(1)

$23.8 = 0.238 \times 10^{2}$

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التقريب القطع والتقريب

Error Analysis:-

we assume that machine numbers are represented in the normalized decimal floating-point form

$\pm 0.d_1 d_2 d_3 \ldots d_k \times 10^{k}$

$1 \leq d_1 \leq 9$, $0 \leq d_i \leq 9$, for each $i = 2, 3, \ldots, k$

Ex:-

$x = 284.60541$, $y = 0.0324807$

$x = 0.28460541 \times 10^{3}$; $y = 0.324807 \times 10^{-1}$

Numbers of this form are called k-digit decimal machine numbers.

Any positive number (real) within a numerical range of the machine can be normalized to the form

$y = 0.d_1 d_2 \ldots d_k d_{k+1} d_{k+2} \ldots \times 10^{n}$

There are two ways of performing this termination

$\frac{1}{3} = 0.33333\ldots \times 10^{0}$

one method, called chopping is to simply (chop-off) the digits d_{k+1}, d_{k+2}

$y^* = 0.d_1 d_2 \ldots d_k * 10^{n}$

The other method called rounding, adds $\boxed{5 \times 10^{n-(k+1)}}$ to y and then chops the result to obtain a number of the form

$y^* = 0.\delta_1 \delta_2 \ldots \delta_k \times 10^{n}$

For rounding when $d_{k+1} \geq 5$ we add 1 to d_k that is round up. when $d_{k+1} < 5$ we simply chop off all $d_{k+1}, d_{k+2}, \ldots$ that is round down. then $\delta_i = d_i$

(8 5-digit k-digit $5\times10^{-n}(4)$

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Ex:- Determinate the five-digit of π by using ⓐ chopping ⓑ rounding

Sol: $\pi = 0.31415|9265\ldots \times 10^{1}$

ⓐ $\pi^* = 0.31415\times10^{1} = 3.1415$ ونفلتها كلها باسم

ⓑ $\pi^* = 0.31416\times10^{1} = 3.1416$ ← التقريب

Def^n:- The error that is Produced when a calculator or computer is used to Perform real-number calculations is called round-off error

Def^n:- Suppose that x^* is approximation of x.

① The absolute error (e_x) is defined by

$$e_x = |x - x^*|$$ الخطأ المطلق

② The relative error (R_x) is defined by أو الخطأ النسبي

$$R_x = \frac{|x-x^*|}{|x|} = \frac{e_x}{|x|}, \quad x \neq 0$$

or

$$R_x = \frac{|x-x^*|}{|x^*|} = \frac{e_x}{|x^*|}, \quad x \neq 0$$

Ex:- Determine the absolute and relative errors, when approximating x by x^* for the following

ⓐ $x = 0.3000\times10^{1}$ and $x^* = 0.3100\times10^{1}$

ⓑ $x = 0.3000\times10^{-3}$ and $x^* = 0.3100\times10^{-3}$

ⓒ $x = 0.3000\times10^{4}$ and $x^* = 0.3100\times10^{4}$

Sol ⓐ $e_x = |x - x^*| = 0.1\times10^{0}$, $R_x = \frac{e_x}{|x|} = 0.3333\bar{3}\times10^{-1}$

ⓑ $e_x = |x - x^*| = 0.1\times10^{-4}$, $R_x = \frac{e_x}{|x|} = 0.3333\bar{3}\times10^{-1}$

ⓒ $e_x = 0.1\times10^{3}$, $R_x = 0.333\bar{3}\times10^{-1}$

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Effect of round off error on the operation of arithmetic

Let x^* and y^* are approximations of x and y respectively

اقتراب

1) The addition

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$$e_{(x+y)} = |(x+y)-(x^*+y^*)| = |x-x^*+y-y^*|$$
$$\le |x-x^*|+|y-y^*| = e_x+e_y$$

$$e_{(x+y)} \le e_x+e_y$$

$$R_{(x+y)} = \frac{e_{(x+y)}}{|x+y|} \le \frac{1}{|x+y|}(e_x+e_y) * \frac{|x\cdot y|}{|x\cdot y|}$$

$$= \frac{|x|\cdot|y|}{|x+y|}\left(\overbrace{\frac{e_x}{|x|\cdot|y|}}^{R_x}+\overbrace{\frac{e_y}{|x|\cdot|y|}}^{R_y}\right)$$

$$= \frac{|x|\cdot|y|}{|x+y|}\left(\frac{1}{|y|}R_x+\frac{R_y}{|x|}\right)$$

$$= \frac{1}{|x+y|}(|x|\cdot R_x+|y|\cdot R_y)$$

$$R_{(x+y)} \le \frac{1}{|x+y|}(|x|\cdot R_x+|y|\cdot R_y)$$

2) The subtract

الطرح

متأكده او صح

$$e_{(x-y)} \le e_x+e_y$$

$$R_{(x-y)} \le \frac{1}{|x-y|}(|x|R_x+|y|R_y)$$

$$R_{(x-y)} = \frac{e_{(x-y)}}{|x-y|} \le \frac{1}{|x-y|}(e_x-e_y)*\frac{|x\cdot y|}{|x\cdot y|} = \frac{|x\cdot y|}{|x-y|}\cdot\left(\frac{e_x}{|x||y|}-\frac{e_y}{|x|\cdot|y|}\right)$$

$$= \frac{|x||y|}{|x-y|}\cdot\left(\frac{R_x}{|y|}-\frac{R_y}{|x|}\right) = \frac{1}{|x-y|}\cdot(|x|R_x-|y|R_y)$$

$$\therefore R_{(x-y)} \le \frac{1}{|x-y|}\cdot(|x|R_x-|y|R_y)$$

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Ex:- Find the bound of exact value ⓐ $x+y$, ⓑ $x-y$ if $x^* = 23.86$ and $y^* = 0.01762$ where x and y rounding four digit

Sol:

$x^* = 0.2386 \times 10^{2}$

$y^* = 0.1762 \times 10^{-1}$

$x^* + y^* = 0.2386 \times 10^{2} + 0.1762 \times 10^{-1} = 23.87762$

$e_{x+y} \leq e_x + e_y$

$e_x = 5 * 10^{x-(k+1)} = 5 * 10^{2-(4+1)} = 5 * 10^{-3}$ نقرب

$e_y = 5 * 10^{-1-(4+1)} = 5 * 10^{-6}$

$e_{x+y} = 5 * 10^{-3} + 5 * 10^{-6} \Rightarrow e_{x+y} = 0.005005$

$x+y = (x^* + y^*) \pm e_{x+y} = 23.87762 \pm 0.005005$

$\therefore (x+y) \in [(x^* + y^*) - e_{x+y}, (x^* + y^*) + e_{x+y}]$

$(x+y) \in [23.872615, 23.882625]$

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③ the multiplication

$$e_x = |x - x^*| = \begin{cases} x - x^* & ; x \geq x^* \\ -(x - x^*) & ; x < x^* \end{cases}$$

$$= \begin{cases} x - x^* & ; x \geq x^* \\ x^* - x & ; x < x^* \end{cases}$$

$$e_y = |y - y^*| = \begin{cases} y - y^* & ; y \geq y^* \\ -(y - y^*) & ; y < y^* \end{cases}$$

$$= \begin{cases} y - y^* & ; y \geq y^* \\ y^* - y & ; y < y^* \end{cases}$$

$$e_{(x.y)} = |x.y - x^*.y^*|$$

① if $x \geq x^*$ and $y \geq y^* \Rightarrow$

$e_x = x - x^*$ and $e_y = y - y^*$

$\Rightarrow x = e_x + x^*$ and $y = e_y + y^*$

$$\therefore e_{(x.y)} = |(e_x + x^*)(e_y + y^*) - x^* y^*|$$

$$= |e_x e_y + x^* e_y + y^* e_x + \cancel{x^* y^*} - \cancel{x^* y^*}|$$

$$= |e_x e_y + x^* e_y + y^* e_x| \leq e_x . e_y + |x^*| e_y + |y^*| e_x$$

Since $0 < e_x \lll 1$ and $0 < e_y \lll 1 \Rightarrow e_x . e_y \simeq 0$

$$e_{(x.y)} = |x^*| e_y + |y^*| e_x$$

② if $x \geq x^*$ and $y < y^*$

$\rightarrow x = e_x + x^*$ and $y = y^* - e_y$

$$e_{x.y} = |(e_x + x^*)(y^* - e_y) - x^* y^*|$$

$$= |y^* e_x - e_x . e_y + \cancel{x^* y^*} - x^* e_y - \cancel{x^* y^*}|$$

$$\leq |y^*| e_x + \underbrace{e_x . e_y}_{\simeq 0} + |x^*| e_y$$

$$\therefore e_{x.y} \leq |y^*| e_x + |x^*| e_y$$

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if $x < x^*$ and $y \geq y^*$

$e_x = x^* - x$ and $e_y = y - y^*$

$x = x^* - e_x$ and $y = e_y + y^*$

$e_{x.y} = |(x^* - e_x).(e_y + y^*) - x^* y^*|$

$= |x^* e_y + \cancel{x^* y^*} - e_x.e_y - y^* e_x - \cancel{x^* y^*}|$

$\leq |x^*| e_y + \underbrace{e_x . e_y}_{=0} + |y^*| e_x$

$\therefore e_{x.y} \leq |x^*| e_y + |y^*| e_x$

if $x < x^*$ and $y < y^*$

$e_x = x^* - x$ and $e_y = y^* - y$

$x = x^* - e_x$ and $y = y^* - e_y$

$e_{x.y} = |(x^* - e_x).(y^* - e_y) - x^* . y^*|$

$= |\cancel{x^* y^*} - x^* e_y - y^* e_x + e_x . e_y - \cancel{x^* . y^*}|$

$\leq |x^*| e_y + |y^*| e_x + \underbrace{e_x . e_y}_{=0}$

$\therefore e_{x.y} \leq |x^*| e_y + |y^*| e_x$

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$$R_{x\cdot y} = \frac{e_{x\cdot y}}{|x^* y^*|} \le \frac{|y^*| e_x + |x^*|}{|x^*|\,|y^*|}$$

$$= \frac{e_x}{|x^*|} + \frac{e_y}{|y^*|} = R_x + R_y$$

$$\boxed{\therefore R_{x\cdot y} \le R_x + R_y}$$

القسمة

4 The Division

$$e_{x/y} = \left| \frac{x}{y} - \frac{x^*}{y^*} \right|$$

$$e_x = \begin{cases} x - x^*, & x \ge x^* \\ x^* - x, & x < x^* \end{cases} \qquad e_y = \begin{cases} y - y^*, & y \ge y^* \\ y^* - y, & y < y^* \end{cases}$$

if $x \ge x^*$ and $y \ge y^* \Rightarrow x = e_x + x^*$ and $y = e_y + y^*$

$$\therefore e_{x/y} = \left| \frac{e_x + x^*}{e_y + y^*} - \frac{x^*}{y^*} \right| = \left| \frac{e_x + x^*}{y^*\left(\frac{e_y}{y^*} + 1\right)} - \frac{x^*}{y^*} \right|$$

$$= \left| \left(\frac{e_x}{y^*} + \frac{x^*}{y^*} \right) \left(1 + \frac{e_y}{y^*} \right)^{-1} - \frac{x^*}{y^*} \right| \quad \text{--- (1)}$$

$$(1+z)^{-1} = \frac{1}{1+z} = 1 - z + z^2 - z^3 \dots \quad \text{(Binormal theorem)}$$

$$\therefore \left(1 + \frac{e_y}{y^*}\right)^{-1} = 1 - \frac{e_y}{y^*} + \frac{e_y^2}{y^{*2}} - \frac{e_y^3}{y^{*3}} + \dots$$

Since $0 \le e_y \ll 1 \to e_y^2 \simeq 0,\ e_y^3 \simeq 0, \dots$

Substitute (2) into (1) we have

باقي الحالات
دا اكتشفوها

$$e_{x/y} = \left| \left(\frac{e_x}{y^*} + \frac{x^*}{y^*} \right) \left(1 - \frac{e_y}{y^*} \right) - \frac{x^*}{y^*} \right|$$

$$e_{x/y} = \left| \frac{e_x}{y^*} - \frac{e_x \cdot e_y}{y^{*2}} + \cancel{\frac{x^*}{y^*}} - \frac{x^*}{y^{*2}} e_y - \cancel{\frac{x^*}{y^*}} \right|$$

$$\boxed{e_{x/y} \le \frac{e_x}{|y^*|} + \frac{|x^*|}{|y^{*2}|} e_y}$$

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$$R_{x/y} = \frac{e_{x/y}}{|x^*/y^*|} \leqslant \frac{\frac{e_x}{|y^*|} + \frac{|x^*|}{|y^{*2}|} e_y}{\frac{|x^*|}{|y^*|}}$$

$\therefore$ $\boxed{R_{x/y} \leqslant R_x + R_y}$

Ex:- find the bound of exact value ⓐ $x-y$ ⓑ x/y if $x^* = 2.1956$ and $y^* = 3.4781$ where x and y rounding five-digit.

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The Error in function Evaluation

If x^* is approximation of x and e_x is absolute error in x^*. Suppose e_f is absolute error in value of function f at point x, that is

$$e_f = |f(x) - f(x^*)| = |f(x^* + e_x) - f(x^*)| \quad (1)$$

$x = x^* + e$

$$f(x^* + e_x) = f(x^*) + \frac{e_x}{1!} f'(x^*) + \frac{e_x^2}{2!} f''(x^*) + \dots$$

$$f(x^* + e_x) \simeq f(x^*) + e_x f'(x^*) \quad (2)$$

$$e_f = |\cancel{f(x^*)} + e_x f'(x^*) - \cancel{f(x^*)}|$$

$$\boxed{\therefore e_f = e_x |f'(x^*)|}$$

Ex:- Find bounds of exact value of the function $f(x) = \cos x$ where $x^* = 0.359$

$$e_f = e_x |f'(x^*)|$$

$$f(x) = \cos x \Rightarrow f'(x) = -\sin(x)$$

$$f(x^*) \pm e_f$$

$$f(x) \in [f(x^*) - e_f, f(x^*) + e_f]$$

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Chapter 2

Solution of Nonlinear Equations

حل المعادلات غير الخطية

Given a function $f: \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x)=0$.

In this chapter we determined roots (solutions) of equation $f(x)=0$ or (Zeros of function)

Definition (1): Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of real numbers that converges to x. If there are positive constants c and β and a integer $n \geq 1$ such that $|x_{n+1}-x| \leq c|x_n - x|^{\beta}$ then we say the rate of convergence is of order β.

Remarks:-

1. If $\beta=1$ then we say that the rate of convergence is linear.
2. If there exists a sequence $\{c_n\}$, $c_n \longrightarrow 0$ as $n \longrightarrow \infty$ such that $|x_{n+1}-x| = c_n|x_n - x|$ or $\lim\limits_{n\to\infty} \frac{|x_{n+1}-x|}{|x_n - x|} = 0$, then we say that the rate of convergence superlinear.
3. If $\beta=2$ then we say that the rate of convergence is quadratic.

Def:-(2) suppose $\{B_n\}_{n=1}^{\infty}$ is a sequence known to converge to zero and $\{x_n\}_{n=1}^{\infty}$ converge to x. If there exist a positive constant c and an integer $n \geq 1$ such that $\boxed{|x_n - x| \leq c|B_n|}$, then we say $\{x_n\}$ converges to x with rate convergence $O(B_n)$ and write $\boxed{x_n = x + O(B_n)}$

(This real "big ~~th~~ oh of B_n)

Ex:- Compar the convergence behavior the sequences $\{x_n\} = \frac{n+1}{n^2}$ and $\{y_n\} = \frac{n+3}{n^3}$

Sol. Note that both $\lim_{n\to\infty} x_n = 0$ and $\lim_{n\to\infty} y_n = 0$.

Let $\alpha_n = \frac{1}{n}$ and $\beta_n = \frac{1}{n^2}$

$$|x_n - 0| = \frac{n+1}{n^2} \leq \frac{n+n}{n^2} = \frac{2}{n} = 2\alpha_n$$

$$|y_n - 0| = \frac{n+3}{n^3} \leq \frac{n+3n}{n^3} \leq \frac{4}{n^2} = 4\beta_n$$

$$|x_n - 0| \leq 2|\alpha_n| = 2 \cdot \frac{1}{n}$$

$$|y_n - 0| \leq 4|\beta_n| = 4 \cdot \frac{1}{n^2}$$

Hence $\{x_n\}$ with rate convergence $o(\frac{1}{n})$ and $\{y_n\}$ with rate convergence $o(\frac{1}{n^2})$. This show that the sequence $\{y_n\}$ convergence to 0 much faster than the sequence $\{x_n\}$.

* In the following sections we discusse the (numerical methodes) can be used to approximate solutions (roots) of nonlinear equations

طريقة تقسيم (اي تقسيم الفترة)

① Bi section Method :-

The Bisection method is based on the intermediate value theorem the idea behaind the mathod is that $f(x) \in C[a,b]$ and $f(a) \cdot f(b) < 0$ then there exist a root $P \in (a,b)$ such that $f(P) = 0$

خوارزمية طريقة التقسيم

(Algorthim (Bisection method))

① Input a, b and ϵ

② If $f(a) \cdot f(b) > 0$ then stop (does not exist root)

③ $P = \frac{a+b}{2}$

④ If $f(a) \cdot f(P) \leq 0$ then $b = P$

⑤ If $f(a) \cdot f(P) > 0$ then $a = P$

⑥ If $|b-a| \geq \epsilon$ then go to ③

or $f(P) \geq \epsilon$

⑦ Print P

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Example:: find the root (solution) of the equation $x^3+4x^2-10=0$ on $[1,2]$ where $\epsilon=10^{-3}$.

