# Geometric Optics

[اكتب العنوان الفرعي للمستند]

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# Optics II

## Reflection of Light by Spherical Mirrors

A spherical mirror can be defined as a mirror that has a reflecting surface part of a hollow sphere of glass. In general, spherical mirrors can be divided into two types, concave mirrors and convex mirrors.

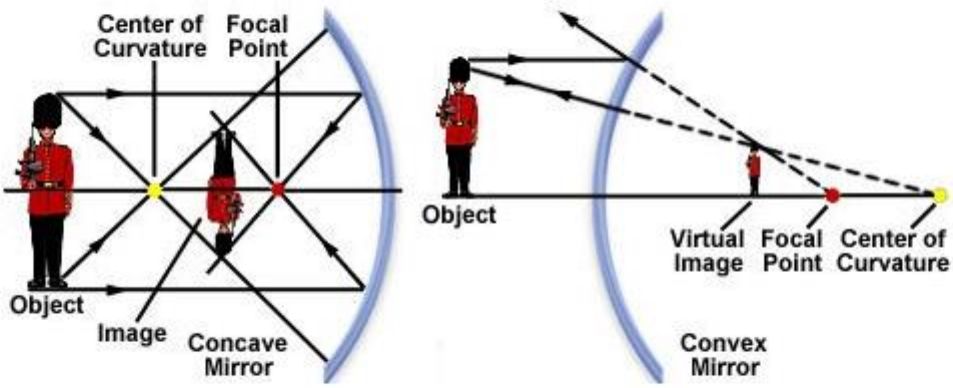



**Fig. (6): Image formed by concave and convex mirror.**

## Terminology of Spherical Mirrors

**1)** Radius of curvature (**r**): It is the distance between the pole and the center of curvature.

**2)** Center of curvature (**C**): The center of curvature of a spherical mirror is the point at the center of the mirror that passes through the curve of the mirror and has the same tangent and curvature at this point.

**3)** Aperture: This is the point where light reflection actually occurs.

**4)** **vertex** (**V**) (or Pole **P**): The pole is the midpoint of the mirror. It is twice the focus.

**5)** Focal point (**F**): Any point where light rays parallel to the main axis converge after reflecting off the mirror.

**6)** Principal axis (**A**): It is an imaginary line which passes through the optical center and the center of curvature of the spherical mirror.

**7)** Focal length (**f**): On the axis of the mirror, rays parallel to the axis converge after being reflected or refracted.

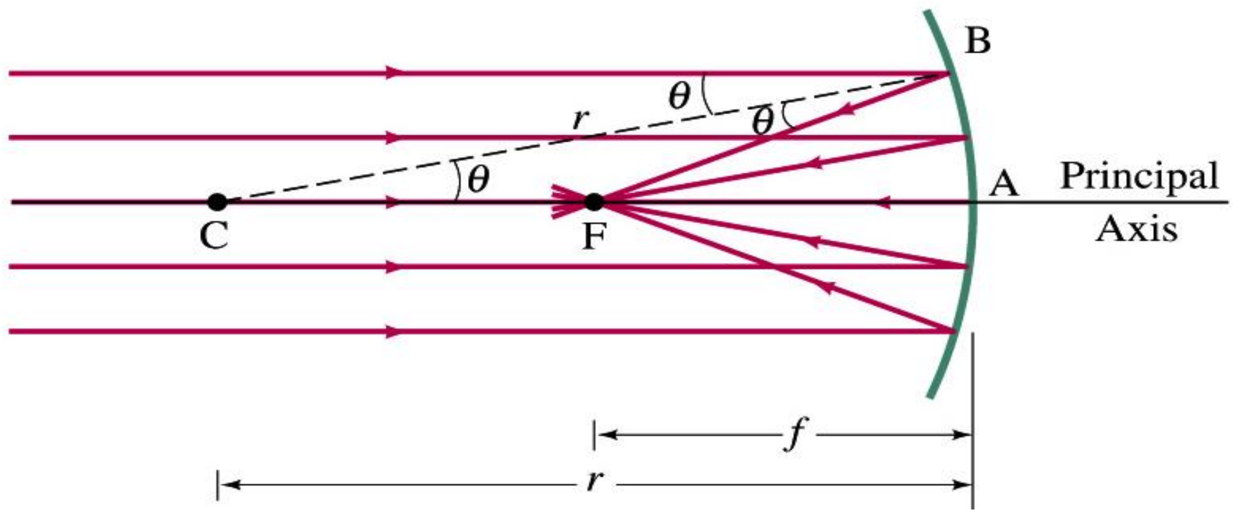



**Fig. (7): Spherical mirrors.**

## Types of Spherical Mirrors

There are two types of spherical mirrors:

**1) Concave Mirror**: Concave mirrors are also called converging mirrors, because in these types of mirrors, light rays converge at a point after impact and reflect back from the reflective surface of the mirror. If these reflected rays extend behind the mirror through the dotted line, they will meet at one point.

**2) Convex Mirror:** The convex mirror has a reflective surface that is curved outward. Regardless of the distance between the subject and the mirrors, these mirrors are "always" virtual, upright and reduced. When parallel rays hit the mirror, they are reflected in a diffuse or divergent manner. Therefore, the convex mirror is also a divergent mirror.

## Types of Images

**1)** Real image: When the image and the mirror are formed on the same side, the image is called a real image. In other words, if the image can be thrown onto the screen, it is a real image.

**2)** Virtual image: If the image cannot be thrown onto the screen, but is formed on the other side of the mirror by the external extension of the light, the formed image is called a virtual image.

