$$R = \left(\frac{8\pi^5 k^4}{15 c^3 h^3}\right) T^4$$

But the total emmi (power radiated per unit area) is,

$$R_B = \frac{c}{4} \times R$$

$$= \frac{c}{4} \times \left(\frac{8\pi^5 k^4}{15 c^3 h^3}\right) T^4$$

قانون ستيفان

$$= \left(\frac{2\pi^5 k^4}{15 c^2 h^3}\right) T^4 \Rightarrow \boxed{R_B = \delta T^4}$$

where $\delta = 5.625 \times 10^{-8}\ W m^{-2} K^{-4}$ (stefan's const)

Thus, we see that planck's theory Successfully explains all the facts of black body Radiation.

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photons and wave-particle Duality الطبيعة الازدواجية

Einstein (1905) who proposed the photon concept and postulated, $E = h\nu$, and verified it by photoelectric effect. The Compton effect also indicated the particle property of radiation. When dealing with radiations of visible wavelength, we are not normally able to measure the frequency it is rather too high, that:

$$\nu = \frac{c}{\lambda} = \frac{3\times10^{8}\, m/s}{5000\times10^{-10}\, m} = 10^{15}\ Hz$$

But we can measure wavelength through the use of diffraction gratings. Thus it is often convenient to write:

$$E_{photon} = h\nu = h\frac{c}{\lambda} \quad \text{——— (1)}$$

Since photons have to move at the speed of light, their rest mass is zero and,

$$E = pc \quad (\text{for a relativistic particle where } E^2 = p^2c^2 + m^2c^4)$$

$$\text{or } h\nu = pc \Rightarrow p = \frac{h\nu}{c} \quad \text{——— (2)}$$

$$\boxed{\text{or}} \quad h\frac{c}{\lambda} = pc$$

$$\Rightarrow \boxed{p = \frac{h}{\lambda}} \quad \text{——— (3)}$$

That is ~~p~~ from equation (2), $m_{eff.}c = \frac{h\nu}{c}$

or effective mass of ~~a~~ a photon, $\boxed{m_{eff} = \frac{h\nu}{c^2}}$

and light of a given frequency has a fixed momentum (particle like property) and equation (3) implies it is also a wave having a definite wavelength. This is expressed by stating that a photon is a wave or a particle.

- Heisenberg's Uncertainty Relation :

The product of ~~of~~ the uncertainities Δx and Δp in the position coordinate and its conjugate momentum coordinate respectively are such that

$$\Delta x \cdot \Delta p \geq \hbar \; ; \; \hbar = \frac{h}{2\pi}$$

Derivation of Uncertainty Relation from de Broglie's Concept :

In general, a wave packet is formed by a number of waves of slightly different frequencies. When a wave packet is formed, the size of the wave packet and the range of the wave numbers (K) required to form this are related according to Fourier ~~and~~ analysis by :

$$\Delta x \Delta k \simeq 1 \qquad (1)$$

Now, according to de Broglie, wavelength is related to momentum associated with the wave by,

$$\lambda = \frac{h}{p} \Rightarrow \frac{p}{h} = \frac{1}{\lambda} \qquad (2)$$

Thus, a range of wave numbers Δk can be related to a range of momenta Δp,

$$\Delta k = 2\pi \Delta\left(\frac{1}{\lambda}\right) \qquad \left\{k = \frac{2\pi}{\lambda}\right\} \qquad (3)$$

Substituting eq. (2) into eq. (3):

$$\Delta k = 2\pi \Delta\left(\frac{p}{h}\right)$$

$$= 2\pi \frac{\Delta p}{h} \qquad (4)$$

from eq. (1), $\Delta x \simeq \frac{1}{\Delta k}$, then eq. (4) becomes

$$\Delta x = \frac{h}{2\pi \Delta p} \Rightarrow \boxed{\Delta x \Delta p = \hbar}$$ [illegible]

The more accurately the position of the particle is specified, the greater will be the spread of possible momenta, and, in the limit where the position is specified with the infinite accuracy, the momentum of the particle can have any value, i.e., it would be indeterminate and vice versa.

The uncertainty principle can also be expressed in the time domain. So, we know that for any wave train:

$$\Delta\omega \cdot \Delta t \simeq 1; \quad \omega = 2\pi\nu \qquad (5)$$

Since $E = h\nu$, so we have $\Delta E = h\Delta\nu$

$$\Delta\nu = \frac{\Delta E}{h}$$

$$\Rightarrow \Delta\omega = 2\pi\Delta\nu = \frac{2\pi\Delta E}{h} \qquad (6)$$

eq. (6) into eq (5):

$$\frac{2\pi\Delta E}{h} \cdot \Delta t \simeq 1$$

$$\boxed{\Delta E \cdot \Delta t = \hbar}$$

احفظها جيداً.