Solution:- $a=1$, $b=2$, $\epsilon=10^{-3}$

$f(x)=x^3+4x^2-10$

$f(a)=f(1)=-5$, $f(b)=14$

$f(a)\cdot f(b)=f(1)\cdot f(2)<0$

$p=\frac{1+2}{2}=1.5$

$f(p)=2.375$

$f(a)\cdot f(p)=f(1)\cdot f(1.5)=-5*2.375<0$

$b=1.5$

$f(p)=2.375>10^{-3}$

$p=\frac{a+b}{2}=\frac{1+1.5}{2}=1.25$

$f(p)=-1.79687$

$f(a)\cdot f(p)=f(1)\, f(1.25)>0$

$a=1.25$

$|f(p)|=|-1.79687|>\epsilon=10^{-3}$

$p=1.3652\cancel{3}4175$

$f(p)=0.000072\Rightarrow |f(p)|<\epsilon=10^{-3}$

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Theorem ①:- Suppose that $f\in C[a,b]$ and $f(a)\cdot f(b)<0$. The bisection method generates a sequence $\{P_n\}_{n=1}^{\infty}$ approximating to p a zero p of f $(f(p)=0)$ with

$$|P_n-p|\leq\frac{b-a}{2^n} \quad \text{when } n\geq 1$$

Proof:-

$b_2-a_2=\frac{1}{2}(a-b)$

$b_3-a_3=\frac{1}{2}(b_2-a_2)=\frac{1}{2}\left(\frac{1}{2}(b-a)\right)$

$b_4-a_4=\frac{1}{2^3}(b-a)$ $=\frac{1}{2^{n-1}}(b-a)$

$b_n-a_n=\frac{1}{2^{n-1}}(b-a)$

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Since $b_2 - a_2 = \frac{1}{2}(b_1 - a_1) = \frac{1}{2}(b-a)$

$b_3 - a_3 = \frac{1}{2}(b_2 - a_2) = \frac{1}{2^2}(b-a)$

$\vdots$

$b_n - a_n = \frac{1}{2}(b_{n-1} - a_{n-1}) = \frac{1}{2^{n-1}}(b-a)$

Then for each $n \geq 1$, we have $b_n - a_n = \frac{1}{2^{n-1}}(b-a)$ and $P \in (b_n, a_n)$.

Since $P_n = \frac{1}{2}(a_n + b_n)$ for all $n \geq 1$, it follow that

$|P_n - P| \leq \frac{1}{2}|(b_n - a_n)| = \frac{1}{2}\left|\left(\frac{1}{2^{n-1}}(b-a)\right)\right| = \frac{1}{2^n}|b-a|$

$\therefore \boxed{|P_n - P| \leq \frac{1}{2^n}|b-a|}$

Note:-

① Since $|P_n - P| \leq \frac{1}{2^n}|b-a|$ then the sequence $\{P_n\}_{n=1}^{\infty}$ converges to P with rate of convergence $O(\frac{1}{2^n})$ that is $P_n = P + O(\frac{1}{2^n})$

② The rate of convergence is linear رسم التقارب خطي

③ Since $\frac{|b-a|}{2^n} \leq \epsilon$

$\Rightarrow \frac{|b-a|}{\epsilon} \leq 2^n$ $\quad \ln\left(\frac{|b-a|}{\epsilon}\right) \leq \ln(2^n)$

$\Rightarrow n\ln(2) \geq \ln\left(\frac{|b-a|}{\epsilon}\right) \Rightarrow n \geq \frac{\ln\left(\frac{|b-a|}{\epsilon}\right)}{\ln(2)}$

* $f(x) = x^3 + 4x^2 - 10$, $[1,2]$, $\epsilon = 10^{-3}$

$n \geq \frac{\ln\left(\frac{|2-1|}{10^{-3}}\right)}{\ln(2)} = \frac{3\ln(10)}{\ln(2)} \Rightarrow n \geq 9.9658$ عدد التكرارات

$\boxed{n \simeq 10}$

شرح طريقة نيوتن هندسياً تكتب هذه الرسم

نقترب للحل بشكل خط مستقيم

طريقة نيوتن (خط المماس)

② **Newton's Method**

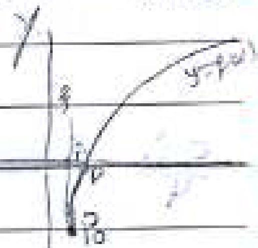
التفسير الهندسي لنيوتن

$m = \frac{f(P_1) - f(P_0)}{P_1 - P_0}$

$P_2 = P_1 - \frac{f(P_1)}{f'(P_1)}$

$f'(P_0) = \frac{f(P_1) - f(P_0)}{P_1 - P_0}$

$f'(P_0) = \frac{-f(P_0)}{P_1 - P_0}$



$\boxed{P_{n+1} = P_n - \frac{f(P_n)}{f'(P_n)}, \; n \geq 0}$ القانون

$P_1 = P_0 - \frac{f(P_0)}{f'(P_0)}$

$f'(P_n) \neq 0$

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تقريب الجذر ونبني الدقيقة مشتق

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تقريب

Suppose that $f \in C^2[a,b]$, let $P_0 \in [a,b]$ be approximation to P (P is exact root, zero of f). Such that $f'(P_0) \neq 0$ and $|P-P_0|$ is small. Consider the first Tylor's polynomial for $f(x)$ expand about P_0 and evaluated at $x = P$

$$f(P) = f(P_0) + \frac{(P-P_0)}{1!} f'(P_0) + \frac{(P-P_0)^2}{2!} f''(\xi(P_0))$$

where $\xi(P_0)$ lies between P and P_0

Since $f(P)=0$ (P is zero of f), then the equation gives

$$0 = f(P_0) + (P-P_0) f'(P_0) + \frac{(P-P_0)^2}{2} f''(\xi(P_0))$$

Newton's method is derived by assuming that $|P-P_0|$ is small then the term involving $(P-P_0)^2$ is much small

$$\Rightarrow 0 \simeq f(P_0) + (P-P_0) f'(P_0) \rightarrow P \simeq P_0 - \frac{f(P_0)}{f'(P_0)} \equiv P_1$$

$$\boxed{P_1 = P_0 - \frac{f(P_0)}{f'(P_0)}}$$

In general

$$\boxed{P_{n+1} = P_n - \frac{f(P_n)}{f'(P_n)}}, \quad n \geq 0$$

Algorithm (Newton's method)

① Input P_0, ϵ

② $P_1 = P_0 - \dfrac{f(P_0)}{f'(P_0)}$

0.001155356

10.92364005

③ $P_0 = P_1$

④ if $(f(P_0) \geq \epsilon)$ then go to ②

⑤ Print P_0

$2x^2+3x$

Example: find the approximate solution to $x^3+4x^2-10=0$ on $[1,2]$, $\epsilon = 10^{-3}$

Sol: $P_0 \in [a,b] \equiv [1,2]$

choose $P_0 = 1.5$

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$P_1 = P_0 - \frac{f(P_0)}{f'(P_0)} = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.37733$

$P_0 = P_1 = 1.3733$

$f(P_0) = f(1.3733) = 0.1343$

$\rightarrow |f(P_0)| = 0.1343 > \epsilon = 10^{-3}$

$P_1 = 1.3733 - \frac{f(1.3733)}{f'(1.3733)} = 1.3653$

$P_0 = 1.3653$

$f(P_0) = 0.00052846$

$|f(P_0)| = 0.00052846 < 10^{-3}$

$\therefore P = 1.3653$

(72)

Sub:

Date:

ملاحظة/ for تكرار كم شكل if

١- اطبع قيمة a

```
clc
clear
a=input('a=')
disp(a)
```

while مشروطة. التكرار عدد التكرارات غير معروفة

الاثنين ١٤/١٠/٢٠١٨

س/ اكتب برنامج فرعي بلغة الماتلاب لايجاد حل المعادلة $x^3+4x^2-10=0$ ضمن الفترة [1,2] وبدقة $\epsilon=10^{-3}$ وبين عدد التكرارات

```
clc    -> مسح الشاشة
clear  -> مسح الذاكرة
a=1 ; b=2 ; e=10^-3
if (f(a)*f(b)>0)
'does not exist root'
  break
  end
P=(a+b)/2 ; i=0
while (abs(f(P))>e)
 p=(a+b)/2 ;
 i=i+1 ;
if (f(a)*f(P)<0)
 b=d;
else if ( f(a)*f(P)>0)
 a=P;
else
Print(P,i)
break
end
end
P,i
```

اكتب برنامج يجد حل المعادلة $x^3+4x^2-10=0$ ضمن الفترة [1,2] وبدقة $\epsilon=10^{-3}$

```
clc
clear
a=input('a=');
b=input('b=');
c=input('c=');
if (f(a)*f(b)>0)
"does not exist root"
break
end
P=(a+b)/2
while (f(P)>=c)
P=(a+b)/2;
if (f(a)*f(b)<=0)
  b=P;
else  a=P;
  end
  end
P=1.2500
```

تختبر الدول

برنامج فرعي نفس السؤال

```
function y=f(x)
y=x^3+4*x^2-10;
end
```

اولا
ثم رئيسي
تحفظ باسم f

الغاية من تنصيف الفترة ايجاد جذور لا يمكن ايجادها بالطرق التحليلية

AL-WARAQ
الوراق

اول مناقشة

س1/

$f(x) = (x-1)^{10}$, $P = 1$, $P_n = 1 + \frac{1}{n}$

$|f(P_n)| < 10^{-3} \Rightarrow n > 1$

$|P - P_n| < 10^{-3} \Rightarrow n > 1000$

$|f(P_n)| = |(P_n - 1)^{10}| = (\frac{1}{n})^{10} = n^{-10} < 10^{-3}$

$\ln(n^{-10}) < \ln(10^{-3})$

$-10 * \ln(n) < -3 \ln(10)$

$\ln(n) > \frac{3}{10} \ln(10)$

$n > e^{\frac{3}{10}\ln(10)}$

$n > 1.99 \Rightarrow \boxed{n > 1}$

$|1 - 1 - \frac{1}{n}| < 10^{-3}$

$n^{-1} < 10^{-3}$

$-\ln(n) < -3\ln(10)$

$n > e^{3 \ln(10)}$

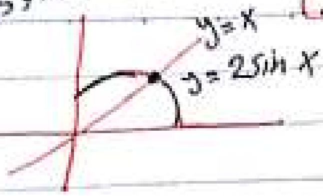
$n > e^{\ln(10^3)} = 10^3 = 1000$

$n > 1000$

س11/ Let $f(x) = (x-1)^{10}$, $P = 1$ & $P_n = 1 + \frac{1}{n}$. show that $|f(P_n)| < 10^{-3}$ whenever $n > 1$ but that $|P - P_n| < 10^{-3}$ requires that $n > 1000$

نقطة التقاطع $[1.5, 2]$ ⓐ

ملاحظات عن س*



س12/ دائماً فقط على ومالاحظات

$x = \sqrt{3}$

$x^2 = 3 \longrightarrow x^3 - 3 = 0$

$f(x) = x^3 - 3$

$\sqrt{1} < \sqrt{3} < \sqrt{4}$

$1 < \sqrt{3} < 2$

∴ $1 < x < 2$

∴ $x \in (1, 2)$

∴ $f(1)$

$f(2)$

وتكمل

الان نجد تقاريب $f(a) * f(b) < 0$ التي تكون الجذر المطلوب

س10/

دائماً فقط جذور تقريبية

∴ يبين الفترة المعطاة نعوض فقط الجذور التي تجعل الدالة اقل ولتي هي

$\{1, 2, -1, -2, 0\}$

(4)

$P \rightarrow$ exact solution (root)

$f(P)=0$

Sub: ٢٠١٧/١٠/٢٨

Date: الاربعاء

Convergence using Newton's Method

Let $e_{n+1} = P - P_{n+1} \Rightarrow P_{n+1} = P - e_{n+1}$ $\frac{e_{n+1}}{e_n^2}$ نيوتن يتقارب بشكل تربيعي

where P is the exact solution

also $P_n = P - e_n$ $e_{n+1} \simeq c\, e_n$

$|x_{n+1} - x| \le C\,|x_n - x|^P$

من نيوتن

$$P_{n+1} = P_n - \frac{f(P_n)}{\bar{f}(P_n)}$$

$$\cancel{P} - e_{n+1} = \cancel{P} - e_n - \frac{f(P-e_n)}{\bar{f}(P-e_n)} \Rightarrow e_{n+1} = e_n + \frac{f(P-e_n)}{\bar{f}(P-e_n)}$$

Since $f(P-e_n) = f(P) - \frac{e_n}{1!}\bar{f}(P) + \frac{e_n^2}{2!}\bar{\bar{f}}(P) + \dots$

and

$$\bar{f}(P-e_n) = \bar{f}(P) - \frac{e_n}{1!}\bar{\bar{f}}(P) + \frac{e_n^2}{2!}\bar{\bar{\bar{f}}}(P) + \dots$$

$$\therefore e_{n+1} = e_n + \frac{f(P) - \frac{e_n}{1!}\bar{f}(P) + \frac{e_n^2}{2!}\bar{\bar{f}}(P) + \dots}{\bar{f}(P) - e_n\bar{\bar{f}}(P) + \frac{e_n^2}{2}\bar{\bar{\bar{f}}}(P) + \dots}$$

$$= e_n + \frac{\overset{0}{f(P)} - e_n\bar{f}(P) + \frac{e_n^2}{2}\bar{\bar{f}}(P) + \dots}{\bar{f}(P)\left(1 - e_n\frac{\bar{\bar{f}}(P)}{\bar{f}(P)} + \frac{e_n^2}{2}\frac{\bar{\bar{\bar{f}}}(P)}{\bar{f}(P)} + \dots\right)}$$

by using Taylor theorem

$$= e_n + \frac{\left(-e_n + \frac{e_n^2}{2}\frac{\bar{\bar{f}}(P)}{\bar{f}(P)} + \dots\right)}{1 - e_n\frac{\bar{\bar{f}}(P)}{\bar{f}(P)} + \frac{e_n^2}{2}\frac{\bar{\bar{\bar{f}}}(P)}{\bar{f}(P)} + \dots}$$

$$= e_n + \left(-e_n + \frac{e_n^2}{2}\frac{\bar{\bar{f}}(P)}{\bar{f}(P)} + \dots\right)\left(1 + e_n\frac{\bar{\bar{f}}(P)}{\bar{f}(P)} + \frac{e_n^2}{2}\frac{\bar{\bar{\bar{f}}}(P)}{\bar{f}(P)} + \dots\right)$$

$$= \cancel{e_n} - \cancel{e_n} - \cancel{e_n^2\frac{\bar{\bar{f}}(P)}{\bar{f}(P)}} \;\; \frac{e_n^3}{2}\frac{\bar{\bar{f}}(P)}{\bar{f}(P)} + \dots + \cancel{\frac{e_n^2}{2}\frac{\bar{\bar{f}}(P)}{\bar{f}(P)}} + \dots$$

نهمل

$$e_{n+1} \simeq -\frac{e_n^2}{2}\frac{\bar{\bar{f}}(P)}{\bar{f}(P)} \Rightarrow \boxed{e_{n+1} \simeq C\,e_n^2}$$

Where $C = \frac{-1}{2}\frac{\bar{\bar{f}}(P)}{\bar{f}(P)}$

$\therefore$ (The rate of convergence is quadratic (تربيعي))

Sub:

Date:

Prove that the rate of convergence of ~~Ne~~ Newton's method is quadratic.-*

طريقة القاطع

(3) The Secant Method

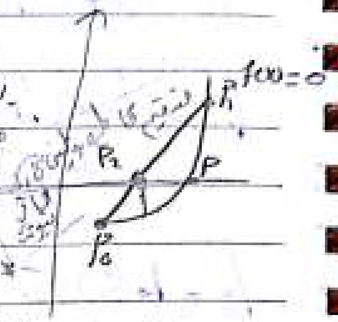
from the graph, we obtain

الفرق بين نيوتن والقاطع نجد في نيوتن نفترض P_0 ... مثل P_2 ...

حذر العامل

* اذا كانت P_2 ... ϵ ...