## Image Formation by Concave Mirror

The object's position in relation to a concave mirror affects the type and characteristics of the image formed. Different scenarios result in different types of images:

1) **Object at Infinity**: A real and inverted image is formed at the focus when the object is placed at infinity. The size of the image is significantly smaller than that of the object.

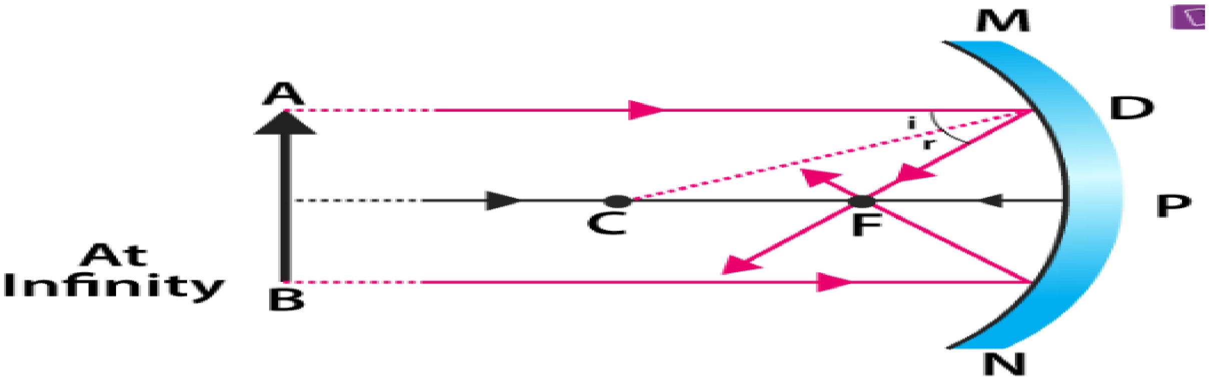



**Fig. (8): first state of concave mirror.**

**2) Object Beyond the Centre of Curvature**: When the object is positioned beyond the center of curvature, a real image is formed between the center of curvature and the focus. The size of the image is smaller compared to that of the object.

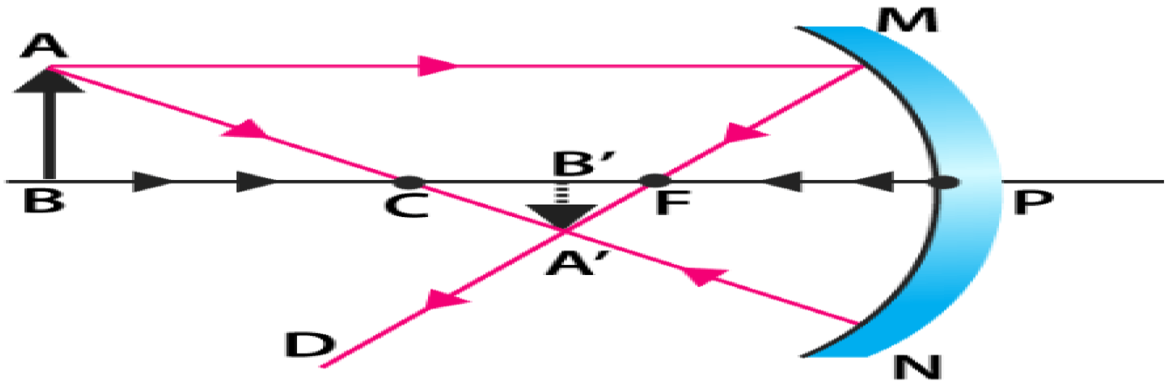



**Fig. (9): secind state of concave mirror.**

**3) Object at the Centre of Curvature or Focus**: When the object is placed at the center of curvature, or the focus, a real image is formed at the center of curvature. The size of the image remains the same as that of the object.

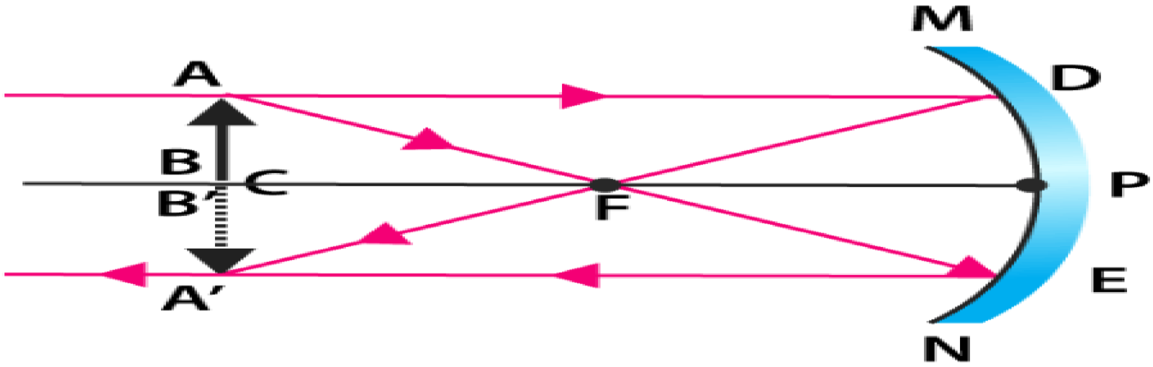



**Fig. (10): third state of concave mirror.**

4) **Object Between the Centre of Curvature and Focus**: If the object is located between the center of curvature and the focus, a real image is formed behind the center of curvature. The size of the image is larger compared to that of the object.