It should be noted that the distinction between classical (large) systems and quantum (small) systems is not made on the basis of spatial extension only, but in units of $\hbar$.

مثال

Examples: "The radius of the Bohr's First orbit" حساب نصف قطر بور الأول لذرة الهيدروجين

If Δq and Δp are the uncertainties in the position and momentum of the electron in the first orbit, then:

$$\Delta q \Delta p \simeq \hbar \quad \text{or} \quad \Delta p \simeq \frac{\hbar}{\Delta q} \qquad \text{——— (1)}$$

The uncertainty in the kinetic energy,

$$\Delta T = \frac{1}{2} m (\Delta v)^2 = \frac{1}{2} \frac{(m \Delta v)^2}{m}$$

$$\Rightarrow \Delta T = \frac{1}{2} \frac{(\Delta p)^2}{m} = \frac{1}{2m} \left(\frac{\hbar}{\Delta q}\right)^2$$

$$= \frac{\hbar^2}{2m (\Delta q)^2} \qquad \text{——— (2)}$$

The uncertainty in the potential energy,

$$\Delta V = \frac{-Ze^2}{\Delta q} \qquad \text{——— (3)}$$

So, the uncertainty in the total energy,

$$\Delta E = \Delta T + \Delta V$$

$$\therefore \Delta E = \frac{\hbar^2}{2m (\Delta q)^2} - \frac{Ze^2}{\Delta q} \qquad \text{——— (4)}$$

ΔE will be minimum if $\dfrac{d(\Delta E)}{d(\Delta q)} = 0$

Equation (4) yields,

$$\frac{d(\Delta E)}{d(\Delta q)} = -\frac{\hbar^2}{m(\Delta q)^3} + \frac{Ze^2}{(\Delta q)^2} \quad \text{——— (5)}$$

For E to be minimum, we must have:

$$-\frac{\hbar^2}{m(\Delta q)^3} + \frac{Ze^2}{(\Delta q)^2} = 0$$

نصف قطر

$$\Rightarrow \boxed{\Delta q = \frac{\hbar^2}{mZe^2}} \simeq r \quad \text{——— (6)}$$

So, eq. (6) represents the condition of minimum uncertainty in the first orbit, hence:

$$r \simeq \Delta q = \frac{\hbar^2}{mZe^2}$$

the radius of Bohr's first orbit.

مساوي تقريباً [illegible] (0.53 Å)

Example / The ground state energy of a linear Harmonic oscillator is given by:

$$E = \frac{P_x^2}{2m} + \frac{1}{2} m\omega^2 x^2$$

Assume that the particle is confined in a region of dimension $\sim a$, i.e.,

$$x \sim \Delta x \cong a$$

then, according to uncertainty principle,

$$p_x \sim \Delta p_x \cong \frac{\hbar}{2a}$$ (displacement in a period $= 2a$)

Thus, we have:

$$E = \frac{\hbar^2}{8ma^2} + \frac{1}{2} m \omega^2 x^2 \quad \text{—— (1)}$$

The ground state will be the state of lowest energy for which $dE/da = 0$, so:

$$\frac{dE}{da} = \frac{-\hbar^2}{4ma^3} + m\omega^2 a = 0$$

$$\therefore \quad a^4 \cong \frac{\hbar^2}{4m^2\omega^2} \quad \boxed{\text{or}} \quad a \cong \left(\frac{\hbar}{2m\omega}\right)^{1/2} \quad \text{—— (2)}$$

The minimum energy is given by:
(Substituting eq (2) into eq (1))

$$E = \frac{\hbar^2}{8m} \times \frac{2m\omega}{\hbar} + \frac{1}{2} m\omega^2 a^2$$

$$= \frac{\hbar\omega}{4} + \frac{1}{2} m\omega^2 \times \frac{\hbar}{2m\omega} \quad ; \quad \boxed{a^2 = \frac{\hbar}{2m\omega}}$$

$$\therefore \quad E = \frac{1}{2}\hbar\omega$$

This is really the ground state energy of the linear harmonic oscillator (known as the zero point energy)

$x \sim \Delta x \cong a$

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