Physical Optics 2018

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Third lecture [Huygens' Principle, Interference of light]

1. Huygens' principle

Long before people understood the electromagnetic character of light, Christian Huygens—a17th-century scientist—came up with a technique for propagating waves from one position to another, determining, in effect, the shapes of the developing wave fronts. This technique is basic to a quantitative study of interference and diffraction, so we cover it here briefly. Huygens claimed that:

Every point on a known wave front in a given medium can be treated as a point source of secondary wavelets (spherical waves "bubbling" out of the point, so to speak) *which spread out in all directions with a wave speed characteristic of that medium. The developing wave front at any subsequent time is the envelope of these advancing spherical wavelets.*

Figure (1) shows how Huygens' principle is used to demonstrate the propagation of successive (a) plane wave fronts and (b) spherical wave fronts. Huygens' technique involves the use of a series of points $P_1 \ldots P_8$, for example, on a given wave front defined at a time $t = 0$. From these points—as many as one wishes, actually—spherical wavelets are assumed to emerge, as shown in Figures (1a) and (1b). Radiating outward from each of the P-points, with a speed v, the series of secondary wavelets of radius $r = vt$ defines a new wave front at some time t later. In Figure (1a) the new wave front is drawn as an *envelope tangent* to the secondary wavelets at a distance $r = vt$ from the initial plane wave front. It is, of course, another *plane* wave front. In Figure (1b), the new wave front at time t is drawn as an *envelope tangent* to the secondary wavelets at a distance $r = vt$ from the initial spherical wave front. It is an advancing *spherical* wave front. While there seems to be no physical basis for the existence of Huygens' "secondary" point sources, Huygens' technique has enjoyed extensive use, since it does predict accurately—with waves, not rays—both the *law of reflection* and *Snell's law of refraction*. In addition, Huygens' principle forms the basis for calculating, for example, the diffraction pattern formed with multiple slits. We shall soon make use of Huygens' secondary sources when we set up the problem for diffraction from a single slit.

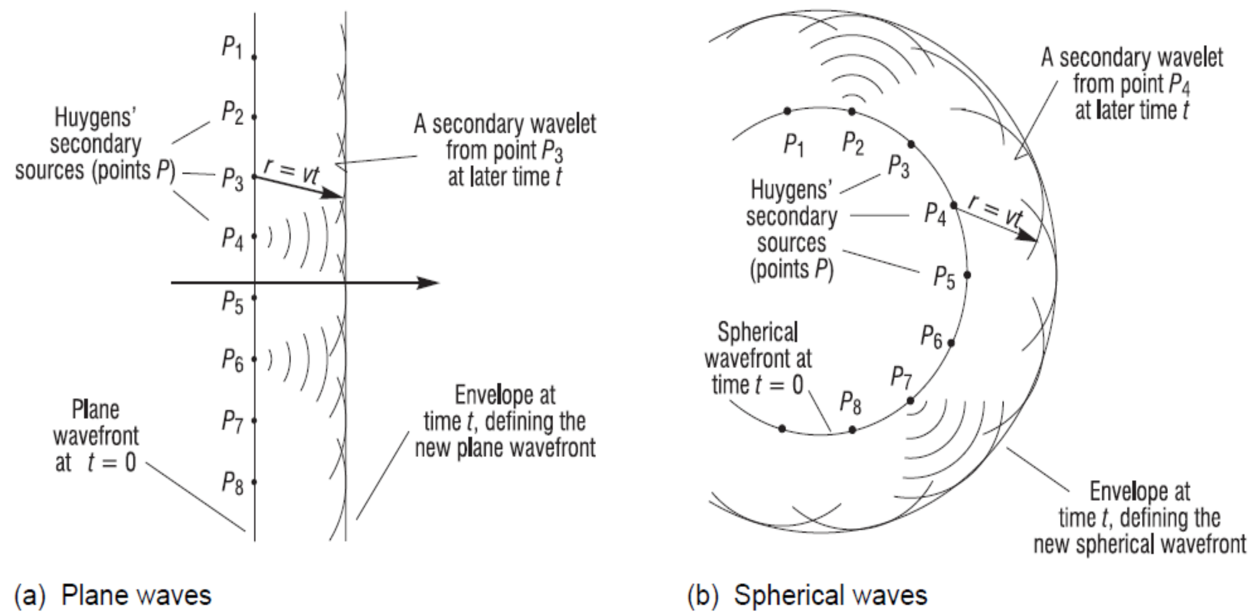

Fig. 1 Huygens' principle applied to the propagation of plane and spherical wave fronts.

2. Interference

All waves show the phenomena of interference and diffraction which arise from the superposition of more than one wave. At each point of observation within the interference or diffraction pattern the phase difference between any two component waves of the same frequency will depend on the different paths they have followed and the resulting amplitude may be greater or less than that of any single component. Although we speak of separate waves the waves contributing to the interference and diffraction pattern must ultimately derive from the same single source. This avoids random phase effects from separate sources and guarantees coherence. Interference effects may be classified in two ways:

1) *Division of wave front*

Here the wavefront from a single source passes simultaneously through two or more apertures each of which contributes a wave at the point of superposition. Diffraction also occurs at each aperture.