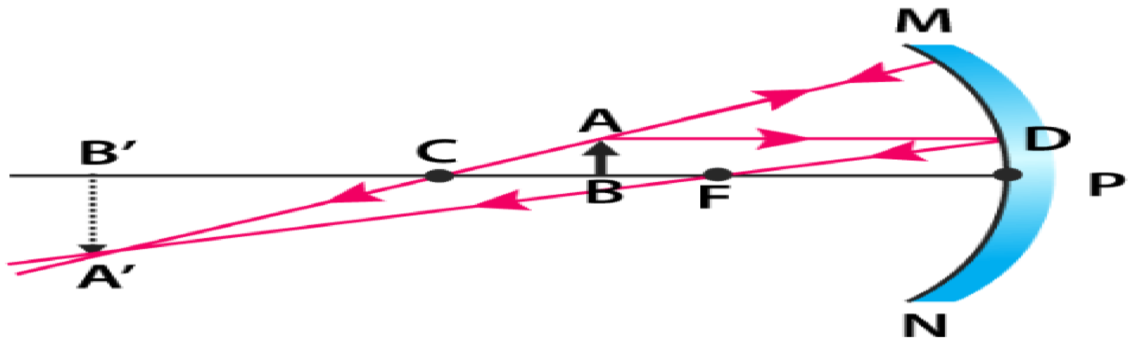



**Fig. (11): fourth state of concave mirror.**

**5) Object at the Focus:** When the object is positioned exactly at the focus, a real image is formed at infinity. The size of the image is much larger than that of the object.

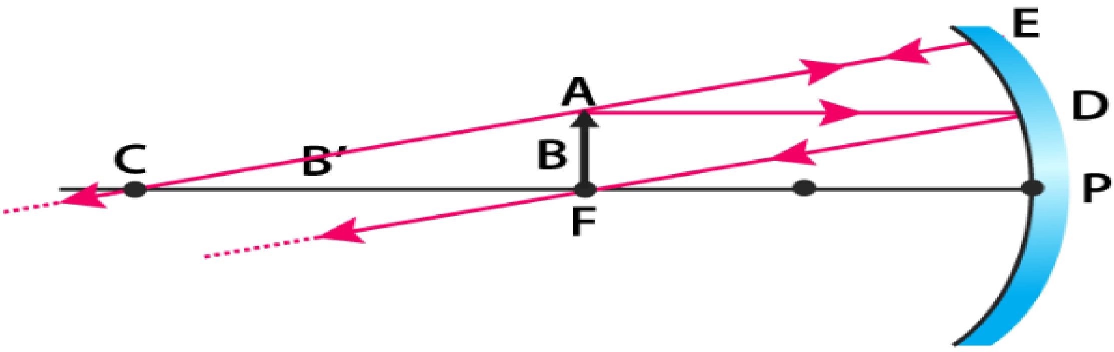



**Fig. (12): fifth state of concave mirror.**

**6) Object Between the Focus and the Pole**: Placing the object between the focus and the pole results in the formation of a virtual and erect image. The size of the image is larger compared to that of the object.

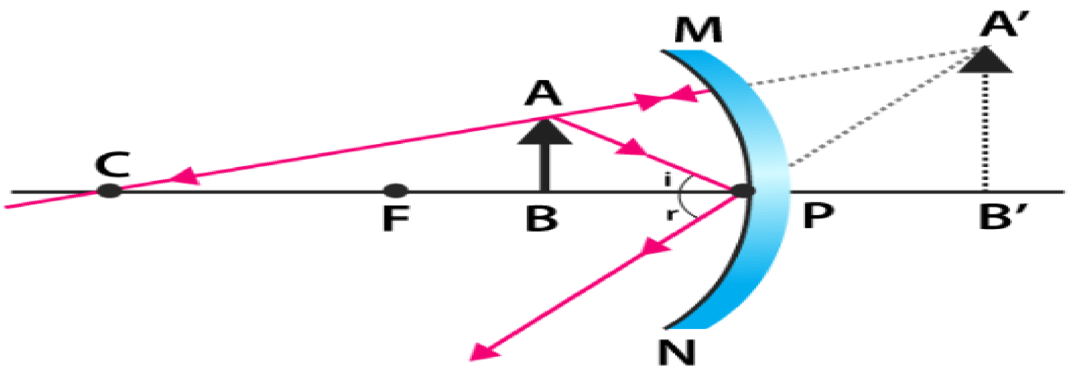



Fig. (13): seventh state of concave mirror.

## Image Formation by Convex Mirror

A convex mirror produces specific characteristics in the images formed. Let's explore the types of images formed by a convex mirror.

**1) Object at Infinity:** When the object is positioned at infinity, a virtual image is formed at the focus of the convex mirror. The size of the image is significantly smaller than that of the object.

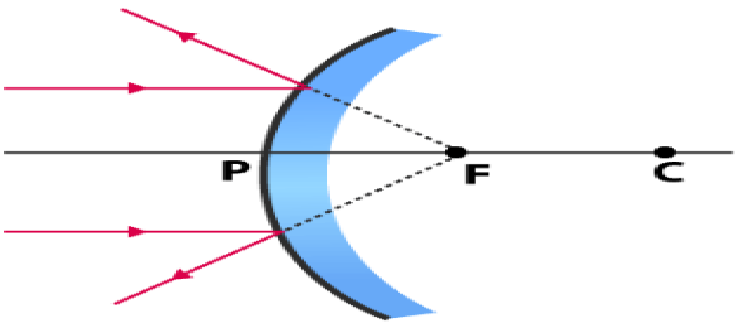



Fig. (14): First state of convex mirror.