* بشكل عام



$$\frac{f(x)-f(P_0)}{x-P_0}=\frac{f(P_1)-f(P_0)}{P_1-P_0}$$

from the graph, we

$$\frac{f(P_2)-f(P_0)}{P_2-P_0}=\frac{f(P_1)-f(P_0)}{P_1-P_0}\Rightarrow\frac{-f(P_0)}{P_2-P_0}=\frac{f(P_1)-f(P_0)}{P_1-P_0}$$

$$-P_1f(P_0)+\cancel{P_0f(P_0)}=P_2(f(P_1)-f(P_0))-P_0f(P_1)+\cancel{P_0f(P_0)}$$

$$P_0f(P_1)-P_1f(P_0)=P_2(f(P_1)-f(P_0))$$

$$P_2=\frac{P_0f(P_1)-P_1f(P_0)}{f(P_1)-f(P_0)}\rightarrow P_3=\frac{P_1f(P_2)-P_2f(P_1)}{f(P_2)-f(P_1)}$$

In general

$$\boxed{P_{n+1}=\frac{P_{n-1}f(P_n)-P_nf(P_{n-1})}{f(P_n)-f(P_{n-1})}}$$

Algorithm (secant method)

خوارزمية القاطع

1. Input P_0, P_1, ϵ
2. $P_2=\frac{P_0f(P_1)-P_1f(P_0)}{f(P_1)-f(P_0)}$
3. $P_0=P_1$, $P_1=P_2$
4. If $|f(P_1)|\geq\epsilon$ then go to (2)
5. Print P_2

Sub:

Date:

Example: find the solution of $x^3+4x^2-10=0$ on $[1,2]$, $\epsilon=10^{-3}$

Sol Let $P_0=1.5$, $P_1=1.9$ $\quad (P_0, P_1 \in [1,2])$

$$P_2=\frac{P_0 f(P_1)-P_1 f(P_0)}{f(P_1)-f(P_0)}=\frac{1.5 * f(1.9)-1.9 * f(1.5)}{f(1.9)-f(1.5)}$$

$P_2=1.3935$

$|f(P_2)|=|f(1.3935)|=|0.4741|=0.4741>\epsilon=10^{-3}$

$P_0=P_1=1.9$, $P_1=P_2=1.3935$

$$P_2=\frac{1.9\ f(1.3935)-1.3935 * f(1.9)}{f(1.3935)-f(1.9)} \Rightarrow P_2=1.3414$$

$P_0=P_1=1.3935$, $P_1=P_2=1.3414$

$|f(P_1)|=0.1016>\epsilon=10^{-3}$

$P_2=1.3653$

$|f(P_2)|=4.1666 * 10^{-6}<\epsilon=10^{-3}$

Convergence of Secant method

Let $e_{n+1}=P-P_{n+1} \rightarrow P_{n+1}=P+e_{n+1}$

$e_n=P-P_n \Rightarrow P_n=P-e_n$

$e_{n-1}=P-P_{n-1} \rightarrow P_{n-1}=P-e_{n-1}$

$$P_{n+1}=\frac{P_{n-1} f(P_n)-P_n f(P_{n-1})}{f(P_n)-f(P_{n-1})}$$

$$P-e_{n+1}=\frac{(P-e_{n-1}) f(P-e_n)-(P-e_n) f(P-e_{n-1})}{f(P-e_n)-f(P-e_{n-1})}$$

By Taylor's expansion of $f(P-e_n)$ and $f(P-e_{n-1})$, we obtain

① الثاني

(27

Sub:

$$\ldots(P-e_n)\left(f(P)+\ldots\right)$$

$$P-e_{n+1}=\frac{(P-e_{n-1})\left(f(P)-e_n\bar f(P)+\frac{e_n^2}{2}\bar{\bar f}(P)-\frac{e_n^3}{6}\bar{\bar{\bar f}}(P)+\ldots\right)-\ldots}{f(P)-e_n\bar f(P)+\frac{e_n^2}{2}\bar{\bar f}(P)+\ldots-\left(f(P)-e_{n-1}\bar f(P)+\frac{e_{n-1}^2}{2}\bar{\bar f}(P)+\ldots\right)}$$

Since $f(P)=0$ (P is exact solution)

$$\therefore P-e_{n+1}=\frac{(P-e_{n-1})\left(-e_n\bar f(P)+\frac{e_n^2}{2}\bar{\bar f}(P)+\ldots\right)-(P-e_n)\left(-e_{n-1}\bar f(P)+\frac{e_{n-1}^2}{2}\bar{\bar f}(P)+\ldots\right)}{-(e_n-e_{n-1})\bar f(P)+\frac{e_n^2-e_{n-1}^2}{2}\bar{\bar f}(P)+\ldots}$$

⋮

$$e_{n+1}\simeq C\,e_{n-1}\cdot e_n$$

مختبر الخميس 30/10/2014

اكتب برنامج فرعي بلغة matlab لايجاد الحل للمعادلة $x^3+4x^2-10=0$ في الفترة [1,2] وبدقة $\epsilon=10^{-3}$ مبين عدد التكرارات ثم

```
clc
clear
a=1 ;b=2 ; e=10^-3 ;
if (f(a)*f(b)>0)
'does not exist root'
break
end
p= (a+b)/2 ; i=0 ;
while (abs(f(p))>e)
p=(a+b)/2 ;
i=i+1 ;
if (f(a)*f(p)<0)
b=p ;
else if (f(a)*f(p)>0)
a=p ;
else
print (p,i)
break
end end → p,i
end
```

```
function y=f(x)
y=x^3+4x^2-10 ;
end
```

الناتج
p=1.3652
i=9

تحتاج البرنامج الى
① البرنامج الفرعي
② عبارة for
③ عبارة if
④ عبارة while
⑤

مثال لبيان الدالة الاتية

$f(x)=e^{x^2}-\ln(x^3+1)$

```
function y=f(x)
y=exp(x^2)-log(x^3+1);
end
```

تحتاج البرنامج وعيشان حساب المشتقة

```
p0= ; e= ;
while (abs(f(p0))>=e)
p1=p0-f(p0)/f1(p0)
p0=p1
end
```

```
function z=f1(x)
z=2*x*exp(x^2)-3x^2/(x^3+1)
end
```

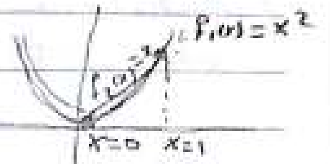
②

Sub: Date: 2012 / 11 / 5

طريقة النقطة الثابتة

④ :- Fixed - Point Iteration Method

Def :- The number P is a fixed Point for a given function g if $g(P) = P$.



ex :- $x^3 + 4x^2 - 10 = 0$, $[1,2]$

$f(x) = 0$

$x = g(x) \rightarrow f(x) = x - g(x)$

$x = g(x)$

$x_{n+1} = g(x_n)$

① $g(x) \in [a,b]$, $\forall x \in [a,b]$

② $|g'(x)| \le 1$

① $x^3 = 10 - 4x^2 \rightarrow x = \sqrt[3]{10 - 4x^2}$

$\therefore x = g_1(x) = \sqrt[3]{10 - 4x^2}$

② $4x^2 = 10 - x^3 \rightarrow x^2 = \dfrac{10 - x^3}{4}$

$x = -\dfrac{\sqrt{10 - x^3}}{2}$

$x = g_2(x) = -\dfrac{\sqrt{10 - x^3}}{2}$

③ $x = \dfrac{\sqrt{10 - x^3}}{2}$

$x = g_3(x) = \dfrac{\sqrt{10 - x^3}}{2}$

④ $x = x^3 + 4x^2 + x - 10$

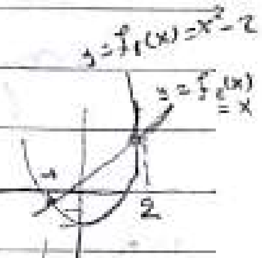
$\therefore x = g_4(x) = x^3 + 4x^2 + x - 10$

① $g_1(x) = \sqrt[3]{10 - 4x^2}$, $[1,2]$

$x = 1 \rightarrow g_1(x) = \sqrt[3]{6} \in [1,2]$

$x = 2 \rightarrow g_1(x) = \sqrt[3]{-6} \notin [1,2]$

Example :- $x^2 - x - 2 = 0$, $[a,b]$



① $x = x^2 - 2 \rightarrow x = g_1(x) = x^2 - 2$

② $x^2 = x + 2 \rightarrow x = -\sqrt{x+2}$

$x = g_2(x) = -\sqrt{x+2}$

نقاط التقاطع لم تتغير

③ $x = \sqrt{x+2}$

$x = g_3(x) = \sqrt{x+2}$

Theorem :- ① If $g \in C[a,b]$ and $g(x) \in [a,b]$ for all $x \in [a,b]$ then g has at least one fixed Point in $[a,b]$

② If in addition, $g'(x)$ exist on (a,b) and a Positive number $K < 1$ exists with $|g'(x)| \le K$ for all $x \in (a,b)$ then there is exactly one fixed Point in $[a,b]$

$0 < K < 1$

①

Sub: ٦/١١/٢٠١٤
Date: الخميس

س/ اكتب برنامج بلغة matlab لإيجاد جذر المعادلة $x^3+4x^2-10=0$ ضمن الفترة [1,2] ① باستخدام طريقة نيوتن وبدقة $\epsilon=10^{-3}$ مبيناً عدد التكرارات ② باستخدام طريقة القاطع

①

```
clc
clear
P=1.5; e=10^-3 ;i=0
while(abs(f(P))>=e)
a=P- f(P)/f1(P);
  P=a;
  i=i+1;
  end
  i
  a
```

```
function y=f(x)
y=x^3+4*x^2-10;
end
```

```
function Z=f1(x)
Z=3*x^2+2*X;
end
```

② القاطع

```
clc
clear
P=1.5; P1=1.9; e=10^-3; i=0;
while (abs(f(P1))>=e)
a=(P*f(P1)-P1*f(P))/(f(P1)-f(P));
  P=P1;
  P1=a;
  i=i+1
  end
  i
  a
```

$P_1=1.9000$

$i=1$

$i=2$

$a=1.3652$

الاثنين فقط fixed point ...
$g(P) = P$

Sub: الاثنين
Date: ٢٠١٢/١٢/٢٥

Theorem ②: — ① ...

Proofs —

If $g(a)=a$ or $g(b)=b$, then g has a fixed Point at the end Point of interval $[a,b]$. If not then $g(a)>a$ and $g(b)<b$. set $f(x) = g(x) - x$ for all $x \in [a,b]$

$f(a) = g(a) - a$ and $f(b) = g(b) - b$

$\Rightarrow f(a) > 0$ and $f(b) < 0$

By intermediate value theorem, we obtain there exists $P \in (a,b)$ for which $f(P)=0$

$\Rightarrow f(P) = g(P) - P = 0 \Rightarrow g(P) = P$

$\therefore P$ is fixed Point for g.

② suPPose, in addition, that $|g'(x)| < K < 1$, and that P and q are both fixed Points for g in $[a,b]$ $(P \neq q)$.

By the mean value theorem, we obtain there exists a number $\eta \in [a,b]$ between P and q such that $g'(\eta) = \dfrac{g(P)-g(q)}{P-q}$

$\Rightarrow |g(P) - g(q)| = |g'(\eta)|\,|P-q| < K\,|P-q| < |P-q|$

$\Rightarrow |g(P) - g(q)| < |P-q|$

Since $g(P)=P$ and $g(q)=q \Rightarrow |g(P)-g(q)| = |P-q|$

$\therefore |P-q| < |P-q|$

which is contradiction, This contradiction must come from only supposition $P \neq q$.

$\therefore P = q \Rightarrow$ The fixed Point is unique.

Example

Show that $g(x) = \dfrac{(x^2-1)}{3}$ has a unique fixed Point on $[-1,1]$

Solution:-

Since $g(x)$ is Continuous in $[-1,1]$

$g(-1) = 0 \in [-1,1]$ and $g(1) = 0 \in [-1,1]$

also for all $x \in [-1,1] \Rightarrow g(x) \in [-1,1]$

$\frac{x^3+4}{5} = \frac{x}{5}$

يونيلاب ...

Sub:

Date:

$g(x) = \frac{x^2-1}{3} \to \bar{g}(x) = \frac{2x}{3}$ the absolute maximum for g at $x=-1$ and $x=1$ and the absolute minimum for g $x=0$

$\Rightarrow |\bar{g}(x)| = \left|\frac{2x}{3}\right| \leq \frac{2}{3} < 1$ for all $x \in [-1,1]$

$\therefore g$ satisfies all the hypotheses of theorem ②

$\Rightarrow g$ has unique fixed Point in $[-1,1]$

شروط كافية وليست ضرورية

Notes:- The hypotheses of theorem ② are sufficient to garantee a unique fixed but are not necessary.

الأربعاء ٢٠١٧/١١/٨

Example:- Show that theorem ② does not ensure a unique fixed point of $g(x) = 3^{-x}$ on $[0,1]$, even though a unique fixed point on this interval does exist

Solution:- g is continuous on $[0,1]$

$g(0) = 1 \in [0,1]$ & $g(1) = \frac{1}{3} \in [0,1]$ also for all $x \in [0,1] \to g(x) \in [0,1]$

$\frac{d}{dx} 2^{x} = 2^x \cdot \ln(2) \cdot$

$\bar{g}(x) = -\ln(3) \cdot 3^{-x}$

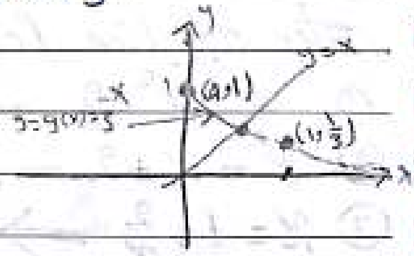
so the first part of theorem ② satisfy

$\bar{g}(0) = -\ln(3) \to |\bar{g}(0)| = \ln 3 = 1.09861 > 1$

K لكبرت

so the second part of theorem ② not satisfy.

But the function g is decreasing and it clear form figure that (نقطة تقاطع $y=x$ مع $y=3^{-x}$) the fixed Point must be unique.



$f(x) = x - g(x) = 0$

$\Rightarrow x = g(x)$ الشكل العام للتكرار

$P_n = g(P_{n-1})$ $n \geq 1$

$P_1 = g(P_0)$, $P_2 = g(P_1)$

$g(x)$ $\quad x^2 - x - 1$ $\quad x^2 = 1 + \frac{1}{x}$ $\quad x = \sqrt{1+\frac{1}{x}}$ $\quad \therefore x = g(x)$

Sub:

Date:

Algorithm of fixed-Point Iteration

① Input P_0, ϵ

② $P_1 = g(P_0)$

③ $P_0 = P_1$

④ If $(|P_0 - g(P_0)| \geq \epsilon)$ then go to ② | ⑤ Print P_1

اذا اكبر يرجع لين ما يتحقق المطلوب

Example:- Find the solution of the equation $x^3 - x - 1 = 0$, $g(x) = \sqrt{1+\frac{1}{x}}$; $P_0 = 1.5$, $\epsilon = 5\times10^{-4}$

Solution:-

$P_n = g(P_{n-1})$; $n \geq 1$

$P_1 = g(P_0) = \sqrt{1+\frac{1}{1.5}} = 1.29094 \Rightarrow |f(P_1)| = |f(1.29094)| = 0.875 > \epsilon$

$P_2 = g(P_1) = 1.33214$

$P_3 = g(P_2) = 1.32313 \Rightarrow |f(P_3)| > \epsilon = 5\times10^{-4}$

$P_4 = g(P_3) = 1.32506$

$P_5 = g(P_4) = 1.32464 \Rightarrow |f(P_5)| < \epsilon$

$P_5 = 1.32464$ هو الحل المطلوب

Example:- The equation $x^2 - x - 2 = 0$ has a unique root in $[1.5, 3]$ by using Fixed Point Iteration method find approximate of this solution accurate within 5×10^{-5}.

Solution:-

The equation $f(x) = x^2 - x - 2$ can be written in following forms

① $x = x^2 - 2 \longrightarrow x = g_1(x) = x^2 - 2$

② $x^2 = x + 2 \longrightarrow x = \sqrt{x+2} \Rightarrow x = g_2(x) = \sqrt{x+2}$

③ $x = 1 + \frac{2}{x} \Longrightarrow x = g_3(x) = 1 + \frac{2}{x}$

$P_n = g_i(P_{n-1})$; $n \geq 1$; $i = 1, 2, 3$

$P_0 = 2.5$

n	$g_1(P_{n-1})$	$g_2(P_{n-1})$	$g_3(P_{n-1})$
1	4.25	2.12132	1.8
2	1.60625	2.	2.11111
⋮			
6	1.84×10^{4}	2.000117	2.006711
7	divergence	2.000025	1.996656
	⋮		

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n	$g_1(P_{n-1})$	$g_2(P_{n-1})$	$g_3(P_{n-1})$
8	⋮	2.000007	2.001675
⋮	---		
15		2.0000000	1.999987

Theorem ③ :- Let $g \in C[a,b]$ be such that $g(x) \in [a,b]$ for all $x \in [a,b]$. Suppose that, in addition, that g' exists on (a,b) and that a constant $0 < K < 1$ exists with $|g'(x)| \leq K$ for all $x \in (a,b)$. Then for any number $P_0 \in [a,b]$, the sequence defined by $P_n = g(P_{n-1})$; $n \geq 1$ converge to the unique fixed-point P in (a,b).

Proof :-

Let $e_n = P - P_n$; $n = 0,1,2,\ldots$

Since $P_n = g(P_{n-1})$ and $P = g(P)$

$\therefore e_n = g(P) - g(P_{n-1})$

$$e_n = \frac{g(P) - g(P_{n-1})}{(P - P_{n-1})} \cdot (P - P_{n-1})$$

By using the mean value theorem, we have

$e_n = g'(\lambda_n) \cdot (P - P_{n-1})$

where $\lambda_n \in (a,b)$ and between P and P_{n-1}

$e_n = |g'(\lambda_n)| \, |P - P_{n-1}| \leq K \; e_{n-1}$ $\quad (|g'(x)| \leq K)$

$\boxed{e_n \leq K \; e_{n-1}}$

The rate of convergence of fixed-Point Iteration method is linear

$e_n \leq K \, e_{n-1}$ Similarly $e_{n-1} \leq K \; e_{n-2}$

Sub: ٢٠١٨/١١/١٧
Date: خمس الخميس

اكتب برنامج فرعي لايجاد حل المعادلة $x^2-3x-e^x+5=0$ ضمن الفترة $[1,2]$ باستخدام طريقة النقطة الثابتة وبدقة $\epsilon=5\times10^{-5}$

بالنقطة الثابتة نجعل الدالة بدلالة x فقط

$3x = x^2 - e^x + 5$
$x = (x^2 - e^x + 5)/3$

```
Function y= g(x)
y= (x^2 - exp(x)+5)/3
end

clc
clear
a=1.5;
c= 5*10^-5;
i=0;

while (abs(a-g(a))>=c)
i=i+1;
b= g(a)
a=b
end
b
i
```

```
b= 1.0752
i=8
>> b^2-3*b-exp(b)+5
>>
ans=
-4.3539e-005.
```

Sub: الأربعاء

Date: ١٩/١١/٢٠١٧

الحلول العددية لنظام المعادلات الخطية **chapter 3:-**

Numerical Solution of linear systems equations:

In this chapter we will solve a linear system of n equation in n variable. Such a system has the form

$$\left.\begin{aligned} E_1 &: a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ E_2 &: a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ &\vdots \\ E_n &: a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{aligned}\right\} \dots (1)$$

In this system we are given the constants a_{ij} for each $i,j = 1,2,\dots,n$ and b_i for each $i = 1,2,\dots,n$, and we are need to determine the unknowns $x_1, x_2, \dots, x_n$.

we can write the linear system (1) as matrix equation

$$AX = b \quad \dots (2)$$

خط يرمز له

with
$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}; \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}; \quad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

There are two kind of method to solve linear system of equations (1) or (2)

تكرارية

(1) Direct Technique

هي طرائق تعطي الحل النظام بعدد من الخطوات المنتهية

Direct techniques are methods that theoretically give the exact solution to the system in a finite number steps

(a) Gaussian Elimination

حذف كاوس

ثلاث عمليات

we used three operations to simplify the linear system given in (1)

لتبسيط نظام المعادلات رقم (1)

والغرض منه تبسيط النظام

(1) Equation E_i can be multiplied by any constant (nonzero) λ with resulting equation used in place of E_i. This operation is denoted $(\lambda E_i) \longrightarrow E_i$

(2) Equation E_j can be multiplied by any constant λ and added to equation E_i with the resulting equation used in place of E_i. This operation denoted $(E_i + \lambda E_j) \longrightarrow E_i$

(3) Equation E_i and E_j can be transposed in order. This operation denoted $E_i \longleftrightarrow E_j$

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By the sequence of these operation a linear system will be systematically transform in a new linear system that is more easily solved and has the same solution

The sequence of operations is illustrated in the following example.

