rays diverge. At normal incidence, cos θ_t = 1 and $\Delta = 2\, n_f\, t$. Thus the condition for bright and dark fringes, Eqs. (12) and (13) is

$2nt = \left(m + \frac{1}{2}\right)\lambda$ ------------------ (14) bright fringes

Where Δ_r is either λ/2 or 0, depending on whether there is or is not a relative phase shift of π between the rays reflected from the top and bottom surfaces of the film. One way of forming a suitable wedge is to use two clean, glass microscope slides, wedge apart at one end by thin space, as in figure (4b). the resulting air layer between the slides shows Fizeau fringes when the slide are illuminated by monochromatic light. For this film, the two reflections are from glass to air (internal reflection) and from air to glass (external reflection), so that $\Delta = \lambda/2$. As t increases from 0 to d, Eq. (14) is satisfied for consecutive orders of m, and a series of equally spaced, alternating bright and dark fringes will be seen by reflected light. These fringes are virtual, localized and cannot be projected onto screen.

3. Newton's Rings

If the fringes of equal thickness are produced in the air film between a convex surface of a long-focus lens and a plane glass surface, the contour lines will be circular. The ring-shaped fringes thus produced were studied in detail by Newton, although he was not able to explain them correctly. For purposes of measurement, the observations are usually made at normal incidence by an arrangement such as that in Fig. (5), where the glass plate G reflects the light down on the plates. After reflection, it is transmitted by G and observed in the low-power microscope T. Under these conditions the positions of the maxima are given by Eq. (l4), where t is the thickness of the air film. Now if we designate by R the radius of curvature of the surface A and assume that A and B are just touching at the center, the value of *t* for any ring of radius, is the sagitta of the arc, given by

$$t = \frac{r^2}{2R} \text{ --------------------------- } (15)$$

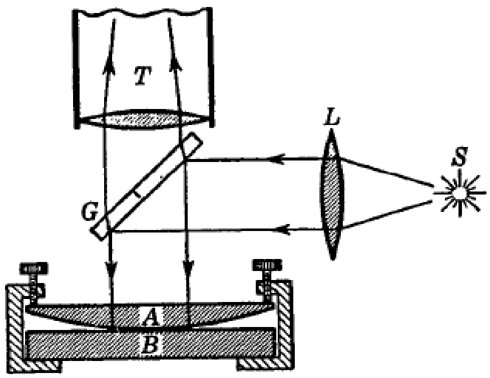


Fig. 5 Experimental arrangement used in viewing and measuring Newton's rings.

Because the ring diameters depend on wavelength, white light will produce only a few colored rings near the point of contact. With monochromatic light, however, an extensive fringe system such as that shown in Figure. (6) is observed. When the contact is perfect, the central spot is black. This is direct evidence of the relative phase change of 1t between the two types of reflection, air-to-glass and glass-to-air. If there were no such phase change, the rays reflected from the two surfaces in contact should be in the same phase and produce a bright spot at the center. In an interesting modification of the experiment, due to Thomas Young, the lower plate has a higher index of refraction than the lens, and the film between is filled with an oil of intermediate index. Then both reflections are at "rare to-dense" surfaces, no relative phase change occurs, and the central fringe of the reflected system is bright. The experiment does not tell us at which surface the phase change in the ordinary arrangement occurs, but it is now definitely known that it occurs at the lower (air-to-glass) surface.