**2) Object at a Finite Distance:** When an object is placed at a finite distance from the mirror, a virtual image is formed between the pole and the focus of the convex mirror. The size of the image is smaller than compared to that of the object.

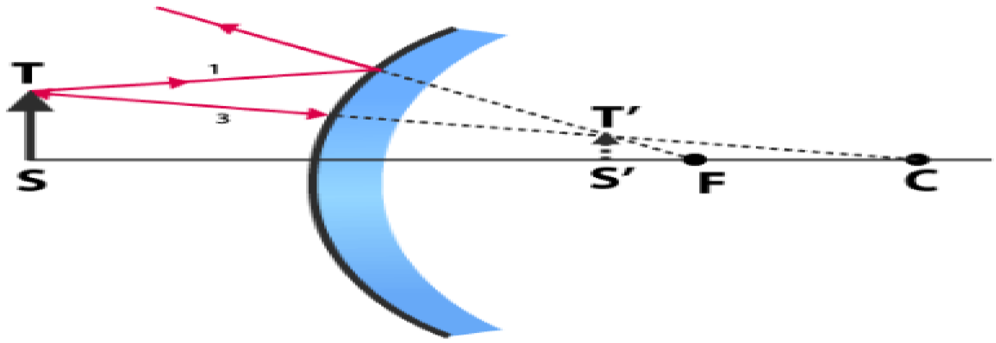



**Fig. (15): second state of convex mirror.**

It's important to note that in both cases, the images formed by a convex mirror are always virtual and erect. The nature of a convex mirror causes light rays to diverge upon reflection, creating virtual images with reduced sizes. Understanding these principles helps us accurately predict the characteristics of images formed by convex mirrors.

## The Mirror Equation

For a plane mirror, the image formed has the same height and orientation as the object, and it is located at the same distance behind the mirror as the object is in front of the mirror. Although the situation is a bit more complicated for curved mirrors, using geometry leads to simple formulas relating the object and image distances to the focal lengths of concave and convex mirrors.

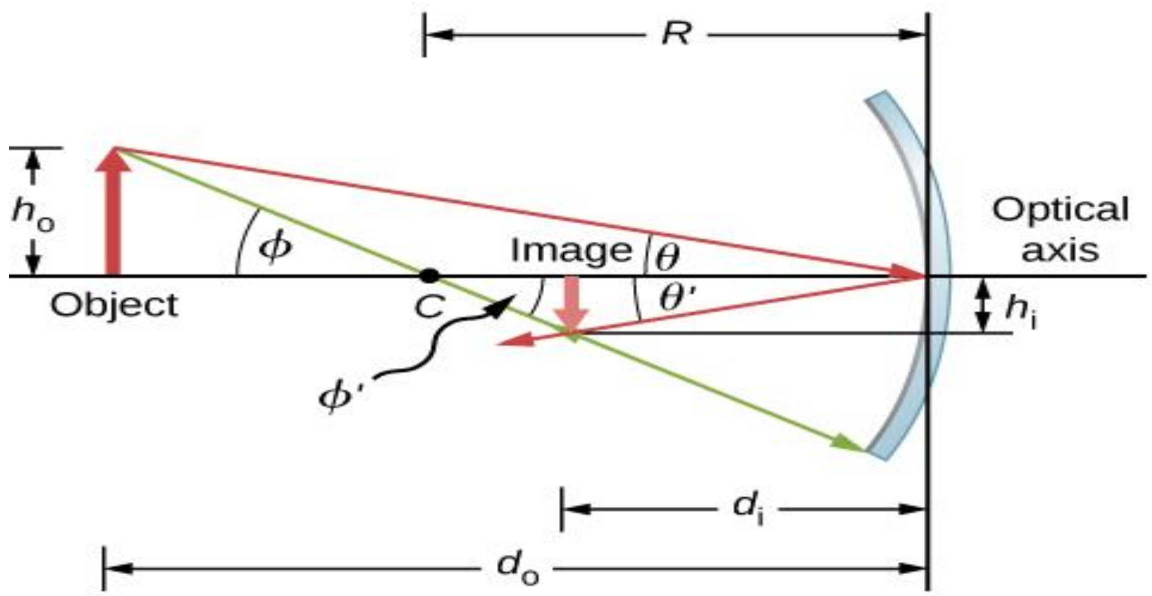



**Fig. (16): Image formed by a concave mirror**

Consider the object (OP) shown in Figure (16). The center of curvature of the mirror is labeled (c) and is a distance (R) from the vertex of the mirror, as marked in the figure. The object and image distances are labeled (do) and (di), and the object and image heights are labeled ($h_o$) and ($h_i$), respectively. Because the angles ($\phi$) and ($\phi'$) are alternate interior angles, we know that they have the same magnitude. However, they must differ in sign if we measure angles from the optical axis, so ($\phi=-\phi'$). An analogous scenario holds for the angles ($\theta$) and ($\theta'$). The law of reflection tells us that they have the same magnitude, but their signs must differ if we measure angles from the optical axis. Thus, ($\theta=-\theta'$). Taking the tangent of the angles ($\theta$) and ($\theta'$), and using the property that ($\tan(-\theta)=-\tan\theta$) gives us:

$$\left.\begin{array}{l} \tan\theta = \dfrac{h_0}{d_0} \\ \tan\theta' = -\tan\theta = \dfrac{h_i}{d_i} \end{array}\right\} = \frac{h_0}{d_0} = -\frac{h_i}{d_i} \qquad (1-5)$$