Example 8– Use Gaussian elimination to solve the system

$$-3x_1+2x_2-x_3=-1$$
$$6x_1-6x_2+7x_3=-7$$
$$3x_1-4x_2+4x_3=-6$$

Solution:– $A\underline{x}=\underline{b}$

$$A=\begin{pmatrix}-3 & 2 & -1\\ 6 & -6 & 7\\ 3 & -4 & 4\end{pmatrix};\quad \underline{x}=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix};\quad \underline{b}=\begin{pmatrix}-1\\ -7\\ -6\end{pmatrix}$$

$$(A:b)^{(0)}=\left(\begin{array}{ccc:c}-3 & 2 & -1 & -1\\ 6 & -6 & 7 & -7\\ 3 & -4 & 4 & -6\end{array}\right)$$

$$E_1\longrightarrow E_1$$

$$\left(E_j-\frac{a_{ji}}{a_{ii}}E_i\right)^{(m)}\longrightarrow E_j^{(m+1)}\quad \text{for } j=i+1,\ i+2,\dots,n$$

خطوات الحل واحفظ هذا القانون

$$E_1^{(0)}\longrightarrow E_1^{(1)}$$

$$\left(E_2-\frac{a_{21}}{a_{11}}E_1\right)^{(0)}=(6\ \ -6\ \ 7:-7)-\frac{6}{-3}(-3\ \ 2\ \ -1:-1)$$
$$=(6\ \ -6\ \ 7:-7)+(-6\ \ 4\ \ -2:-2)=(0\ \ -2\ \ 5:-9)=E_2^{(1)}$$

$$\left(E_3-\frac{a_{31}}{a_{11}}E_1\right)^{(0)}=(3\ \ -4\ \ 4:6)-\frac{3}{-3}(-3\ \ 2\ \ -1:-1)=(0\ \ -2\ \ 3:-7)$$
$$=E_3^{(1)}$$

$$(A:b)^{(1)}=\left(\begin{array}{ccc:c}-3 & 2 & -1 & -1\\ 0 & -2 & 5 & -9\\ 0 & -2 & 3 & -7\end{array}\right)$$

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$E_1^{(1)} \longrightarrow E_1^{(2)}$, $E_2^{(1)} \longrightarrow E_2^{(2)}$

$$\left(E_3 - \frac{a_{32}}{a_{22}} E_2\right)^{(1)} = (0 \;\; -2 \;\; 3 : -7) - \frac{-2}{-2}(0 \;\; -2 \;\; 5 : -9)$$
$$= (0 \;\; 0 \;\; -2 : 2)$$

$$(A:b)^{(2)} = \left(\begin{array}{ccc:c} -3 & 2 & -1 & -1 \\ 0 & -2 & 5 & -9 \\ 0 & 0 & -2 & 2 \end{array}\right)$$

$$\begin{pmatrix} -3 & 2 & -1 \\ 0 & -2 & 5 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -9 \\ 2 \end{pmatrix}$$ by using back substitution

$$x_i = \frac{b_i - \sum_{j=i+1}^{n} a_{ij} x_j}{a_{ii}} \quad , \quad x_n = \frac{b_{nn}}{a_{nn}}$$

$$x_3 = \frac{b_{33}}{a_{33}} = \frac{2}{-2} = -1$$ يطلب ناقص وفقط

$$x_2 = \frac{b_2 - \sum_{j=3}^{3} a_{2j} x_j}{a_{22}} = \frac{-9 - 5 * (-1)}{-2} = 2$$

$$x_1 = \frac{b_1 - \sum_{j=2}^{3} a_{1j} x_j}{a_{11}} = 2$$

$$x_1 = \frac{b_1 - [a_{12} x_2 + a_{13} x_3]}{a_{11}}$$
$$= \frac{-1 - [2(2) + (-1)(-1)]}{-3}$$
$$= \frac{-6}{-3} = 2$$

محتاج أحزرك وصن رتاج

sol. $\underline{x} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

٢٠١٤/١١/٢٠ مختبر الخميس

كتابة ممولتين المشترك

$\Rightarrow a = [R \quad Y] \Leftarrow$

هذا لم نتشغف (تاج الأعطيا) لعمل $A(i,:) = A(j,:)$ لأن عمل ($A(i,:)$) ستساويان

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Date:

كيف نبدل صف مكان صف اخر

طريقة كرامر

التيار برنامج مجال حل النظام

$$-3x_1 + 2x_2 - x_3 = -1$$
$$6x_1 - 6x_2 + 7x_3 = -7$$
$$5x_1 - 4x_2 + 4x_3 = -6$$

$$x_1 = \frac{|A_1|}{|A|}, \quad x_2 = \frac{|A_2|}{|A|}, \quad x_3 = \frac{|A_3|}{|A|}$$

$$A = \begin{pmatrix} -3 & 2 & -1 \\ 6 & -6 & 7 \\ 5 & -4 & 4 \end{pmatrix}, \quad b = \begin{pmatrix} -1 \\ -7 \\ 6 \end{pmatrix}$$

الاربعاء 2014/11/26

Partial Pivoting :- الارتكاز الجزئي

$$x_n = \frac{b_n^{(k)}}{a_{nn}^{(k)}}$$

نستخدم الارتكاز الجزئي تكبير المعادلات حتى تكون عناصر القطر كبيرة

Example :- Solve the system

$$x_2 + x_3 = 6$$
$$x_1 - 2x_2 - x_3 = 4$$
$$x_1 - x_2 + x_3 = 5$$

Using Gaussian elimination with Partial Pivoting

استخدم الاقتراح مع التبديل الجزئي

Solution :-

$$(A:b) = \begin{pmatrix} 0 & 1 & 1 & : & 6 \\ 1 & -2 & -1 & : & 4 \\ 1 & -1 & 1 & : & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & -1 & : & 4 \\ 0 & 1 & 1 & : & 6 \\ 1 & -1 & 1 & : & 5 \end{pmatrix}$$

$$|a_{11}| = 0, \quad |a_{21}| = 1, \quad |a_{31}| = 1$$

نبحث عن اكبر العمود الاول

$$(A:b)^{(0)} \longrightarrow (A:b)^{(1)}$$

الغرض من الارتكاز الجزئي هو جعل عنصر القطر اكبر

$$(A:b)^{(1)} = \begin{pmatrix} 1 & -2 & -1 & : & 4 \\ 0 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & -1 & : & 4 \\ 0 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 1 \end{pmatrix}$$

$$|a_{22}^{(1)}| = 1, \quad |a_{32}^{(1)}| = 1$$

فاختار بشكل طبيعي a_{22} لأن

AL-WARAQ

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يعني ملخص للوتكار ... نأخذ رقم أكبر
رقم العمود لوطه
(2) a_{21} و a_{31} و a_{32} ...
نجيب ان تساوي صفر

$$(A:b)^{(1)} \longrightarrow (A:b)^{(2)}$$

$$(A:b)^{(2)} \longrightarrow \left(\begin{array}{ccc:c} 1 & -2 & -1 & 4 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 1 & -5 \end{array}\right)$$

أهم شي نعتمد على أكبر ما يكون بالعمود الأول

$x_3 = -5$; $x_2 = 11$, $x_1 = 21$

طرائق تجزئة المصفوفة

(b) Decomposition matrix method.

Doolittle's method
Crout's method

$A\underline{x} = \underline{b} \rightarrow L.U.\underline{x} = \underline{b}$

$A\underline{x} = \underline{b}$

$L.(U.\underline{x}) = \underline{b}$

تجزئة A لجزئين

$U.\underline{x} = \underline{y}$ —— ①

$L.\underline{y} = \underline{b}$ —— ②

where L is lower traingular and U is upper trainglular

$L.U.\underline{x} = \underline{b}$

$$L = \begin{pmatrix} l_{11} & & & \\ l_{21} & l_{22} & & \\ l_{31} & l_{32} & l_{33} & \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{pmatrix} ; U = \begin{pmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ & u_{22} & u_{23} \cdots & u_{2n} \\ & & \ddots & \vdots \\ & & & u_{nn} \end{pmatrix}$$

First solve $L.\underline{y} = \underline{b}$ for $\underline{y}$ by forward substitution second solve $U\underline{x} = \underline{y}$ for x by backward substitution

① Doolitle's method

$A\underline{x} = \underline{b} \longrightarrow L.(U.\underline{x}) = \underline{b}$

$$\text{where } L = \begin{pmatrix} 1 & & & \\ l_{21} & 1 & & \\ l_{31} & l_{32} & 1 & \\ l_{n1} & l_{n2} & \cdots & 1 \end{pmatrix} , U = \begin{pmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ & u_{22} & \cdots & u_{2n} \\ & & & \vdots \\ & & & u_{nn} \end{pmatrix}$$

$A = L.U$

Ex:- Solve the system using doolitle's method

$$2x_1 + 4x_2 - 6x_3 = -4$$
$$x_1 + 5x_2 - 3x_3 = 10$$
$$x_1 + 3x_2 + 2x_3 = 5$$

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Solution:-

$$A=\begin{pmatrix}2&4&-6\\1&5&3\\1&3&2\end{pmatrix},\quad b=\begin{pmatrix}-4\\10\\5\end{pmatrix}$$

$$A=L\cdot U$$

$$\begin{pmatrix}2&4&-6\\1&5&3\\1&3&2\end{pmatrix}=\begin{pmatrix}1&0&0\\l_{21}&1&0\\l_{31}&l_{32}&1\end{pmatrix}\cdot\begin{pmatrix}u_{11}&u_{12}&u_{13}\\0&u_{22}&u_{23}\\0&0&u_{33}\end{pmatrix}$$

$$=\begin{pmatrix}u_{11}&u_{12}&u_{13}\\l_{21}\cdot u_{11}&l_{21}\cdot u_{12}+u_{22}&l_{21}\cdot u_{13}+u_{23}\\l_{31}u_{11}&l_{31}\cdot u_{12}+l_{32}u_{22}&l_{31}u_{13}+l_{32}u_{23}+u_{33}\end{pmatrix}$$

$u_{11}=2,\quad u_{12}=4,\quad u_{13}=-6$

$l_{21}u_{11}=1 \rightarrow l_{21}=\frac{1}{2}$

$l_{31}u_{11}=1 \rightarrow l_{31}=\frac{1}{2}$

$l_{21}\cdot u_{12}+u_{22}=5 \rightarrow \frac{1}{2}(4)+u_{22}=5 \rightarrow u_{22}=3$

$$L=\begin{pmatrix}1&0&0\\\frac{1}{2}&1&0\\\frac{1}{2}&\frac{1}{3}&1\end{pmatrix},\quad U=\begin{pmatrix}2&4&-6\\0&3&6\\0&0&3\end{pmatrix}$$

$$L\cdot y=b$$

$$\begin{pmatrix}1&0&0\\\frac{1}{2}&1&0\\\frac{1}{2}&\frac{1}{3}&1\end{pmatrix}\begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}=\begin{pmatrix}-4\\10\\5\end{pmatrix}$$

$y_1=-4,\quad y_2=12,\quad y_3=3$

$$U\cdot x=y \Rightarrow \begin{pmatrix}2&4&-6\\0&3&6\\0&0&3\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}-4\\12\\3\end{pmatrix}$$

$x_3=1,\quad x_2=2,\quad x_1=-3$

$$x=\begin{pmatrix}-3\\2\\1\end{pmatrix}$$

First thank you
must be sure
It's hard word
but I am sure
I am right when
I told you that.

If you want to
be my best
friend don't
make me
hurt from
you
because it's
not only
for me

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Date:

(2) Crout's Method :→

$$A\underline{x} = \underline{b} \longrightarrow L\cdot(U\underline{x}) = \underline{b}$$

$$L = \begin{pmatrix} l_{11} & & & \\ l_{12} & l_{22} & & \\ l_{13} & l_{23} & l_{33} & \\ l_{1n} & l_{2n} & \cdots & l_{nn} \end{pmatrix}, \quad U = \begin{pmatrix} 1 & U_{12} & \cdots & U_{1n} \\ & 1 & U_{23} & U_{2n} \\ & & 1 & U_{3n} \\ & & & 1 \end{pmatrix}$$

Examples :→ نفس المثال السابق واحسب ولكن نأخذ احادي العلوية

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Date:

اكتب برنامج لايجاد حل النظام

$2x_1 + 3x_2 - x_3 = 5$

$4x_1 + 4x_2 - 3x_3 = 3$

$-2x_1 + 3x_2 - x_3 = 1$

باستخدام طريقة كاوس للحذف

الحل/

A=[2 3 -2 ;

b=

C= [a b]

```
>> C(2,:) = C(2,:) - C(2,1)/C(1,1)
>> C(3,:) = C(3,:) - C(3,1)/C(1,1)
for j=1:2;
for i=j+1:3;
C(i,:) = C(i,:) - C(i,j)/C(j,j) * C(j,:)
end
end
C(3,:) = C(3,:) - C(3,2)/C(2,2) * C(2,:)
X(3) = C(3,4)/C(3,3)
X(2)=(C(2,4) - C(2,3) * X(3))/C(2,2)
X(1) = (C(1,4) - C(1,2) * X(2) - C(1,3) * X(3))/C(1,1)
X(3)= C(3,4)/C(3,3)
for j=2:-1:1
X(j)=(C(j,4) - C(j,3) * X(3)) C(j,j)
end
X(j) = (C(j,4) - C(j,j+1:3) * X(j+1:3))/C(j,j)
```

Sub:

Date: تمرين -

Solve the system

$x_1 + x_2 + 3x_4 = 4$

$2x_1 + x_2 - x_3 + x_4 = 1$

$3x_1 - x_2 - x_3 + 2x_4 = -3$

$-x_1 + 2x_2 + 3x_3 - x_4 = 4$

using Gaussian elimination

Solution:-

$$A:B = \left[\begin{array}{cccc:c} 1 & 1 & 0 & 3 & 4 \\ 2 & 1 & -1 & 1 & 1 \\ 3 & -1 & -1 & 2 & -3 \\ -1 & 2 & 3 & -1 & 4 \end{array}\right]$$

$$\left(E_j \ \frac{a_{ji}}{a_{ii}} \ E_i\right)^m$$

$(2 \ 1 \ -1 \ 1 : 1) - \frac{2}{1}(1 \ 1 \ 0 \ 3 : 4) = (0 \ -1 \ -1 \ -5 : -7)$

$(3 \ -1 \ -1 \ 2 : -3) - 3(1 \ 1 \ 0 \ 3 : 4) = (0 \ -4 \ -1 \ -7 : -15)$

$(-1 \ 2 \ 3 \ -1 : 4) + (1 \ 1 \ 0 \ 3 : 4) = (0 \ 3 \ 3 \ 2 : 8)$

$$\left[\begin{array}{cccc:c} 1 & 1 & 0 & 3 & 4 \\ 0 & -1 & 1 & -5 & -7 \\ 0 & -4 & -1 & -7 & -15 \\ 0 & 3 & 3 & 2 & 8 \end{array}\right]$$

أكمل -

يعني نتكلم بالمعلوم $x^t \cdot A \cdot x$ ؟

Sub: الاربعاء ٢٠١٢/١٤٣٤
Date:

③ Cholesky method :-

Definition:-① A matrix A is Positive definite if it is Symmetric and $\underline{x}^t \cdot A \cdot \underline{x} > 0$ for every n-dimensional colomn vector $\underline{x} \neq 0$.

$$(x_1 \quad x_2 \; \cdots \; x_n)\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$= (x_1 \quad x_2 \; \cdots \; x_n)\begin{pmatrix} \sum_{j=1}^{n} a_{1j}x_j \\ \sum_{j=1}^{n} a_{2j}x_j \\ \vdots \\ \sum_{j=1}^{n} a_{nj}x_j \end{pmatrix} = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_i x_j$$

$A = A^t$ متناظرة
$a_{21} = a_{12}$
$a_{32} = a_{23}$

Ex:- The matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ is Positive definite

solution $\underline{x} \neq 0$, $\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$(x_1 \quad x_2 \quad x_3)\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = (x_1 \quad x_2 \quad x_3)\begin{pmatrix} 2x_1 - x_2 \\ -x_1 + 2x_2 - x_3 \\ -x_2 + 2x_3 \end{pmatrix}$$

$$= 2x_1^2 - x_1x_2 - x_1x_2 + 2x_2^2 - x_2x_3 - x_2x_3 + 2x_3^2$$

$$= x_1^2 + (x_1^2 - 2x_1x_2 + x_2^2) + (x_2^2 - 2x_2x_3 + x_3^2) + x_3^2$$

$$= x_1^2 + (x_1 - x_2)^2 + (x_2 - x_3)^2 + x_3^2$$

$\therefore \underline{x}^t \cdot A \cdot \underline{x} = x_1^2 + (x_1 - x_2)^2 + (x_2 - x_3)^2 + x_3^2 > 0$

$\therefore A$ is Positive definite.