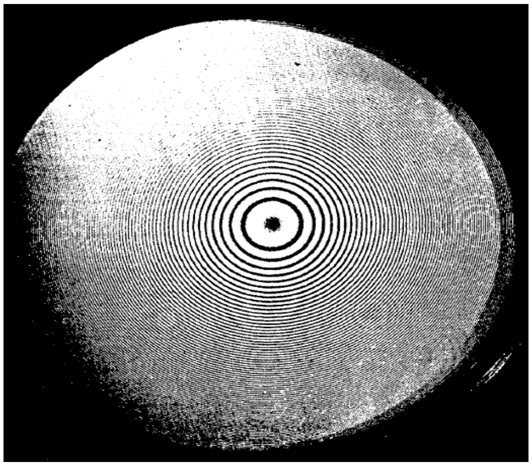
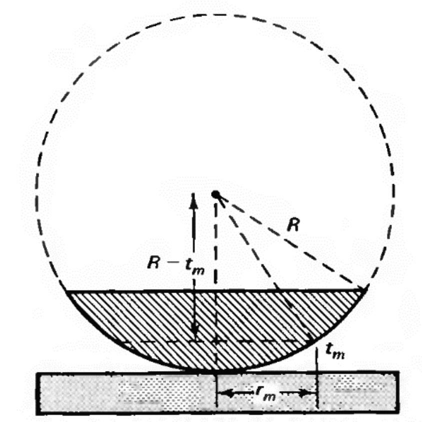


Fig. 6 Newton's rings.

A geometrical relation exists between the radius r_m of the *mth*-order dark fringe, the corresponding air-film thickness t_m and the radius of curvature R of the air film or lens surface.

$R^2 = r_m^2 + (R - t_m)^2$ ----------------- (16)



(b)

Example: A plano-convex lens (n=1.523) of 1/8 diopter power is placed, convex surface down, on an optically flat surface. Using a traveling microscope and sodium light (λ=589.3nm), interference fringes are observed. determine the radii of the first and tenth dark rings?

Air film thickness at *mth* dark ring given by

$t_m = {m\lambda}/{2n_f}$. Since the film is air, n_f= 1 and $t_m = m\lambda/2$. The ring radii given by Eq. (16). On neglecting the very small term t_m^2 , this is $r_m^2 = 2Rt_m$. The radius of curvature of the convex surface of the lens is found from the lens maker's eq.

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

With f=8 m, n=1.523, R_2 =σ, this gives R = 4.184m. then

$r_m^2 = 2Rt_m = 2R\left(\frac{m\lambda}{2}\right) = mR\lambda$

$r_1^2 = (1)(4.184)(589.3\ x\ 10^{-9}) = 2.466x\ 10^{-6}\ m^2$

$r_{10}^2 = (10)(4.184)(589.3\ x\ 10^{-9}) = 24.66\ x\ 10^{-6}\ m^2$

Or

r_1= 1.57 mm and r_{10}= 4.97 mm.

Problems

Q1/ Young's experiment is performed with orange light from a krypton arc. If the fringes are measured with a micrometer eyepiece at a distance 100 cm from the double slit, it is found that 25 of them occupy a distance of 12.87 mm between centers. Find the distance between the centers of the two slits.

Q2/ A double slit with a separation of 0.250 mm between centers is illuminated with green light from a cadmium-arc lamp. How far behind the slits must one go to measure the fringe separation and find it to be 0.80 mm between centers?

Q3/ Two harmonic waves with amplitudes of 1.6 and 2.8 interfere at some point on a screen. What fringe contrast or visibility results there if their electric field vectors are parallel and if they are perpendicular?

Q4/ Two slits are illuminated by light that consist of two wavelengths. one wavelength is known to be 436 nm. On screen, the fourth minimum of the 436nm light coincides with the third maximum of the other light. What is the wavelength of the unknown light?

Q5/ In a Young's experiment, narrow double slits 0.2 mm apart diffract monochromatic light onto a screen 1.5 m away. The distance between the fifth minima on either side of the zeroth- order maxima is measured to be 34.73 mm. determine the wavelength of the light?

Q6/ Sodium light (589.3 nm) from a narrow-slit illuminates a Fresnel biprism made of glass of index 1.5. the biprism is twice as far from a screen on which fringes are observed as it is from the slit. The fringes are observed to be separated by 0.03 cm. what is the biprism angle?

Q7/ A Newton's ring apparatus is illuminated by light with two wavelength components. One of the wavelengths is 546 nm. If the eleventh bright ring of the 546 nm fringe system coincides with the tenth ring of the other, what is the second wavelength? what is the radius at which overlap takes place and the thickness of the air film there? The spherical surface has a radius of 1m.