$$\frac{h_0}{h_i} = -\frac{d_0}{d_i} \qquad (1-6)$$

Similarly, taking the tangent of ($\phi$) and ($\phi'$) gives:

$$\left.\begin{array}{l} \tan\theta = \dfrac{h_0}{d_0 - R} \\ \tan\theta' = -\tan\theta = \dfrac{h_i}{R - d_i} \end{array}\right\} = \frac{h_0}{d_0 - R} = -\frac{h_i}{R - d_i} \qquad (1-7)$$

$$-\frac{h_0}{h_i} = \frac{d_0 - R}{R - d_i} \qquad (1-7)$$

Combining equation (1-5) and (1-7) gives:

$$\frac{d_0}{d_i} = \frac{d_0 - R}{R - d_i} \qquad (\mathbf{H.W}) \qquad (1-8)$$

After a little algebra, this becomes:

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{2}{R} \qquad (1-9)$$

No approximation is required for this result, so it is exact. The focal length of a spherical mirror is one-half the radius of curvature of the mirror, or (f=R/2). Inserting this into equation (1-8) gives the mirror equation:

$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f} \qquad \textbf{Mirror Equation} \qquad (1-10)$$

## Image Magnification

To interpret the derivation of the mirror equation, we found that the object and image heights are related by:

$$-\frac{h_0}{h_i}=\frac{d_0}{d_i} \qquad (1-11)$$

The object and image distances are both positive. The highest point of the object is above the optical axis, so the object height is positive. The image, however, is below the optical axis, so the image height is negative. Thus, this sign convention is consistent with our derivation of the mirror equation.

Equation (1-11) in fact describes the linear magnification (often simply called "**magnification**") of the image in terms of the object and image distances. We thus define the dimensionless magnification (m) as follows:

$$m=\frac{h_i}{h_0} \qquad \textbf{linear magnification} \qquad (1-12)$$

If (m) is positive, the image is upright, and if (m) is negative, the image is inverted, and if (|m|>1) the image is larger than the object, and if (|m|<1) the image is smaller than the object.

**Example 1:** If you find the focal length of the convex mirror formed by the cornea, then you know its radius of curvature (it's twice the focal length). The object distance is (do=12cm) and the magnification is (m=0.032). First find the image distance (di) and then solve for the focal length (f).

Solution:

From the equation for magnification (Equation 12) and solving for (di):

$$d_i=-md_0=-(-0.032)(12\text{ cm})=-0.384\text{ cm}$$

$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f} \quad \Rightarrow$$

$$f=\left(\frac{1}{d_0}+\frac{1}{d_i}\right)^{-1}=\left(\frac{1}{12\ \text{cm}}+\frac{1}{-0.384\ \text{cm}}\right)^{-1}$$

$$f=-40\ \text{cm}$$

The radius of curvature is twice the focal length, so:

$$R=2f=-80\ \text{cm}$$

**Example 2**: A real image formed by a concave mirror. A (2 cm) high object is placed (7.1 cm) from a concave mirror whose radius of curvature is (10.2 cm). Find the location of the image and its size?

Solution:

$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{2}{R} \quad \Rightarrow \quad \frac{2}{10.2}=\frac{1}{7.1}+\frac{1}{d_0}$$

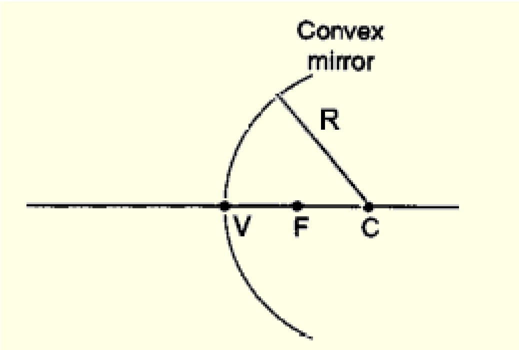
$d_0=18\ \text{cm}$ real image since positive

$$\frac{h_0}{h_i}=-\frac{d_0}{d_i} \quad \Rightarrow \quad \frac{h_0}{2}=-\frac{18}{7.1}$$

$h_0=5.1\ \text{cm} \quad \Rightarrow$ magnified and inverted.

## Homeworks:

1) In the diagram shown above, C is located at the center of the mirror and F is the virtual principal focus. Which equation correctly describes the relationship between the mirror's focal length and its radius?




a) f=R/2

b) f=2R

c) F=R/4

d) f=4R

2) What is the radius of curvature of a concave mirror that magnifies an object by a factor of (+3.2) when the object is placed (20) cm from the mirror?

3) A convex mirror is needed to produce an image one-half the size of an object and located (36) cm behind the mirror. What focal length should the mirror have?

4) A concave mirror has a radius of curvature of (26) cm. An object that is (2.4) cm tall is placed (30) cm from the mirror. a. Where is the image position? b. What is the image height?

5) An object is (30) cm from a concave mirror of (15) cm focal length. The object is (1.8) cm tall. Use the mirror equation to find the image position. What is the image height?

## Fermat principle (Law of reflection)

Assume we want light to get from point (A) to point (B), subject to some boundary condition. For example, we want the light to bounce off a mirror on its way from (A) to (B). Fermat's principle states that Light follows the path of least time.