المصفوفة الجزئية الرئيسية

Definition:-② A leading Principal Submatrix of a matrix A is a matrix of the form

$$A_k = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \vdots & & & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{kk} \end{pmatrix} \text{ for some } 1 \leq k \leq n.$$

$A_1 = (a_{11})$, $A_2 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \rightarrow A_n = A$

Theorem ①:- A symetric matrix A is positive definite if and only if each of its leading principal submatrix has a positive determinat.

$|A_k| > 0$ for some $1 \leq k \leq n$

$|A_1| > 0$, $|A_2| > 0$, ... , $|A_n| > 0$

Example:- The matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ is positive definite

Solution:-

$|A_k| > 0$ for some $1 \leq k \leq n$

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

$|A_1| = |a_{11}| = 2 > 0$

$|A_2| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 2*2 - (-1)*(-1) = 3 > 0$

$|A_3| = \begin{vmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ 0 & 2 \end{vmatrix} + (0)\begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix}$

$= 2*3 + (-2) = 4 > 0$

Theorem ②:- If A is an $n \times n$ positive definite matrix then ⓐ A is nonsigular ($|A| \neq 0$)

ⓑ $a_{ii} > 0$ for each $i = 1, 2, \ldots, n$

ⓒ $\max\limits_{1 \leq k, j \leq n} |a_{kj}| \leq \max\limits_{1 \leq i \leq n} |a_{ii}|$

ⓓ $(a_{ij})^2 < a_{ii} \cdot a_{jj}$ for each $i \neq j$

Theorem 3:- A symetric matrix A is positive definite if and only if A can be factored تجزأ in the form $L \cdot L^t$ where L is lower tringular

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$$A = L.L^t \rightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} =$$

$$\begin{pmatrix} l_{11} \\ l_{21} & l_{22} \\ l_{31} & l_{32} & l_{33} \\ \vdots \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{pmatrix} \cdot \begin{pmatrix} l_{11} & l_{21} & \cdots & l_{n1} \\ & l_{22} & l_{32} \cdots & l_{n2} \\ & & & l_{nn} \end{pmatrix}$$

choleski's Algorithm Produces the $L.L^t$ factorization described in theorem ③. for all $j = 1, 2, \dots, n$

① If $i = j$

$$l_{ii} = \left(a_{jj} - \sum_{k=1}^{j-1} l_{jk}^2\right)^{\frac{1}{2}}$$

② If $i = j+1, j+2, \dots, n$

$$l_{ij} = \frac{1}{l_{jj}} \left[a_{ji} - \sum_{k=1}^{j-1} l_{ik} l_{jk}\right]$$

Ex:- Solve the system using choleski's Method.

$$x_1 - x_2 + x_3 = -2$$
$$-x_1 + 2x_2 - 3x_3 = 6$$
$$x_1 - 3x_2 + 9x_3 = 2$$

$\begin{pmatrix} 5 \\ 6 \\ 3 \end{pmatrix}$

Solution:-

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 2 & -3 \\ 1 & -3 & 9 \end{pmatrix}, \quad \underline{b} = \begin{pmatrix} -2 \\ 6 \\ 2 \end{pmatrix}$$

$A = L.L^t$ for all $j = 1, 2, 3$

$j = 1$

① If $i = j = 1 \Rightarrow l_{11} = \left(a_{11} - \sum_{k=1}^{0} l_{1k}^2\right)^{\frac{1}{2}} \Rightarrow l_{11} = (a_{11})^{\frac{1}{2}} = \sqrt{1} = 1$

② If $i = 2 \Rightarrow l_{21} = \frac{1}{l_{11}} \left(a_{12} - \sum_{k=1}^{0} \text{ del}\right) \Rightarrow l_{21} = \frac{a_{12}}{l_{11}} = -1$

$i = 3 \Rightarrow l_{31} = \frac{1}{l_{11}} \left(a_{13} - \sum_{k=1}^{0} \text{ del}\right) \Rightarrow l_{31} = \frac{a_{13}}{l_{11}} = 1$

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$j=2$

① If $i=j=2 \Rightarrow l_{22} = \left(a_{22} - \sum_{k=1}^{1} l_{2k}^2\right)^{\frac{1}{2}} = \left(a_{22} - l_{21}^2\right)^{\frac{1}{2}}$

$\rightarrow l_{22} = \left(2 - (-1)^2\right)^{\frac{1}{2}} = \sqrt{1} = 1$

② If $i=3 \Rightarrow l_{32} = \frac{1}{l_{22}}\left[a_{23} - \sum_{k=1}^{1} l_{3k} \cdot l_{2k}\right]$

$l_{32} = \frac{1}{l_{22}}\left[a_{23} - l_{31} l_{21}\right] = \frac{1}{1}\left[-3 - (1)(-1)\right] = -2$

$j=3$

① If $i=j=3$
$l_{33} = \left(a_{33} - \sum_{k=1}^{2} l_{3k}^2\right)^{\frac{1}{2}} = \left(a_{33} - l_{31}^2 - l_{32}^2\right)^{\frac{1}{2}}$

$= \left(9 - (1)^2 - (-2)^2\right)^{\frac{1}{2}} = \sqrt{4} = 2$

$L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -2 & 2 \end{pmatrix}, \quad L^t = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{pmatrix}$

$\therefore A\underline{x} = \underline{b} \Longrightarrow L(L^t \cdot \underline{x}) = \underline{b}$

$L\underline{y} = \underline{b} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -2 & 2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \\ 2 \end{pmatrix}$

$L^t \cdot \underline{x} = \underline{y}$

② Iterative Techniques.

Def:- ③ The L_2 and L_∞-norms for the vector $\underline{x} = (x_1, x_2, \ldots, x_n)^t$ are defined by $\|\underline{x}\|_2 = \left\{\sum_{i=1}^{n} x_i^2\right\}^{\frac{1}{2}}$; $\|\underline{x}\|_\infty = \max_{1\le i\le n} |x_i|$

Ex:- The vector $\underline{x} = (-1, 1, 2)^t$ in $\mathbb{R}^3$ has norms

$$\|\underline{x}\|_2 = \sqrt{x_1^2 + x_2^2 + x_3^2} = \sqrt{(-1)^2 + (1)^2 + (2)^2} = \sqrt{6}$$

$$\|\underline{x}\|_\infty = \max_{1\le i\le 3} |x_i| = \max(|x_1|, |x_2|, |x_3|)$$
$$= \max(|-1|, |1|, |2|) = 2$$

Def:- ④ If A is an $n\times n$ matrix then the norm for the matrix is defined by $\|A\|_\infty = \max_{1\le i\le n}\left(\sum_{j=1}^{n} |a_{ij}|\right)$

Ex:- If $A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 5 & -1 & 1 \end{pmatrix}$ then find the norm.

$$\sum_{j=1}^{3} |a_{1j}| = |1| + |2| + |-1| = 4$$
$$\sum_{j=1}^{3} |a_{2j}| = |0| + |3| + |-1| = 4$$
$$\sum_{j=1}^{3} |a_{3j}| = |5| + |-1| + |1| = 7$$

$$\|A\|_\infty = \max_{1\le i\le 3}\left(\sum_{j=1}^{3} a_{ij}\right)$$
$$= \max(4, 4, 7) = 7$$

ⓐ Jacobi Iterative method :-

Let A is $n\times n$ matrix $A\underline{x} = b$

$$\Rightarrow \sum_{j=1}^{n} a_{ij} x_j = b_i \quad (i = 1, 2, \ldots, n)$$

$$\sum_{\substack{j=1\\ j\ne i}}^{n} a_{ij} x_j + a_{ii} x_i = b_i$$

$$x_i = \frac{1}{a_{ii}}\left[b_i - \sum_{\substack{j=1\\ j\ne i}}^{n} a_{ij} x_j\right]$$

$$\boxed{x_i^{(k)} = \frac{1}{a_{ii}}\left[b_i - \sum_{\substack{j=1\\ j\ne i}}^{n} a_{ij} x_j^{(k-1)}\right]} \quad i = 1, 2, \ldots, n \quad a_{ii} \ne 0$$

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Ex: Solve the system

$$3x_1 + x_2 + x_3 = 13$$
$$x_1 + 2x_2 - x_3 = 4$$
$$x_1 + x_2 + x_3 = 9$$

where $\underline{x}^{(0)} = 0$ with accurate $\epsilon = 10^{-5}$

solution:

$$x_1^{(k)} = \frac{1}{a_{11}}\Big[b_1 - \sum_{j=2}^{3} a_{1j} x_j^{(k-1)}\Big] = \frac{1}{a_{11}}\Big[b_1 - a_{12}x_2^{(k-1)} - a_{13}x_3^{(k-1)}\Big]$$

$$x_1^{(k)} = \frac{1}{3}\Big[13 - x_2^{(k-1)} - x_3^{(k-1)}\Big] \quad \text{--- (1)}$$

$$x_2^{(k)} = \frac{1}{a_{22}}\Big[b_2 - \sum_{\substack{j=1 \\ j\neq 2}}^{3} a_{2j} x_j^{(k-1)}\Big] = \frac{1}{a_{22}}\Big[b_2 - a_{21}x_1^{(k-1)} - a_{23}x_3^{(k-1)}\Big]$$

$$x_2^{(k)} = \frac{1}{2}\Big[4 - x_1^{(k-1)} + x_3^{(k-1)}\Big] \quad \text{--- (2)}$$

$$x_3^{(k)} = \frac{1}{a_{33}}\Big[b_3 - \sum_{j=1}^{2} a_{3j} x_j^{(k-1)}\Big] = \frac{1}{a_{33}}\Big[b_3 - a_{31}x_1^{(k-1)} - a_{32}x_2^{(k-1)}\Big]$$

$$x_3^{(k)} = 9 - x_1^{(k-1)} - x_2^{(k-1)} \longrightarrow (3)$$

$$x_1^{(1)} = \frac{1}{3}\Big[13 - x_2^{(0)} - x_3^{(0)}\Big] = \frac{1}{3}\Big[13 - 0 - 0\Big] = \frac{13}{3}$$

$$x_2^{(1)} = \frac{1}{2}\Big[4 - x_1^{(0)} + x_3^{(0)}\Big] = 2$$

$$x_3^{(1)} = 9 - x_1^{(0)} - x_2^{(0)} = 9$$

$$\underline{x}^{(1)} = \begin{pmatrix} \frac{13}{3} \\ 2 \\ 9 \end{pmatrix}$$

$$\|\underline{x}^{(1)} - \underline{x}^{(0)}\|_2 = \left\| \begin{pmatrix} \frac{13}{3} \\ 2 \\ 9 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\|_2 = \left\| \begin{pmatrix} \frac{13}{3} \\ 2 \\ 9 \end{pmatrix} \right\|_2$$

$$= \sqrt{\left(\tfrac{13}{3}\right)^2 + (2)^2 + (9)^2} = 10.187 > \epsilon = 10^{-5}$$

$$x_1^{(2)} = \frac{1}{3}\Big[13 - x_2^{(1)} - x_3^{(1)}\Big] = \frac{1}{3}\Big[13 - 2 - 9\Big] = 0.6667$$

$$x_2^{(2)} = \frac{1}{2}\left[4 - x_1^{(1)} + x_3^{(1)}\right] = \frac{1}{2}\left[4 - \frac{13}{3} + 9\right] = 4.3333$$

$$x_3^{(2)} = 9 - x_1^{(1)} - x_2^{(1)} = 9 - \frac{13}{3} - 2 = 2.6667$$

$$\underline{x}^{(2)} = \begin{pmatrix} 0.6667 \\ 4.3333 \\ 2.6667 \end{pmatrix}$$

$$\|\underline{x}^{(2)} - \underline{x}^{(1)}\|_2 = 2.304 > \epsilon$$

$$\underline{x}^{(3)} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \rightarrow \|\underline{x}^{(3)} - \underline{x}^{(2)}\| = 1.1957 \times 10^{-15} < \epsilon = 10^{-5}$$

$$\therefore \underline{x} = \underline{x}^{(3)} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

(b) Gauss - Seidel method

$$AX = b \rightarrow \sum_{j=1}^{n} a_{ij}x_j = b_i \quad (i = 1, 2, \ldots, n)$$

$$\sum_{j=1}^{i-1} a_{ij}x_j + a_{ii}x_i + \sum_{j=i+1}^{n} a_{ij}x_j = b_i$$

$$x_i = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}x_j - \sum_{j=i+1}^{n} a_{ij}x_j\right]$$

$$x_i^{(k)} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k-1)}\right] \quad i = 1, 2, \ldots, n, \quad a_{ii} \neq 0$$

Ex. Find the first seven iterations for Gauss - Seidel method of the following linear system

$$-x_2 + 4x_3 = -24$$
$$3x_1 + 4x_2 - x_3 = 30$$
$$4x_1 + 3x_2 = 24$$

with $\underline{x}^{(0)} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Sol.:
$$4x_1 + 3x_2 = 24$$
$$3x_1 + 4x_2 - x_3 = 30$$
$$-x_2 + 4x_3 = -24$$

$$x_1^{(k)} = \frac{1}{a_{11}}\left[b_1 - \sum_{j=1}^{0} a_{1j}x_j^{(k)} - \sum_{j=2}^{3} a_{1j}x_j^{(k-1)}\right]$$

$$= \frac{1}{a_{11}}\left[b_1 - a_{12}x_2^{(k-1)} - a_{13}x_3^{(k-1)}\right]$$

$$x_1^{(k)} = \frac{1}{4}\left[24 - 3x_2^{(k-1)}\right] \quad \text{---} (1)$$

$$x_2^{(k)} = \frac{1}{a_{22}}\left[b_2 - \sum_{j=1}^{1} a_{2j}x_j^{(k)} - \sum_{j=3}^{3} a_{2j}x_j^{(k-1)}\right]$$

$$x_2^{(k)} = \frac{1}{4}\left[30 - \sum_{j=1}^{1} a_{2j}x_j^{(k)} - \sum_{j=3}^{3} a_{2j}x_j^{(k-1)}\right] \Rightarrow x_2^{(k)} = \frac{1}{4}\left[30 - 3x_1^{(k)} + x_3^{(k-1)}\right] \quad \text{---} (2)$$

$$x_3^{(k)} = \frac{1}{a_{33}}\left[b_3 - \sum_{j=1}^{2} a_{3j}x_j^{(k)} - \sum_{j=4}^{3} a_{3j}x_j^{(k-1)}\right]$$

$$= \frac{1}{4}\left[-24 + x_2^{(k)}\right] \quad (3)$$

$$x_1^{(1)} = \frac{1}{4}\left[24 - 3 * x_2^{(0)}\right] = \frac{1}{4}\left[24 - 3 * 1\right] = \frac{21}{4} = 5.25$$

$$x_2^{(1)} = \frac{1}{4}\left[30 - 3 * x_1^{(1)} + x_3^{(0)}\right] = \frac{1}{4}\left[30 - 3 * 5.25 + 1\right] = 3.8125$$

$$x_3^{(1)} = \frac{1}{4}\left[-24 + x_2^{(1)}\right] = \frac{1}{4}\left[-24 + 3.8125\right] = -5.046375$$

$$X^{(1)} = \begin{pmatrix} 5.25 \\ 3.8125 \\ -5.046375 \end{pmatrix}$$

$$X^{(7)} = \begin{pmatrix} 3.013411 \\ 3.988241 \\ -5.002794 \end{pmatrix} \quad ; \quad x = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \text{ exact Solution}$$

$$\begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ -2 & 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$$

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مختبر الخميس (حل بكتابة البرنامج) بالماتلاب

تعميم النظام

```
clc
clear
a=[2 3 -1 ; 4 4 -3 ; -2 3 -1]
b=[5 ; 3 ; 1]
n=length(b)
c=[a b]
x=zeros(n,1)
for j=1:n-1 ;
  for i=j+1:n ;
c(i,:)=c(i,:)-c(i,j)/c(j,j)*c(j,:)
  end
end
x(n)=c(n,n+1)/c(n,n)
for j=n-1 ; -1 ; 1
x(j)=(c(j,n+1)-c(j,j+1:n)*
       x(j+1:n))/c(j,j)
end
x
```

x مستويين استخرج الـ n

x اذا كان النظام متكون من ٣ معادلات

صفر يرجعه بالاعمدة n

يعني يدل ع طول n

اسطر ٥.٢

١/ n=len

صفر ٣ خطوات n

٢ خطوات n-1

٤ كسر n+1

او تستخدم [m,n]=size

$$3x_1 + 4x_2 - 2x_3 + x_4 = 5$$
$$7x_1 + 7x_2 - 9x_3 + 2x_4 = 9$$
$$x_1 + 5x_2 + x_3 - x_4 = 4$$
$$2x_1 - 4x_2 - x_3 + 2x_4 = -2$$