2) Division of amplitude

Here a beam of light or ray is reflected and transmitted at a boundary between media of different refractive indices. The incident, reflected and transmitted components form separate waves and follow different optical paths. They interfere when they are recombined.

3. Interference of waves from two sources

We consider first the interference of two waves, represented by E_1 *and* E_2, where we take into account the vector property of the electric fields. In case of interference, both waves typically originate from a single source and reunite after travelling along different paths. The direction of travel of the waves need not be the same when the come together, however, they maintain the same frequency, they generally do not have the same propagation vector K. we may express the wave equations by:

$E_1 = E_{01}\cos(k_1.r - \omega t + \epsilon_1)$	1

$E_2 = E_{02}\cos(k_2.r - \omega t + \epsilon_2)$	2

At some general point P, defined by position vector r, the waves intersect to produce a disturbance whose electric field Ep is given by the principle of superposition.

$E_p = E_1 + E_2$	3

now E_1 and E_2 are varying functions with optical frequencies of the order of 10^{14} to 10^{15}Hz for visible light. Thus, both E_1 and E_2 average to zero over very short time intervals. Measurement of the waves by their effect on the eye or some other light detector depends on the energy of the light beam. The radiant power density, or irradiance, E_e (W/m^2), measures the time average of the square of the wave amplitude. To avoid confusion with electric field symbol, we use the symbol *I* for irradiance.

$I = \epsilon_0\, c\, \langle E^2 \rangle$	4

Thus, the resulting irradiance at P given by:

$I = \epsilon_0\, c\, \langle E_p^2 \rangle = \epsilon_0\, c\, \langle E_p \,.\, E_p \rangle = \epsilon_0\, c\, \langle (E_1 + E_2).(E_1 + E_2) \rangle = \epsilon_0\, c\, \langle E_1^2 + E_2^2 + 2\, E_1\, E_2 \rangle$	5

From Eq. (4), the first two terms correspond to the irradiances of the individual waves, $\boldsymbol{I_1}$, $\boldsymbol{I_2}$. The last term depends on interaction of the waves and is called (*Interference term)*, I_{12}, we may then write

$I = I_1 + I_2 + I_{12}$	6

If light behaved without interference, like classical particles, we would expect then $I=I_1+I_2$. The presence of the third term I_{12} is indicative of the wave nature of light, which can produce enhancement or diminution of the irradiance through interference. Notice that when E_1 *and* E_2 are orthogonal, so that their *dot product* vanishes, no interference results. When the electric fields are parallel, the interference makes its maximum contribution. Consider the interference term is:

$I_{12} = 2\,\epsilon_0\, c\,\langle E_1 . E_2\rangle$	7

Where E_1 *and* E_2 are given by Eqs. (1) and (2), their dot product,

$E_1 . E_2 = E_{01} . E_{02} \cos(k_1 . r - \omega t + \epsilon_1) \cos(k_2 . r - \omega t + \epsilon_2)$	8

$\alpha = k_1 . r - \omega t + \epsilon_1\ ,\ \beta = k_2 . r - \omega t + \epsilon_2$	9

$E_1 . E_2 = E_{01} . E_{02} \cos(\alpha - \omega t) \cos(\beta - \omega t)$	10

$\langle E_1 . E_2\rangle = E_{01} . E_{02}[\cos\alpha \cos\beta\ \langle cos^2\ \omega t\rangle$ $+ \sin\alpha \sin\beta \langle sin^2\ \omega t\rangle + (\cos\alpha \sin\beta + \sin\alpha \cos\beta)\langle \sin\omega t \cos\omega t\rangle]$	11

Over any number of complete cycles,

$\langle cos^2 \omega t\rangle = \frac{1}{2}\ ,\ \langle sin^2 \omega t\rangle = \frac{1}{2}$ and $\langle \sin\omega t \cos\omega t\rangle = 0$

$\langle E_1 . E_2\rangle = \frac{1}{2}\ E_{01} . E_{02} \cos(\alpha - \beta)$	12

Or

$\langle E_1 . E_2\rangle = \frac{1}{2}\ E_{01} . E_{02} \cos\ [(k_1 . k_2).\ r + (\epsilon_1 - \epsilon_2)]$	13

Where the expression in brackets is the phase difference between E_1 *and* E_2

$\delta = (k_1 - k_2). r + (\epsilon_1 - \epsilon_2)$	14

Combining Eqs. (7), (13) and (14)

$I_{12} = \epsilon_0\, c\, E_{01} \,.\, E_{02} \cos\delta$	15

Similarly, I_1 *and* I_2 in Eq. (6) can be shown to produce

$I_1 = \epsilon_0\, c \langle E_1^2 \rangle = \frac{1}{2}\, \epsilon_0\, c\, E_{01}^2$	16

$I_2 = \epsilon_0\, c \langle E_2^2 \rangle = \frac{1}{2}\, \epsilon_0\, c\, E_{02}^2$	17

In the case $E_{01} \| E_{02}$, their dot product in Eq. 15 is identical with their magnitudes.