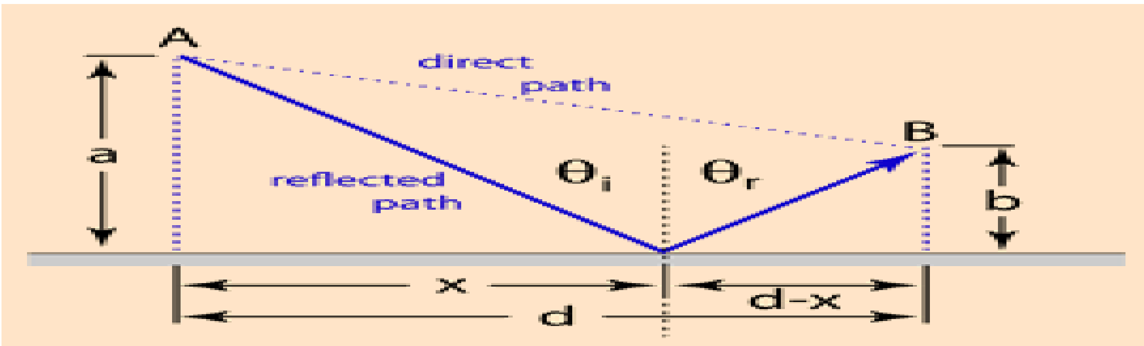



**Fig. (18): Fermat principle for law of reflection.**

The path length from A to B is:

$$L = \sqrt{a^2 + x^2} + \sqrt{b^2 + (d - x)^2} \qquad (1-15)$$

Since the speed is constant, the minimum time path is simply the minimum distance path. This may be found by setting the derivative of (L) with respect to (x) equal to zero.

$$\frac{dL}{dx} = \frac{1}{2}\frac{2x}{\sqrt{a^2 + x^2}} + \frac{1}{2}\frac{2(d - x)(-1)}{\sqrt{b^2 + (d - x)^2}} = 0 \qquad (1-16)$$

This reduces to:

$$\frac{x}{\sqrt{a^2+x^2}} = \frac{(d-x)}{\sqrt{b^2+(d-x)^2}}, \quad \text{which is } (\sin\theta_i = \sin\theta_r) \qquad (1-17)$$

$$\theta_i = \theta_r \qquad \textbf{Law of reflection} \qquad (1-18)$$

## Total Internal Reflection of Light (TIR)

When light passes from an optically denser to a rarer medium, then at the interface of the two media one part of the light reflect back in denser medium and other part passes through the rarer medium. If the angle of incidence is gradually increased then the angle of refraction also increases and for a particular value of angle of incidence, the angle of refraction becomes ($90^0$). That particular value of angle of incidence in the denser medium is called as critical angle and is denoted by ($\theta_c$).

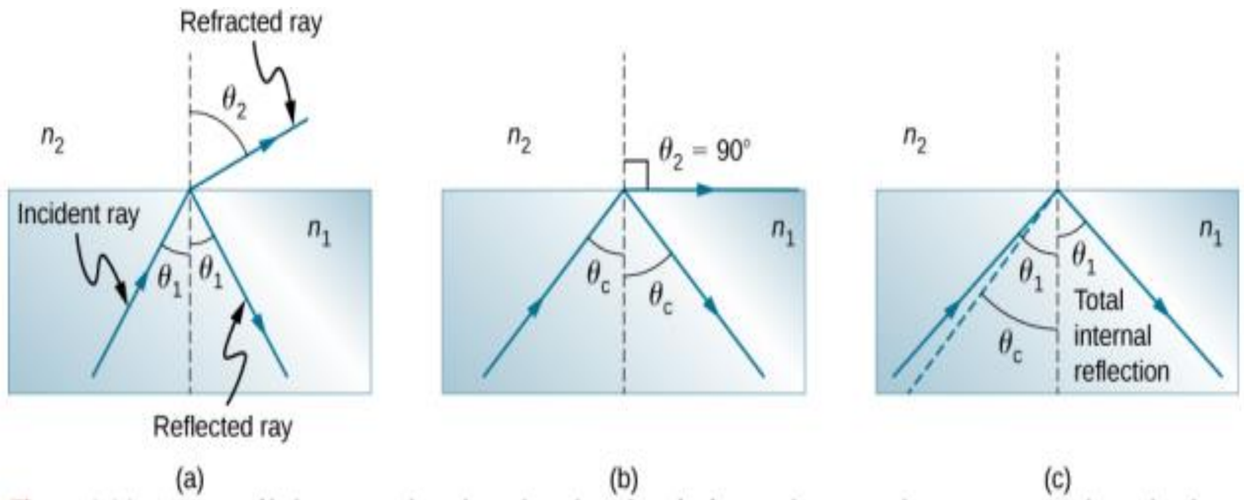



**Fig. (19): (a) A ray of light crosses a boundary where the index of refraction decreases. That is (n2 < n1). The ray bends away from the perpendicular. (b) The critical angle (θc) is the angle of incidence for which the angle of refraction is (90°). (c) Total internal reflection occurs when the incident angle is greater than the critical angle.**

Snell's law states the relationship between angles and indices of refraction. It is given by:

$$n_1 \sin\theta_i = n_2 \sin\theta_r$$

When the incident angle equals the critical angle ($\theta_1 = \theta_c$), the angle of refraction is ($\theta_2 = 90^0$). Noting that ($\sin 90^0 = 1$), Snell's law in this case becomes:

$$n_1 \sin\theta_c = n_2 \sin$$

The critical angle ($\theta_c$) for a given combination of materials is thus:

$$\theta_c = \sin^{-1}(\frac{n_2}{n_1}), \text{for } n_1 > n_2 \qquad (1-19)$$

When the angle of incidence becomes greater than the critical angle then the light ray reflects back totally to the denser medium following the laws of reflection. This phenomenon is known as total internal reflection. Necessary conditions of total internal reflection:

i) Light must travel from denser to rarer medium.
ii) The angle of incidence must be greater than the critical angle.