حل نظام واطفا الحل بالماتلاب

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```
clc
clear
a=[                                              ];
b=[                        ];
n= length(b)
c= [a   b];
x = zeros(n,1);
for j= n-1 : 1;
for i= j+1 :n;
c(i,:) = c(i,:) - c(i,j) / c(j,j) * c(j,:)
 end
 end
x(n) = c(n,n+1) / c(n,n)
 for j = n-1 : -1 : 1
  x(j) =(c(j,n+1) - c(j,j+1:n)*x(j+1:n))/c(j,j)
   end
   x
```

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© Successive over-Relaxation (SOR)

$$x_i^{(k)} = (1-w)\,x_i^{(k-1)} + \frac{w}{a_{ii}}\Big[b_i - \sum_{j=1}^{i-1} a_{ij}\,x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij}\,x_j^{(k-1)}\Big]$$

if $w=1 \rightarrow$ SOR is Gauss-seidel

$1 < w < 2$

under-relaxation

Ex:- Find the first seven iterations of the SOR method for the following linear systems

$$\begin{aligned} 4x_1 + 3x_2 \quad &= 24 \\ 3x_1 + 4x_2 - x_3 &= 30 \\ -x_2 + 4x_3 &= -24 \end{aligned}$$

where $x^{(0)} = \begin{pmatrix}1\\1\\1\end{pmatrix}$ and $w = 1.25$

Solution:-

$$x_1^{(k)} = -0.25\,x_1^{(k-1)} + \frac{1.25}{a_{11}}\Big[b_1 - \sum_{j=2}^{3} a_{1j}\,x_j^{(k-1)}\Big]$$

$$x_1^{(k)} = -0.25\,x_1^{(k-1)} + \frac{1.25}{4}\big[24 - 3x_2^{(k-1)}\big] \quad (1)$$

$$x_2^{(k)} = -0.25\,x_2^{(k-1)} + \frac{1.25}{a_{22}}\Big[b_2 - \sum_{j=1}^{1} a_{2j}\,x_j^{(k)} - \sum_{j=3}^{3} a_{2j}\,x_j^{(k-1)}\Big]$$

$$= 0.25\,x_2^{(k-1)} + \frac{1.25}{4}\big[30 - a_{21}x_1^{(k)} - a_{23}x_3^{(k-1)}\big]$$

$$x_2^{(k)} = -0.25\,x_2^{(k-1)} + \frac{1.25}{4}\big[30 - 3x_1^{(k)} + x_3^{(k-1)}\big] \quad (2)$$

$$x_3^{(k)} = -0.25\,x_3^{(k-1)} + \frac{1.25}{4}\big[-24 + x_2^{(k)}\big] \quad (3)$$

$$x_1^{(1)} = -0.25\,x_1^{(0)} + \frac{1.25}{4}\big[24 - 3x_2^{(0)}\big]$$

$$= -0.25 + \frac{1.25}{4}[24 - 3] = 6.3125$$

$$x_2^{(1)} = -0.25 * x_2^{(0)} + \frac{1.25}{4}\big[30 - 3 * x_1^{(1)} + x_3^{(0)}\big]$$

$$= -0.25 + \frac{1.25}{4}[30 - 3 * 6.3125 + 1] = 3.519531$$

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$$x_3^{(1)} = -0.25\,x_3^{(0)} + \frac{1.25}{4}\left[-24 + x_2^{(1)}\right] = -0.25 + \frac{1.25}{4}\left[-24 + 3.5195313\right]$$

$$x_3^{(1)} = -6.6501465$$

$$\underline{X}^{(1)} = \begin{pmatrix} 6.3125 \\ 3.5195313 \\ -6.6501465 \end{pmatrix}$$

$$\underline{X}^{(7)} = \begin{pmatrix} 3.0000498 \\ 4.0002586 \\ -5.0003486 \end{pmatrix}$$

$$\underline{X} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$$

مختبر الخميس