$I_{12} = 2\sqrt{I_1 I_2} \cos\delta$	18

So that we may writ, finally

$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\delta$	19

When $\cos\delta = +1$, constructive interference produces the max irradiance:

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$	20

This condition occurs whenever the phase difference $\delta = 2m\pi$, where *m* is any integer or zero. On the other hand, when $\cos\delta = -1$, destructive interference produced the minimum, or irradiance.

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$	21

A condition that occurs whenever $\delta = (2m + 1)\pi$. A plot of irradiance *I* versus phase δ, in figure (1a), exhibits periodic fringes. Destructive interference is complete, that is, cancellation is complete, when $I_1 = I_2 = I_0$. Then Eqs. (18) and (19) give:

$$I_{max} = 4\, I_0 \text{ and } I_{min} = 0$$

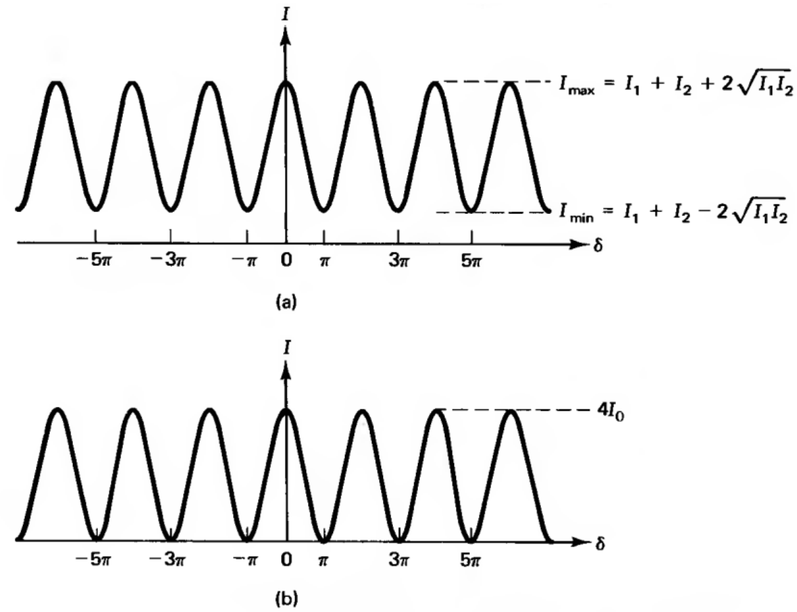

Fig. 1 irradiance of interference fringes as a function of phase. b) $I_{min}=0$, when $I_1 = I_2$

Resulting fringes, shown in figure (1b), now exhibit contrast. A measure of *fringes contrast*, also called *visibility*, with values between 0 and 1, is given by the quantity

$fringe\ contrast\ = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$	22

Another useful form of Eq. (19), for the case of interfering beams of equal amplitude, is found by writing:

$$I = I_0 + I_0 + 2\sqrt{I_0^2}\cos\delta = 2\,I_0\,(1+\cos\delta)$$

$$1 + \cos\delta \equiv 2cos^2\left(\frac{\delta}{2}\right)$$

The irradiance of two equal beam is

$I = 4I_0\,cos^2\left(\frac{\delta}{2}\right)$	23

Example: two interfering beams with parallel electric fields are given by

$$E_1 = 2\cos\left(k_1.r - \omega t + \frac{\pi}{3}\right) \text{(KV/m)} \quad , E_2 = 5\cos\left(k_2.r - \omega t + \frac{\pi}{4}\right) \text{(KV/m)}$$

Let us determine the irradiance contributed by each beam acting along that due to their mutual interference at a point where their path difference is zero. We have:

$$I_1 = \frac{1}{2}\,\epsilon_0\, c\, E_{01}^2 = \frac{1}{2}\,\epsilon_0\, c\,(2000)^2 = 5309\; W/m^2$$

$$I_2 = \frac{1}{2}\,\epsilon_0\, c\, E_{02}^2 = \frac{1}{2}\,\epsilon_0\, c\,(5000)^2 = 33{,}180\; W/m^2$$

$$I_{12} = 2\sqrt{I_1 I_2}\,\cos\delta = 2\sqrt{5309 \times 33180}\;\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = 24640\; W/m^2$$

To find the fringe contrast near the region of superposition we must calculate

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = 5309 + 33180 + 2\sqrt{(5309 \times 33180)} = 65034\; W/m^2$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = 5309 + 33180 - 2\sqrt{(5309 \times 33180)} = 11945\; W/m^2$$

$$fring\ contrast = \frac{65034 - 11945}{65034 + 11945} = 0.690$$

If the amplitudes of the two waves equal, then $I_{max} = 4I_0$, $I_{min} = 0$, and the fringe contrast would be 1