**Example 1:** What is the critical angle for light traveling in a polystyrene (a type of plastic) pipe surrounded by air? The index of refraction for polystyrene is (1.49).

Solution: The index of refraction of air can be taken to be (1.00). Thus, the condition that the second medium (air) has an index of refraction less than the first (plastic) is satisfied, and we can use the equation:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\theta_c = \sin^{-1}(\frac{1.00}{1.49}) = \sin^{-1}(0.671) = 42.2^0$$

**Example 2:** What is the critical angle at the air water interface?

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}(\frac{1.00}{1.333}) = \sin^{-1}(0.750) = 49.6^0$$

## Optical Fibers (Application of total internal reflection)

An optical fiber is a flexible, transparent strand of very pure glass that acts as a light pipe to transmit light between two ends of the fiber and find wide usage in fiber-optic communications, where it permits transmission over longer distances and at higher bandwidths (data rates) than electrical cables.

## Basic Structure of an Optical Fiber

The basic structure of an optical fiber consists of three parts; the core, the cladding, and the coating or buffer as shown in figures (1).

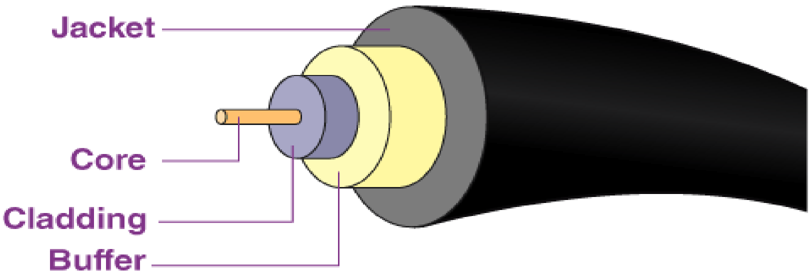



**Fig. (20): Optical fiber.**

**The core**: is a cylindrical rod of dielectric material. Dielectric material conducts no electricity. Light propagates mainly along the core of the fiber. The core is generally made of glass. The core is described as having a radius of (a) and an index of refraction (n1). The core is surrounded by a layer of material called the cladding. Even though light will propagate along the fiber core without the layer of cladding material, the cladding does perform some necessary functions. The light is "guided" down (figure 1) the core of the fiber by the optical "cladding" which has a lower refractive index that traps light in the core through (TIR).

**Cladding**: The cladding: Cladding is added to the outside of an optical fiber to reduce the amount of light that is lost. It does this by giving the light rays a second chance at (TIR) as seen in the diagram. It does increase the critical angle but the shortest path through the optical fiber is straight through, so only letting light which stays in the core means the signal is transmitted quicker. The cladding made of a dielectric material with an index of refraction ($n_2$). The index of refraction of the cladding material is less than that of the core material. The cladding is generally made of glass or plastic. The cladding is made of a dielectric material with an index of refraction (n2). The index of refraction of the cladding material is less than that of the core material. The cladding is generally made of glass or plastic.

**without clouding** $\quad n_2 = 1 \implies \sin\theta_c = \left(\frac{n_2}{n_1}\right) \implies \sin\theta_c = \frac{1}{1.5} \implies \theta_c = 41.8^0$

**with clouding** $\quad n_2 = 1.4 \implies \sin\theta_c = \left(\frac{n_2}{n_1}\right) \implies \sin\theta_c = \frac{1.4}{1.5} \implies \theta_c = 69^0$

If the cladding had a lower refractive index than the core it is easier for light to travel through so the light would bend away from the normal, Total Internal Reflection. If the cladding had a higher refractive index than the core it is harder for light to travel through so the light would bend towards the normal.

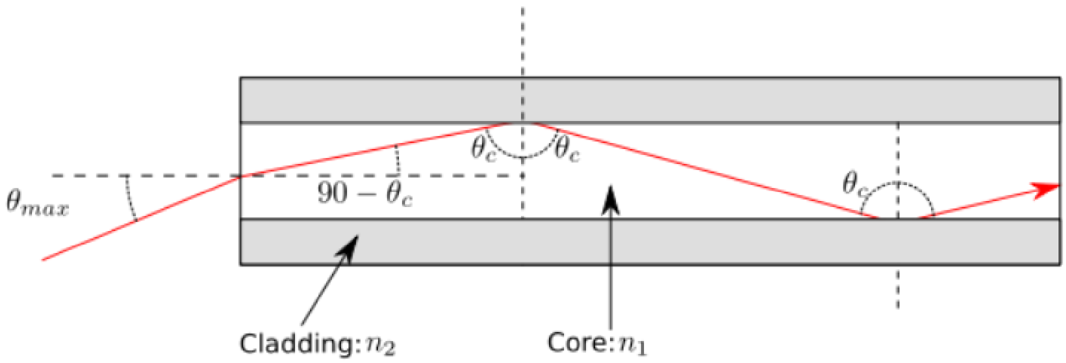



**Fig. (21): Cladding of optical fiber.**

The cladding performs the following functions:

1) Reduces loss of light from the core into the surrounding air.
2) Protects the fiber from absorbing surface contaminants .
3) Reduces scattering loss at the surface of the core.
4) Adds mechanical strength.

For extra protection, the cladding is enclosed in an additional layer called the coating or buffer.

**The coating or buffer**: is a layer of material used to protect an optical fiber from physical damage. The material used for a buffer is a type of plastic. The buffer is elastic in nature and prevents abrasions. The buffer also prevents the optical fiber from scattering losses caused by microbends. Microbends occur when an optical fiber is placed on a rough and distorted surface.