```
clc
clear
a=[3 4 -2 1; 7 7 -9 2; 15 1 -1 9 2, -4 -1 2];
b= [5;9;4;-2];
n= length(b)
c=[a b];
x= zeros(n,1)
for j=1:n-1;
for i=j+1:n;
c(i,:)= c(i,:) - c(i,j)/c(j,j)*c(j,:)
end   end
x(n)= c(n,n+1)/c(n,n)
for j=n-1:-1:1;
x(j)=(c(j,n+1)-c(j,j+1:n)*x(j+1:n))/c(j,j)
end  x
```

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[illegible]

Find the first two iteration of the Jacobi method for linear system , $x^{(0)} = 0$

$$10x_1 - x_2 = 9$$
$$-x_1 + 10x_2 - 2x_3 = 7$$
$$-2x_2 + 10x_3 = 6$$

$$x_i^{(k)} = \frac{1}{a_{ii}} [b_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ij} x_j^{(k-1)}]$$

$$x_1^{(k)} = \frac{1}{a_{11}} [b_1 - a_{12} x_2^{(k-1)} - a_{13} x_3^{(k-1)}]$$

$$= \frac{1}{10} [9 + x_2^{(k-1)}] \quad \text{—— 1)}$$

$$x_2^{(k)} = \frac{1}{a_{22}} [b_2 - a_{21} x_1^{(k-1)} - a_{23} x_3^{(k-1)}]$$
$$= \frac{1}{10} [7 + x_1^{(k-1)} + 2x_3^{(k-1)}] \quad \text{—— 2)}$$

$$x_3^{(k)} = \frac{1}{a_{33}} [b_3 - a_{31} x_1 - a_{32} x_2^{(k-1)}]$$
$$= \frac{1}{10} [6 + 2x_2^{(k-1)}] \quad \text{—— 3)}$$

$k=1$

$$x_1^{(1)} = \frac{1}{10} [9 + x_2^{(0)}] = \frac{9}{10} = 0.9$$

$$x_2^{(0)} = \frac{1}{10} [7 + x_1^{(0)} + 2x_3^{(0)}] = \frac{7}{10} = 0.7$$

$$x_3^{(1)} = \frac{1}{10} [6 + 2x_2^{(0)}] = \frac{6}{10} = 0.6$$

$k=2$

$$x_1^{(2)} = \frac{1}{10} [9 + x_2^{(1)}] = \frac{1}{10} [9 + 0.7] = \frac{9.7}{10} = 0.97$$

$$x_2^{(2)} = \frac{1}{10} [7 + x_1^{(1)} + 2x_3^{(1)}] = \frac{1}{10} [7 + 0.9 + 2 * 0.6]$$
$$= \frac{1}{10} [7 + 0.9 + 1.2] = \frac{9.1}{10} = 0.91$$

$$x_3^{(2)} = \frac{1}{10} [6 + 2x_2^{(1)}] = \frac{1}{10} [6 + 2 * 0.7]$$

$$= \frac{1}{10} [6 + 1.4] = \frac{7.4}{10} = 0.74$$

$$x^{(2)} = \begin{pmatrix} 0.97 \\ 0.91 \\ 0.74 \end{pmatrix}$$

نفس السؤال السابق بطريقة كاوس سايدل

$$x_i^{(k)} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k-1)}\right]$$

$$x_1^{(k)} = \frac{1}{10}\left[9 - \sum_{j=2}^{3} a_{ij}x_j^{(k-1)}\right] = \frac{1}{10}\left[9 + x_2^{(k-1)}\right] \quad \text{--- } (1)$$

$$x_2^{(k)} = \frac{1}{10}\left[7 - \sum_{j=1}^{1} a_{2j}x_j^{(k)} - \sum_{j=3}^{3} a_{2j}x_j^{(k-1)}\right]$$

$$= \frac{1}{10}\left[7 + x_1^{(k)} + 2x_3^{(k-1)}\right] \quad \text{--- } (2)$$

$$x_3^{(k)} = \frac{1}{10}\left[6 - \sum_{j=1}^{2} a_{3j}x_j^{(k)}\right]$$

$$= \frac{1}{10}\left[6 + 2x_2^{(k)}\right] \quad \text{--- } (3)$$

$k=1$

$$x_1^{(1)} = \frac{1}{10}\left[9 + x_2^{(0)}\right] = \frac{9}{10} = 0.9$$

$$x_2^{(1)} = \frac{1}{10}\left[7 + x_1^{(1)} + 2x_3^{(0)}\right] = \frac{1}{10}[7 + 0.9] = 0.79$$

$$x_3^{(1)} = \frac{1}{10}\left[6 + 2x_2^{(1)}\right] = \frac{1}{10}[6 + 1.58] = 0.758$$

$k=2$

$$x_1^{(2)} = \frac{1}{10}\left[9 - x_2^{(1)}\right] = \frac{1}{10}[9 - 0.79] = 0.979$$

$$x_2^{(2)} = \frac{1}{10}\left[7 + x_1^{(2)} + 2x_3^{(1)}\right] = \frac{1}{10}[7 + 0.979 + 2*0.758]$$

$$= \frac{1}{10}[7.979 + 1.516]$$

$$= \frac{1}{10}[9.495] = 0.9495$$

$x_2^{(3)} =$

Convergence of Iterative methods :-

$A\underline{x} = \underline{b} \longrightarrow \underline{x}^{(k)} = T\,\underline{x}^{(k-1)} + C$

Theorem:- If $\|T\| < 1$ then the sequence $\{\underline{x}^{(k)}\}_{k=0}^{\infty}$ to the exact vector $\underline{x}$ and the following inequalities hold

$x_1 = \frac{-a_{12}}{a_{11}} x_2 - \frac{a_{13}x_3}{a_{11}} - \frac{a_{1n}}{a_{11}} x_n + \frac{b_1}{a_{11}}$

$x_2 = \frac{-a_{21}}{a_{22}} x_1 - \frac{a_{23}}{a_{22}} - \frac{a_{2n}}{a_{22}} x_n + \frac{b_2}{a_{22}}$

$x_n =$

$x^{(k)} = T x^{(k-1)} + C$

① $\|\underline{x} - \underline{x}^{(k)}\| \leq \|T\|^k \, \|\underline{x} - \underline{x}^{(0)}\|$

② $\|\underline{x} - \underline{x}^{(k)}\| \leq \dfrac{\|T\|^k}{1 - \|T\|} \cdot \|\underline{x}^{(1)} - \underline{x}^{(0)}\|$

$\|A \cdot B\| \leq \|A\| \, \|B\|$

Proof:-

$\underline{x} = T\underline{x} + \underline{c}$ ——— ①

$\underline{x}^{(k)} = T\,\underline{x}^{(k-1)} + \underline{c}$ ——— ②

$\underline{x} - \underline{x}^{(k)} = T(\underline{x} - \underline{x}^{(k-1)})$

$= T(T(\underline{x} - \underline{x}^{(k-2)})) = T^2(\underline{x} - \underline{x}^{(k-2)})$

$= T^3(\underline{x} - \underline{x}^{(k-3)})$

$\vdots$

$\therefore \underline{x} - \underline{x}^{(k)} = T^k(\underline{x} - \underline{x}^{(0)})$

$\|\underline{x} - \underline{x}^{(k)}\| = \|T^k \cdot (\underline{x} - \underline{x}^{(0)})\|$

$\|\underline{x} - \underline{x}^{(k)}\| \leq \|T^k\| \cdot \|\underline{x} - \underline{x}^{(0)}\|$

Since $\|T\| < 1 \Rightarrow \lim_{k \to \infty} \|T\|^k = 0$

$\lim_{k\to\infty} \|\underline{x} - \underline{x}^{(k)}\| = \lim_{k\to\infty} \|T\|^k \cdot \|\underline{x} - \underline{x}^{(0)}\|$

$\therefore \lim_{k\to\infty} \|\underline{x} - \underline{x}^{(k)}\| = 0$

$\Rightarrow \lim_{k\to\infty} \underline{x}^{(k)} = \underline{x}$

$\therefore \{\underline{x}^{(k)}\}_{k=0}^{\infty}$ converges to $\underline{x}$ and we have

$\|\underline{x} - \underline{x}^{(k)}\| \leq \|T\|^k \cdot \|\underline{x} - \underline{x}^{(0)}\| \Rightarrow$ ① holds

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

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to Prove ②

$$\|\underline{X}^{(k)} - \underline{X}^{(k-1)}\| \leq \|T\| \, \|\underline{X}^{(k-1)} - \underline{X}^{(k-2)}\| \leq \|T\|^2 \|\underline{X}^{(k-2)} - \underline{X}^{(k-3)}\|$$

$$\|\underline{X}^{(k)} - \underline{X}^{(k-1)}\| \leq \|T\|^{k-1} \|\underline{X}^{(1)} - \underline{X}^{(0)}\|$$

Thus for $m > k \geq 1$

$$\|X^{(m)} - X^{(k)}\| = \|X^{m} - X^{(m-1)} + X^{(m-1)} - X^{(m-2)} + X^{(m-2)} + \dots + X^{(k+1)} - X^{(k)}\|$$

$$\leq \|X^{(m)} - X^{(m-1)}\| + \|X^{(m-1)} - X^{(m-2)}\| + \dots + \|X^{(k+1)} - X^{(k)}\|$$

$$\leq \|T\|^{m-1} \|X^{(1)} - X^{(0)}\| + \|T\|^{m-2} \|X^{(1)} - X^{(0)}\| + \dots + \|T\|^{k} \|X^{(1)} - X^{(0)}\|$$

$$= \|T\|^{k} \left(\|T\|^{m-k-1} + \|T\|^{m-k-2} + \dots + \|T\|^2 + \|T\| + 1 \right) \|X^{(1)} - X^{(0)}\|$$

$$\therefore \|\cancel{X}^{(m)} - X^{(k)}\| \leq \|T\|^{k} \left(1 + \|T\| + \|T\|^2 + \|T\|^3 + \dots + \|T\|^{m-k-1}\right) \|X^{(1)} - X^{(0)}\|$$

$$\lim_{m \to \infty} \underline{X}^{(m)} = \underline{X}$$

$$\therefore \|X - X^{(k)}\| \leq \|T\|^{k} \left(1 + \|T\| + \|T\|^2 + \|T\|^3 + \dots\right) \|X^{(1)} - X^{(0)}\|$$

$$\therefore \|X - X^{k}\| \leq \frac{\|T\|^{k}}{1 - \|T\|} \cdot \|X^{(1)} - X^{(0)}\|$$

Ex: →
$$10x_1 - x_2 + 2x_3 = 6$$
$$-x_1 + 11x_2 - x_3 + 3x_4 = 25$$
$$2x_1 - x_2 + 10x_3 - x_4 = -11$$
$$3x_2 - x_3 + 8x_4 = 15$$

Solution:

$$x_1^{(k)} = \frac{1}{10} x_2^{(k-1)} - \frac{1}{5} x_3^{(k-1)} + \frac{3}{5}$$

$$x_2^{(k)} = \frac{1}{11} x_1^{(k-1)} + \frac{1}{11} x_3^{(k-1)} - \frac{3}{11} x_4^{(k-1)} + \frac{25}{11}$$

$$x_3^{(k)} = \frac{-1}{5} x_1^{(k-1)} + \frac{1}{10} x_2^{(k-1)} + \frac{1}{10} x_4^{(k-1)} - \frac{11}{10}$$

$$x_4^{(k)} = \frac{-3}{8} x_2^{(k-1)} + \frac{1}{8} x_3^{(k-1)} + \frac{15}{8}$$

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$$\begin{pmatrix} x_1^{(k)} \\ x_2^{(k)} \\ x_3^{(k)} \\ x_4^{(k)} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{10} & \frac{-1}{5} & 0 \\ \frac{1}{11} & 0 & \frac{1}{11} & \frac{-3}{11} \\ \frac{-1}{5} & \frac{1}{10} & 0 & \frac{1}{10} \\ 0 & \frac{-3}{8} & \frac{1}{8} & 0 \end{pmatrix} \begin{pmatrix} x_1^{(k-1)} \\ x_2^{(k-1)} \\ x_3^{(k-1)} \\ x_4^{(k-1)} \end{pmatrix} + \begin{pmatrix} \frac{3}{5} \\ \frac{25}{11} \\ \frac{-11}{10} \\ \frac{15}{8} \end{pmatrix}$$

$$T = \begin{pmatrix} \quad \end{pmatrix}, \quad C = \begin{pmatrix} \quad \end{pmatrix}$$

$$\|T\| = \|T\|_\infty = \max_{1 \le i \le 4} \left(\sum_{j=1}^{4} |a_{ij}| \right)$$

$$= \max \left(\sum_{j=1}^{4} |a_{1j}|, \sum_{j=1}^{4} |a_{2j}|, \sum_{j=1}^{4} |a_{3j}|, \sum_{j=1}^{4} |a_{4j}| \right)$$

$$= \max \left(\frac{3}{10}, \frac{5}{11}, \frac{2}{5}, \frac{1}{2} \right)$$

$$= \frac{1}{2} < 1$$

$$\boxed{\|T\| < 1}$$

اضطراب

Error Estimates :→ For which Perturbation in Coefficients of A or $\underline{b}$ Produce changes in $\underline{X}$?

اضطراب b

① **Perturbation in $\underline{b}$**

Let $\underline{b}' = \underline{b} + \delta \underline{b}$ ($\delta \underline{b}$ small change)

$\underline{X}' = \underline{X} + \delta \underline{X}$ ($\delta \underline{X}$ small change)

$A\underline{X} = \underline{b}$ ——— ①

$A(\underline{X} + \delta \underline{X}) = (\underline{b} + \delta \underline{b})$

$A \cdot \underline{X} + A \cdot \delta \underline{X} = \underline{b} + \delta \underline{b}$

$\cancel{A\underline{X}} + A \cdot \delta \underline{X} = \cancel{A\underline{X}} + \delta \underline{b}$

$A \cdot \delta \underline{X} = \delta \underline{b}$

$\Rightarrow \delta \underline{X} = A^{-1} \cdot \delta \underline{b}$

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$$\|\delta \underline{x}\| = \|A^{-1} \cdot \delta b\|$$

$$\|\delta \underline{x}\| \leq \|A^{-1}\| \cdot \|\delta \underline{b}\|$$

$$\frac{\|\delta \underline{x}\|}{\|\underline{x}\|} \leq \frac{\|A^{-1}\|}{\|\underline{x}\|} \cdot \|\delta \underline{b}\| \quad \text{--- } (2)$$

$$A\underline{x} = \underline{b} \Rightarrow \|A \cdot \underline{x}\| = \|\underline{b}\|$$

$$\Rightarrow \|A\| \cdot \|\underline{x}\| \geq \|\underline{b}\|$$

$$\|\underline{x}\| \geq \frac{\|\underline{b}\|}{\|A\|}$$

$$\frac{\|\delta \underline{x}\|}{\|\underline{x}\|} \leq \|A^{-1}\| \cdot \|A\| \cdot \frac{\|\delta \underline{b}\|}{\|\underline{b}\|}$$

$$\frac{\|\delta \underline{x}\|}{\|\underline{x}\|} \leq K(A) \cdot \frac{\|\delta \underline{b}\|}{\|\underline{b}\|}$$

where $K(A) = \|A^{-1}\| \cdot \|A\|$ ($K(A)$ is condition number)

(2) Perturbation in A :-

Let $A^{\wedge} = A + \delta A$ (δA is small change)

$\quad \underline{x}^{\wedge} = \underline{x} + \delta \underline{x}$

$$(A + \delta A)(\underline{x} + \delta \underline{x}) = \underline{b}$$

$$A\underline{x} + \delta A \cdot \underline{x} + (A + \delta A)\, \delta \underline{x} = \underline{b}$$

$$(A + \delta A)\, \delta \underline{x} = -\delta A \cdot \underline{x}$$

$$\delta \underline{x} = -(A + \delta A)^{-1} \cdot \delta A \cdot \underline{x}$$

$$\delta \underline{x} = -(A(I + A^{-1}\delta A))^{-1} \cdot \delta A \cdot \underline{x}$$

$$\delta \underline{x} = -(I + A^{-1}\delta A)^{-1} \cdot A^{-1} \cdot \delta A \cdot \underline{x}$$

$$\|\delta \underline{x}\| \leq \|(I + A^{-1}\delta A)^{-1}\| \cdot \|A^{-1}\| \cdot \|\delta A\| \cdot \|\underline{x}\|$$

$$\leq \frac{1}{1-\|A^{-1}\|\cdot\|\delta A\|}\,\|A^{-1}\|\;\|\delta A\|\cdot\|\underline{X}\|$$

$$\frac{\|\delta \underline{X}\|}{\|\underline{X}\|} \leq \frac{1}{1-\|A^{-1}\|\cdot\|\delta A\|}\cdot\|A^{-1}\|\;\|\delta A\|$$

$$\frac{\|\delta \underline{X}\|}{\|\underline{X}\|} \ll$$ كذك

Location

الاربعاء

Date: ٢٠١٥/١٢/١٨

$x = \tan x$

Positive ②

$|A_1| = |a_{11}| > 0$

$|A_2| = 20 > 0$

$|A_3| = 74 > 0$

$|A| = |1| > 0$

chapter 4 :

التعريف الداخلي ومتعددة الحدود التقريبية

Interpolation and Polynomial Approximation

$$\boxed{e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots}$$

$$e^{1.5} = 1 + 1.5 + \frac{(1.5)^2}{2!} + \frac{(1.5)^3}{3!} + \cdots$$

① Lagrange Polynomial :

Let $(x_0, f(x_0))$ and $(x_1, f(x_1))$ are two Points with $x_0 \neq x_1$

$(x_0, f(x_0))$ $(x_1, f(x_1))$

$$\Rightarrow \frac{f(x) - f(x_0)}{x - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$\rightarrow (x_1 - x_0) f(x) - (x_1 - x_0) f(x_0) = (x - x_0) f(x_1) - (x - x_0) f(x_0)$$

$$\Rightarrow (x_1 - x_0) f(x) - x_1 f(x_0) + x_0 f(x_0) = (x - x_0) f(x_1) - x f(x_0) + x_0 f(x_0)$$

$$(x_1 - x_0) f(x) = x_1 f(x_0) - x f(x_0) + (x - x_0) f(x_1)$$

$$(x_1 - x_0) f(x) = (x_1 - x) f(x_0) + (x - x_0) f(x_1)$$

$$\Rightarrow f(x) = \frac{(x_1 - x)}{(x_1 - x_0)} f(x_0) + \frac{(x - x_0)}{(x_1 - x_0)} f(x_1)$$

$$\Rightarrow f(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{(x - x_0)}{(x_1 - x_0)} f(x_1)$$

$$f(x) = P_1(x) = L_0(x)\, f(x_0) + L_1(x)\, f(x_1)$$

$$P_1(x) = \sum_{i=0}^{1} L_i(x)\, f(x_i)$$

where $L_0(x) = \dfrac{x - x_1}{x_0 - x_1}$ and $L_1(x) = \dfrac{x - x_0}{x_1 - x_0}$

In general case for each $i = 0, 1, 2, \ldots, n$. The Polynomial Passing through the Points $(x_0, f(x_0))$, $(x_1, f(x_1))$, ..., $(x_n, f(x_n))$ is given by

$$f(x) = P_n(x) = \sum_{i=0}^{n} L_i(x)\, f(x_i) \quad \text{--- (1)}$$

where $L_i(x) = \dfrac{(x - x_0)(x - x_1) \cdots (x - x_{i-1})(x - x_{i+1}) \cdots (x - x_n)}{(x_i - x_0)(x_i - x_1) \cdots (x_i - x_{i-1})(x_i - x_{i+1}) \cdots (x_i - x_n)}$

الابعاد [illegible] k [illegible]

where $L_i(x)$ is called the nth lagrange Polynomial.

$L_i(x_j) = \begin{cases} 1 & \text{where } j = i \\ 0 & \text{where } j \neq i \end{cases}$ $L_0(x_0) = 1$, $L_0(x_1) = 0$

Theorem (1): If $x_0, x_1, x_2, \ldots, x_n$ are $n+1$ distinct numbers and f is a function whose values is given at these number, then there exist a unique polynomial $P(x)$ of degree n with the Property that $f(x_i) = P(x_i)$ ربما

This Polynomial given by (1) [illegible]

Ex: From the table find $f(1.5)$, using interpolating of Lagrange Polynomial [illegible]

[illegible] للبولنوميل 3 لانه

في هذا السؤال اكبر ما يصير

اعلى شي 3

X	0	1	2	3
f(x)	-5	-6	-1	16

Solution: $n + 1 = 4 \rightarrow n = 3$

[illegible] lagrange interpolating polynomial

$$f(x) \simeq P_3(x) = \sum_{i=0}^{3} L_i(x) \cdot f(x_i) =$$

$$L_0(x) f(x_0) + L_1(x) f(x_1) + L_2(x) f(x_2) + L_3(x) f(x_3)$$

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)}$$

$$= \frac{-1}{6}(x-1)(x-2)(x-3)$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}$$

$$= \frac{1}{2} x(x-2)(x-3)$$

كلما تزيد عدد النقاط تزيد الدقة والتقريب للحل

$$L_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)}$$

$$= \frac{-1}{2} x(x-1)(x-3)$$

$$L_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}$$

$$= \frac{1}{6} x(x-1)(x-2)$$

$$\therefore f(x) = P_3(x) = +\frac{5}{6}(x-1)(x-2)(x-3) - 3x(x-2)(x-3) + \frac{1}{2}x(x-1)(x-3) + \frac{16}{6}x(x-1)(x-3)$$

$$f(x) \simeq x^3 - 2x - 5$$

$$f(1.5) \simeq (1.5)^3 - 2(1.5) - 5$$

(٢)

الفروقات المقسمة

② Divided Differences: The zeroth divided differences is given by $f[x_i] = f(x_i)$. The first divided diff of f is given by

$$f[x_i, x_{i+1}] = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}, \quad i = 0, 1, 2, \dots, n-1$$

The second divided difference of f is given by

$$f[x_i, x_{i+1}, x_{i+2}] = \frac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i-1}]}{x_{i+2} - x_i}$$

The kth divided difference of f is given by

$$f[x_i, x_{i+1}, x_{i+2}, \dots, x_{i+k}] = \frac{f[x_{i+1}, x_{i+2}, \dots, x_{i+k}] - f[x_i, x_{i+1}, \dots, x_{i+k-1}]}{x_{i+k} - x_i}$$

Ex: Find the first, second, third and fourth divided differences from table

x	1	3	4	6	7
$f(x)$	2	10	15	8	12

Heba

Solution: The first divided differences are given by

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{10-2}{3-1} = 4$$

$$f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{15-10}{4-3} = 5$$

$$f[x_2, x_3] = \frac{f(x_3) - f(x_2)}{x_3 - x_2} = \frac{8-15}{6-4} = \frac{-7}{2} = -3.5$$

$$f[x_3, x_4] = \frac{f(x_4) - f(x_3)}{x_4 - x_3} = \frac{12-8}{7-6} = 4$$

The second divided differences are given by

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$$f[x_0,x_1,x_2] = \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0} = \frac{5-4}{4-1} = 0.3333$$

$$f[x_1,x_2,x_3] = \frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1} = \frac{-3.5-5}{6-3} = -2.8333$$

$$f[x_2,x_3,x_4] = \frac{f[x_3,x_4]-f[x_2,x_3]}{x_4-x_2} = \frac{4+(3.5)}{7-4} = 2.5$$

The third divided differences are given by

$$f[x_0,x_1,x_2,x_3] = \frac{f[x_1,x_2,x_3]-f[x_0,x_1,x_2]}{x_3-x_0}$$

$$= \frac{-2.83333-0.33333}{6-1} = -0.63333$$

$$f[x_1,x_2,x_3,x_4] = \frac{f[x_2,x_3,x_4]-f[x_1,x_2,x_3]}{x_4-x_1}$$