They have three very big advantages over old-style copper cables:

1) Less attenuation: (signal loss) Information travels roughly (10) times further before it needs amplifying which makes fiber networks simpler and cheaper to operate and maintain.

2) No interference: Unlike with copper cables, there's no "crosstalk" (electromagnetic interference) between optical fibers, so they transmit information more reliably with better signal quality.

**3)** Higher bandwidth: As we've already seen, fiber-optic cables can carry far more data than copper cables of the same diameter.

## Types of optical fibers:

Optical fibers carry light signals down them in what are called modes.

**Single-mode:** It is the simplest type of optical fiber. It has a very thin core about (5-10) microns (millionths of a meter) in diameter. In a single-mode fiber, all signals travel straight down the middle without bouncing off the edges. Cable TV, Internet, and telephone signals are generally carried by single-mode fibers, wrapped together into a huge bundle. Cables like this can send information over (100 km).

**Multi-mode:** Each optical fiber in a multi-mode cable is about (10) times bigger than one in a single-mode cable. This means light beams can travel through the core by following a variety of different paths in other words, in multiple different modes. Multi-mode cables can send information only over relatively short distances and are used to link computer networks together.

**Graded-index:** is a compromise between the large core diameter and N.A. of multimode fiber and the higher bandwidth of single-mode fiber. With creation of a core whose index of refraction decreases parabolically from the core center toward the cladding, light travelling through the center of the fiber experiences a higher index than light travelling in the higher modes. This means that the higher-order modes travel faster than the lower-order modes, which allows them to "catch up" to the lower-order modes, thus decreasing the amount of modal dispersion, which increases the bandwidth of the fiber.

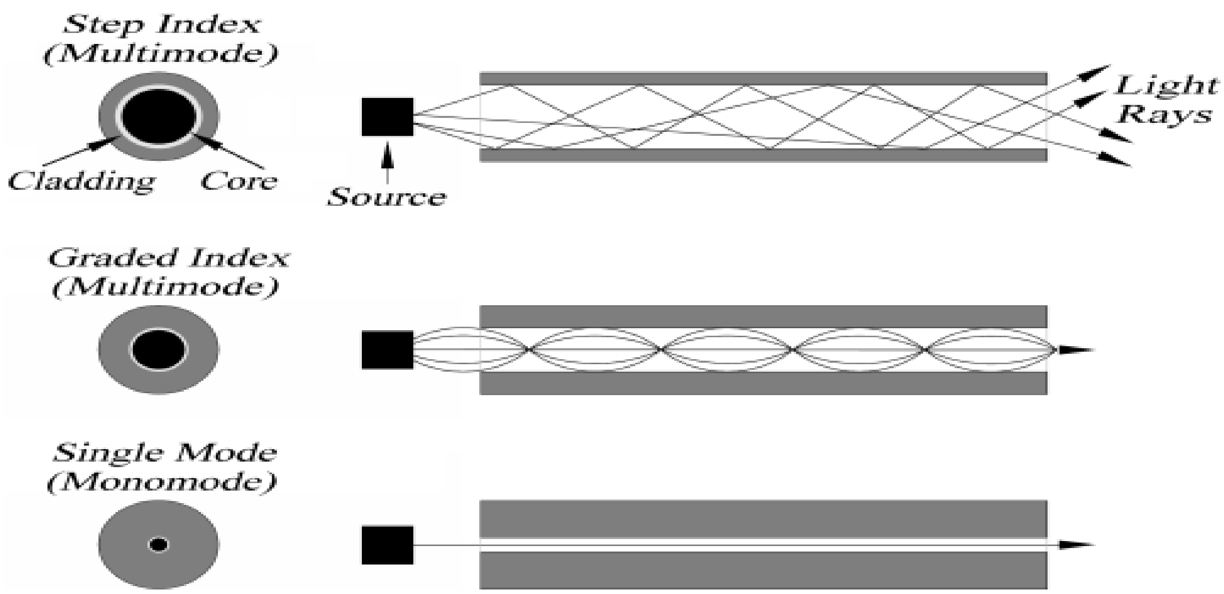



**Fig. (22): Modes of optical fibers.**

## Optical fibers applications:

1) **Medical industry**: Because of the extremely thin and flexible nature, it used in various instruments to view internal body parts by inserting into hollow spaces in the body. It is used as lasers during surgeries, endoscopy, microscopy and biomedical research.
2) **Communication**: Telecommunication to copper wires, fiber optics cables are has major uses of optical fiber cables for transmitting and receiving purposes. It is used in various networking fields and even increases the speed and accuracy of the transmission data. Compared.
3) **Defense**: Fiber optics is used for data transmission in high level data security fields of military and aerospace applications. These are used in wirings in aircrafts, hydrophones for SONARs and Seismic applications.
4) **Lightening and Decorations**: economical and easy way to illuminate the area and that is why; it is widely used in decorations and christmas trees.
5) **Mechanical Inspections**: engineers use optical fibers to detect damages and faults which are at hard to reach places. Even plumbers use optical fibers for inspection of pipes.

## Advantages of optical fibers

1) Less attenuation (order of 0.2 db/km).

2) Small in diameter and size and light weight.

3) Low cost as compared to copper (as glass is made from sand, the raw material used to make optical fiber is free.

4) Greater safety and immune to moisture and corrosion.

5) Flexible and easy to install in tight conducts.

6) Zero resale value (so theft is less).