$$= \frac{2.5-(-2.83333)}{7-3} = 1.33333$$

The fourth divided differences are given by

$$f[x_0,x_1,x_2,x_4] = \frac{f[x_1,x_2,x_3,x_4]-f[x_0,x_1,x_2,x_3]}{x_4-x_0}$$

$$= \frac{1.33333-(0.633333)}{7-1} = 0.32778$$

او بواسطة الجدول

x	f	f[1]	f[2]	f[3]	f[4]
1	2				
		4			
3	10		0.3333		
		5		-0.6333	
4	15		-2.83333		0.32778
		-3.5		1.3333	
6	8		2.5		
		4			
7	12				

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صيغة نيوتن للفروقات المقسمة الأمامية

Newton Interpolatory divided differences formula

Let the Polynomial of Newton's interpolating divided differences formula is written by $f(x) = P_n(x) =$

$$* \quad a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \dots + a_n(x-x_0)(x-x_1)\dots(x-x_n)$$

if $x = x_0$

$$f(x_0) \simeq P_n(x_0) = a_0 + a_1 \cdot 0 + a_2 \cdot 0 + \dots + a_n \cdot 0$$

$$f(x_0) = a_0$$

$x = x_0$

$$\therefore f(x) \simeq P_n(x) = f(x_0) + a_1(x-x_0) + \dots + a_n(x-x_0)(x-x_1)\dots(x-x_n)$$

if $x = x_1$

$$f(x_1) = f(x_0) + a_1(x_1 - x_0) + 0 + \dots + 0$$

$$\therefore f(x_1) = f(x_0) + a_1(x_1 - x_0)$$

$$a_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = f[x_0, x_1]$$

$$\therefore f(x) = P_n(x) = f(x_0) + f[x_0, x_1](x-x_0) + \dots + a_n(x-x_0)(x-x_1)\dots(x-x_n)$$

In general

$$a_k = f[x_0, x_1, \dots, x_k] \quad \text{for each } k = 0, 1, 2, \dots, n$$

The $P_n(x)$ can be written as

صيغة نيوتن للفروقات المقسمة الأمامية

$$f(x) \simeq P_n(x) = f(x_0) + \sum_{k=1}^{n} f[x_0, x_1, \dots, x_k](x-x_0)(x-x_1)\dots(x-x_{k-1}) \quad (2)$$

$$f(x) \simeq P_n(x) = f(x_0) + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1) + \dots + f[x_0, x_1, x_2, \dots, x_n](x-x_0)(x-x_1)\dots(x-x_{n-1})$$

(2) is called Newton forward-divided difference formula

formula (N.F.D.D)

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الفروقات المقسمة الامامية

Ex:- Find $f(1.2)$ using Newton forward-divided difference formula (F.D.D) from the following table

x	1	3	4	6	7
$f(x)$	2	10	15	8	12

Solution: $n+1=5 \rightarrow n=4$

$$f(x) \simeq P_4(x) = f(x_0) + \sum_{k=1}^{4} f[x_0, x_1, \dots, x_4](x-x_0)(x-x_1)\cdots(x-x_{k-1})$$

$$f(x) \simeq f(x_0) + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1) + f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2) + f[x_0,x_1,x_2,x_3,x_4]\cdot(x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

x	f	$f[1]$	$f[2]$	$f[3]$	$f[4]$
1	2				
		4			
3	10		0.3333		
		5		-0.63333	
4	15		-2.83333		0.32778
		-3.5		1.3333	
6	8		2.5		
		4			
7	12				

$$f(x) \simeq 2 + 4(x-1) + 0.3333(x-1)(x-3) + (-0.6333)(x-1)(x-3)(x-4) + 0.32778\cdot(x-1)(x-3)(x-4)(x-6)$$

$$f(1.2) \simeq 0.45569$$

إذا كان الفرق بين النقطتين متساوي

Now if $x_{i+1} - x_i = h \quad \forall i = 0, 1, \dots, n-1$

Then the first divided differences are given by

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_1) - f(x_0)}{h} = \frac{\Delta f(x_0)}{h} = \frac{1}{h}\Delta f_0$$

$$f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\Delta f_1}{h}$$

$\vdots$

In general $\Delta f_k = f_k - f_{k-1}$, $\forall k = 1, 2, \ldots, n$

The second divided differences are given by:

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{1}{2h}(\Delta f_1 - \Delta f_0)$$

$$= \frac{1}{2h^2} \Delta^2 f_0$$

$\vdots$

In general $\Delta^2 f_i = \Delta(\Delta f_i)$, $i = 0, 1, 2, \ldots, n-2$

The kth divided difference are given by

$$\Delta^k f_i = \Delta^{k-1}(\Delta f_i) = \Delta^{k-1}(f_{i+1} - f_i)$$

$$= \Delta^{k-1} f_{i+1} - \Delta^{k-1} f_i \quad , \quad i = 0, 1, 2, \ldots, n-k$$

For example:

x	x_0	x_1	x_2	x_3	x_4
$f(x)$	f_0	f_1	f_2	f_3	f_4

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
x_0	f_0				
		$\Delta f_0 = f_1 - f_0$			
x_1	f_1		$\Delta^2 f_0 = \Delta f_1 - \Delta f_0$		
		$\Delta f_1 = f_2 - f_1$		$\Delta^3 f_0 = \Delta^2 f_1 - \Delta^2 f_0$	
x_2	f_2		$\Delta^2 f_1 = \Delta f_2 - \Delta f_1$		$\Delta^4 f_0 = \Delta^3 f_1 - \Delta^3 f_0$
		$\Delta f_2 = f_3 - f_2$		$\Delta^3 f_1 = \Delta^2 f_2 - \Delta^2 f_1$	
x_3	f_3		$\Delta^2 f_2 = \Delta f_3 - \Delta f_2$		
		$\Delta f_3 = f_4 - f_3$			
x_4	f_4				

From formula ② we have

$$f(x) \simeq P_n(x) = f(x_0) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$
$$+ \ldots + f[x_0, x_1, \ldots, x_n](x - x_0)(x - x_1) \ldots (x - x_{n-1})$$

(7)

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$$= f(x_0) + \frac{\Delta f_0}{h}(x-x_0) + \frac{\Delta^2 f_0}{2h^2}(x-x_0)(x-x_1) +$$

$$\frac{\Delta^3 f_0}{3*2*1\,h^3}(x-x_0)(x-x_1)(x-x_2) + \cdots + \frac{\Delta^n f_0}{n*(n-1)\cdots 3*2*1\,h^n}(x-x_0)\cdot(x-x_1)\cdots(x-x_{n-1})$$

$$f(x) \simeq P_n(x) = f(x_0) + \sum_{k=1}^{n} \frac{\Delta^k f_0}{k!\,h^k}(x-x_0)\cdots(x-x_{k-1}) \quad \text{——— (3)}$$

The formula (3) is called Newton forward difference formula (F.F.D)

2015/3/5 الخميس عشر

Sub: ٢٠١٠/١٢/١١

Date: الاربعاء

فروقات امامية قريبة نقطة ابتدائية

Ex: Find $f(2.5)$ from the following table.

فروقات امامية ومسافات متساوية

x	2	3	4	5	6
$f(x)$	5	10	17	26	37

Solution: To find $f(2.5)$, we use Newton forward difference (formula ②)

$n+1=5 \Rightarrow n=4$

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
2	5				
		5			
3	10		2		
		7		0	
4	17		2		0
		9		0	
5	26		2		
		11			
6	37				

$$f(x) \simeq P(x) = f(x_0) + \sum_{k=1}^{n=4} \frac{\Delta^k f_0}{k!\,h^k}(x-x_0)(x-x_1)\ldots(x-x_{k-1})$$

$$= f(x_0) + \frac{\Delta f_0}{1!\,h}(x-x_0) + \frac{\Delta^2 f_0}{2!\,h^2}(x-x_0)(x-x_1) + \frac{\Delta^3 f_0}{3!\,h^3}(x-x_0)(x-x_1)(x-x_2)$$

$$+ \frac{\Delta^4 f_0}{4!\,h^4}(x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

$$f(x) \simeq 5 + \frac{5}{1!\,(1)}(x-2) + \frac{2}{2!\,(1)^2}(x-2)(x-3) + \frac{0}{3!\,(1)^3}(x-2)(x-3)(x-4) + \frac{0}{4!\,(1)^4}(x-2)(x-3)(x-4)(x-5)$$

$$= 5 + 5x - 10 + x^2 - 5x + 6$$

$$f(x) = x^2 + 1$$

$$f(2.5) = (2.5)^2 + 1 = 6.25 + 1 = 7.25$$

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اذا كانت النقطة المطلوب حساب قيمتها قريبة من نهاية الجدول

Now, if the Polynomial is written by

$$f(x) \simeq P_n(x) = a_0 + a_1(x - x_n) + a_2(x - x_n)(x - x_{n-1}) + \dots + a_n(x - x_n)(x - x_{n-1}) \cdots (x - x_0)$$

if $x = x_n \Rightarrow f(x_n) = a_0$

$$f(x) \simeq f(x_n) + a_1(x - x_n) + a_2(x - x_n)(x - x_{n-1}) + \dots + a_n(x - x_n)(x - x_{n-1}) \cdots (x - x_0)$$

when $x = x_{n-1}$

$\Rightarrow a_1 = f[x_{n-1}, x_n]$

$\vdots$

$a_k = f[x_0, x_1, \dots, x_k]$

Then we have

$$f(x) \simeq P_n(x) = f(x_n) + f[x_{n-1}, x_n](x - x_n) + f[x_{n-2}, x_{n-1}, x_n](x - x_n)(x - x_{n-1}) + \dots + f[x_0, x_1, \dots, x_n](x - x_n)(x - x_{n-1}) \cdots (x - x_0) \quad (4)$$

E(x) find $f(5.5)$ from the following table

x	1	3	4	6	7
$f(x)$	2	10	15	8	12

Solution

To find $f(5.5)$ we used the formula (4) $n+1 = 5$

$n = 4$

x	f	$f[1]$	$f[2]$	$f[3]$	$f[4]$
1	2				
		4			
3	10		0.3333		
		5		−0.63333	
4	15		−2.83333		0.32778
		−3.5		1.3333	
6	8		2.5		
		4			
7	12				

from formula (4), we obtain

∇ خلفي

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$$f(x) \simeq P_4(x) = 12 + 4*(x-7) + 2.5(x-7)(x-6) + 1.3333(x-7)(x-6)(x-4) + 0.32778(x-7)(x-6)(x-4)(x-3)$$

$$f(5.5) = ?$$

If $x_{i+1} - x_i = h$ — اذا كانت المسافات متساوية ونبدأ من آخر الجدول

$$f(x) \simeq P_n(x) = f(x_n) + (x-x_n)\frac{\nabla f_n}{1!h} + (x-x_n)(x-x_{n-1})\cdot\frac{\nabla^2 f_n}{2!h^2} + \dots + (x-x_n)(x-x_{n-1})\dots(x-x_1)\cdot\frac{\nabla^n f_n}{n!h^n}$$

$$= f(x_n) + \sum_{k=0}^{n} \frac{\nabla^k f_n}{n!h^n}(x-x_n)(x-x_{n-1})\dots(x-x_1) \quad (5)$$

Ex: Find $f(5.5)$ from the following table

x	2	3	4	5	6
$f(x)$	5	10	17	26	37

Solution: $n=4$, $x_{i+1} - x_i = h = 1$

x	f	∇f	$\nabla^2 f$	$\nabla^3 f$	$\nabla^4 f$
2	5				
		5			
3	10		2		
		7		0	
4	17		2		0
		9		0	
5	26		2		
		11			
6	37				

from (5) we have

$$f(x) \simeq f(x_4) + (x-x_4)\cdot\frac{\nabla f_4}{1!h} + (x-x_4)(x-x_3)\frac{\nabla^2 f_4}{2!h^2} + (x-x_4)(x-x_3)(x-x_2)\frac{\nabla^3 f_4}{3!h^3} + (x-x_4)(x-x_3)(x-x_2)(x-x_1)\frac{\nabla^4 f_4}{4!h^4}$$

$$f(x) \simeq 37 + (x-6)\cdot\frac{11}{1!(1)} + (x-6)(x-5)\cdot\frac{2}{2!(1)^2} + (x-6)(x-5)(x-4)\cdot\frac{0}{3!(1)^3} + (x-6)(x-5)(x-4)(x-3)\cdot\frac{0}{4!(1)^4}$$

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$= 37 + 11x - 66 + x^2 - 11x + 30 + 0 + 0$

$\boxed{f(x) = x^2+1}$

$f(5.5) = (5.5)^2 + 1 = 31.25$

الخميس ٢٠١٩/١٢/١٨

الاكمالات قريبه الخانقه المتصف

Gauss formulas

a: Gauss forward formula — صيغة كاوس الأمامية

$$f(x) \simeq P_n(x) = f(x_0) + (x-x_0)\frac{\delta f_0}{1!h} + (x-x_0)(x-x_1)\frac{\delta^2 f_0}{2!h^2} + (x-x_0)(x-x_1)(x-x_{-1})\frac{\delta^3 f_1}{3!h^3} + (x-x_0)(x-x_1)(x-x_{-1})(x-x_2)\frac{\delta^4 f_0}{4!h^4} \quad (6)$$

x	f	δf	$\delta^2 f$	$\delta^3 f$	$\delta^4 f$
x_{-3}	f_{-3}	$f_{-2}-f_{-3}$			
x_{-2}	f_{-2}	$f_{-1}-f_{-2}$			
x_{-1}	f_{-1}	f_0-f_{-1}			
x_0	f_0	f_1-f_0			
x_1	f_1	f_2-f_1			
x_2	f_2	f_3-f_2			
x_3	f_3				

Example: Find $P(5.5)$ from the following table

x	1	3	5	7	9
$f(x)$	2	10	15	18	20

Solution: $n+1 = 5 \rightarrow n = 4$

To find $f(5.5)$, we used formula (6)

لان النقطة قريبة للمنتصف ولبعد النقطة عن المنتصف نستعمل صيغة كاوس الامامية

x	f	δf	$\delta^2 f$	$\delta^3 f$	$\delta^4 f$
x_{-2} 1	2				
		8			
x_{-1} 3	10		-3		
		5		1	
x_0 5	15		-2		0
		3		1	
x_1 7	18		-1		
		2			
x_2 9	20				

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$$f(x) \simeq P(x) = 15 + (x-5)\cdot\frac{3}{1!(2)} + (x-5)(x-7)\cdot\frac{-2}{2!(2)^2} + (x-5)(x-7)(x-3)\cdot\frac{1}{3!(2)^3} + (x-5)(x-7)(x-3)(x-9)\cdot\frac{0}{4!(2)^4}$$

$$f(5.5) = 15.89843$$

(6) Gauss Backward formula صيغة كاوس التراجعية

$$f(x) \simeq P_n(x) = f(x_0) + (x-x_0)\frac{\delta f_{-1}}{1!h} + (x-x_0)(x-x_{-1})\frac{\delta^2 f_0}{2!h^2} + (x-x_0)(x-x_1)(x-x_{-1})\frac{\delta^3 f_{-1}}{3!h^3} + (x-x_0)(x-x_{-1})(x-x_1)(x-x_{-2})\frac{\delta^4 f_0}{4!h^4} \quad --- (7)$$

Ex:— Find $f(4.5)$ from the following table

x	1	3	5	7	9
$f(x)$	2	10	15	18	20

Solution: $n+1=5 \Rightarrow n=4$

To Find $f(5.5)$, we used formula (7)

x	f	δf	$\delta^2 f$	$\delta^3 f$	$\delta^4 f$
$x_{-2}=1$	2				
		8			
$x_{-1}=3$	10		-3		
		5		1	
$x_0=5$	15		-2		0
		3		1	
$x_1=7$	18		-1		
		2			
$x_2=9$	20				

$$f(x) \simeq P(x) = 15 + (x-5)\frac{5}{1!(2)} + (x-5)(x-3)\frac{-2}{2!(2)^2} + (x-5)(x-7)\frac{1}{3!(2)^3} + (x-5)(x-3)(x-7)(x-1)\cdot\frac{0}{4!(2)^4}$$

$$f(x) = \frac{1}{x-2}$$

$$f(4.5) = 13.6015625$$

$$x - x_2 = x - (x_1 + h) = x - (x_0 + h + h) = x - (x_0 + 2h)$$

Sub: الاعداد

Date: ١٨/١٢/ ٢٠١٠

— Chapter 5 —

Numerical Differentiation and Integration

التفاضل والتكامل العددي

① If $x_{i+1} - x_i = h$ — اذا كانت المسافات متساوية

ⓐ numerical differentiation of Newton's forward formula — صيغة نيوتن الامامية

$$f(x) = f(x_0) + (x-x_0)\frac{\Delta f_0}{1!\,h} + (x-x_0)(x-x_1)\frac{\Delta^2 f_0}{2!\,h^2} + (x-x_0)(x-x_1)(x-x_2)\frac{\Delta^3 f_0}{3!\,h^3} + \dots + (x-x_0)(x-x_1)\cdots(x-x_{n-1})\frac{\Delta^n f_0}{n!\,h^n} \quad ①$$

Let $\frac{x-x_0}{h} = q \Rightarrow x - x_0 = hq$

$$x - x_1 = x - (x_0 + h) = x - x_0 - h = hq - h = h(q-1)$$

$$x - x_2 = x - (x_1 + h) = x - x_0 - 2h = hq - 2h = h(q-2)$$

In General

$$x - x_k = h(q-k) \quad , \quad k = 0, 1, 2, \dots$$

Substitute the last relations into ①, we obtain

$$f(x) = f(x_0) + q\Delta f_0 + q(q-1)\frac{\Delta^2 f_0}{2} + q(q-1)(q-2)\frac{\Delta^3 f_0}{6} + q(q-1)(q-2)(q-3)\frac{\Delta^4 f_0}{24} + \dots$$

$$f'(x) = \frac{df}{dq}\cdot\frac{dq}{dx} = \frac{1}{h}\frac{df}{dq}$$

مهم ونشتقها

$$f(x) = f(x_0) + q\Delta f_0 + \frac{1}{2}(q^2 - q)\Delta^2 f_0 + \frac{1}{6}(q^3 - 3q^2 + 2q)\Delta^3 f_0 + \frac{1}{24}(q^4 - 6q^3 + 11q^2 - 6q)\Delta^4 f_0 + \dots$$

$$f'(x) = \frac{1}{h}\frac{df}{dq} = \frac{1}{h}\left[\Delta f_0 + \frac{1}{2}(2q-1)\Delta^2 f_0 + \frac{1}{6}(3q^2 - 6q + 2)\Delta^3 f_0 + \frac{1}{24}(4q^3 - 18q^2 + 22q - 6)\Delta^4 f_0\right]$$

$$f'(x) = \frac{1}{h}\left[\Delta f_0 + \frac{1}{2}(2q-1)\Delta^2 f_0 + \frac{1}{6}(3q^2 - 6q + 2)\Delta^3 f_0 + \frac{1}{12}(2q^3 - 9q^2 + 11q - 3)\Delta^4 f_0 + \dots\right] \quad ②$$

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Date:

$$f''(x) = \frac{d^2 f}{dq^2} \cdot \frac{d^2 q}{dx^2}$$

$$f''(x) = \frac{1}{h^2}\left[\Delta^2 f_0 + (q-1)\Delta^3 f_0 + \frac{1}{12}(6q^2 - 18q + 11)\Delta^4 f_0 + \dots\right] \quad (3)$$

If $x = x_i$ نقطة من نقاط الجدول

set $x_i = x_0 \Rightarrow q = 0$, $f(x_i) = f(x_0)$

$$f'(x_0) = \frac{1}{h}\left[\Delta f_0 - \frac{1}{2}\Delta^2 f_0 + \frac{1}{3}\Delta^3 f_0 - \frac{1}{4}\Delta^4 f_0 + \dots\right] \quad (4)$$

$$f''(x_0) = \frac{1}{h^2}\left[\Delta^2 f_0 - \Delta^3 f_0 + \frac{11}{12}\Delta^4 f_0 - \frac{5}{6}\Delta^5 f_0 + \dots\right] \quad (5)$$

Example: — find $f'(2.5)$, $f''(2.5)$ and $f'(3)$ from the following table

x	2	3	4	5	6
$f(x)$	5	10	17	26	37

$n+1 = 5 \Rightarrow n = 4$

Solution:

x	f	Δf_0	$\Delta^2 f_0$	$\Delta^3 f_0$	$\Delta^4 f_0$
2	5				
		5			
3	10		2		
		7		0	
4	17		2		0
		9		0	
5	26		2		
		11			
6	37				

$$q = \frac{x - x_0}{h} = \frac{2.5 - 2}{1} = 0.5$$

$$f'(2.5) = \frac{1}{1}\left[5 + \frac{1}{2}(2(0.5-1)) * 2 + \frac{1}{6}(3(0.5)^2 - 6(0.5) + 2) * 0 + 0\right]$$

$f'(2.5) = 5 \Rightarrow f''(2.5) = \frac{1}{(1)^2}[2 + 0 + 0] = 2$

$f'(3) = \frac{1}{1}\left[7 - \frac{1}{2} * 2 + \frac{1}{3} * 0 + \frac{1}{4} * 0\right] \Rightarrow f'(3) = 6$

$x_{n-1} = x_n - h$

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(b) Numerical Differentiation of Newton – Backward formula

$$f(x) = f(x_n) + (x-x_n)\frac{\nabla f_n}{1!\,h} + (x-x_n)(x-x_{n-1})\frac{\nabla^2 f_n}{2!\,h^2} + \cdots + (x-x_n)(x-x_{n-1})\cdots(x-x_0)\frac{\nabla^n f_n}{n!\,h^n} \quad (6)$$

Let $\frac{x-x_n}{h} = q \Rightarrow x - x_n = hq$

$$x - x_{n-1} = x - (x_n - h) = x - x_n + h = hq + h = h(q+1)$$
$$x - x_{n-2} = x - (x_n - 2h) = h(q+2)$$

In general

$$x - x_{n-k} = h(q+k), \quad k = 0, 1, 2, \ldots$$

Substitute the last relations, we have

$$f(x) = f(x_n) + q\nabla f_n + q(q+1)\frac{\nabla^2 f_n}{2!} + q(q+1)(q+2)\frac{\nabla^3 f_n}{3!} + q(q+1)(q+2)(q+3)\frac{\nabla^4 f_n}{4!} + \cdots$$

$$f(x) = f(x_n) + q\nabla f_n + \tfrac{1}{2}(q^2+q)\nabla^2 f_n + \tfrac{1}{6}(q^3+3q^2+2q)\nabla^3 f_n + \tfrac{1}{24}(q^4+6q^3+11q^2+6q)\nabla^4 f_n + \cdots$$

$$f'(x) = \frac{df}{dq}\cdot\frac{dq}{dx} = \frac{1}{h}\frac{df}{dq}$$

$$= \frac{1}{h}\left(\nabla f_n + \tfrac{1}{2}(2q+1)\nabla^2 f_n + \tfrac{1}{6}(3q^2+6q+2)\nabla^3 f_n + \tfrac{1}{12}(2q^3+9q^2+11q+3)\nabla^4 f_n + \cdots\right) \quad (7)$$

$$f''(x) = \frac{1}{h^2}\left[\nabla^2 f_n + (q+1)\nabla^3 f_n + \tfrac{1}{12}(6q^2+18q+11)\nabla^4 f_n + \cdots\right] \quad (8)$$

2

Sub:

Date:

If $x = x_i$ نقطة منتهاها الجدول

set $x_i = x_n \Rightarrow q = 0$, $f(x_i) = f(x_n)$

$$f'(x_n) = \frac{1}{h}\left[\nabla f_n + \frac{1}{2}\nabla^2 f_n + \frac{1}{3}\nabla^3 f_n + \frac{1}{4}\nabla^4 f_n + \dots\right] \quad (9)$$

$$f''(x_n) = \frac{1}{h^2}\left[\nabla^2 f_n + \nabla^3 f_n + \frac{11}{12}\nabla^4 f_n + \frac{5}{6}\nabla^5 f_n + \dots\right] \quad (10)$$

Example: find $f'(5.5)$, $f''(5.5)$ and $f'(6)$

x	f	∇f	$\nabla^2 f$	$\nabla^3 f$	$\nabla^4 f$
2	5				
3	10	5	2		
4	17	7	2	0	
5	26	9	2	0	0
6	37	11			

x	2	3	4	5	6
f(x)	5	10	17	26	37

$$q = \frac{x - x_n}{h} = \frac{5.5 - 6}{1} = -0.5$$

$$f'(5.5) = \frac{1}{1}\left[11 + \frac{1}{2}(2(-0.5)+1) \times 2 + 0 + 0\right]$$

$f'(5.5) = 11$

$f''(5.5) = 2$

$$f'(6) = \frac{1}{1}\left[11 + \frac{1}{2} \times 2 + 0 + 0\right] = 12$$

(3)

Sub: الخميس «تحضير»
Date: ٢٠١٥ ١٣/١٨

اكتب برنامج لإيجاد حل النظام

$$3x_1 + x_2 + x_3 = 13$$
$$x_1 + 2x_2 - x_3 = 4$$
$$x_1 + x_2 + x_3 = 9$$

إذا أن $x^{(0)} = \begin{pmatrix}0\\0\\0\end{pmatrix}$ وان $\epsilon = 10^{-5}$ باستخدام طريقة كاوس سايدل

```
a=[3 1 1;1 2 -1;1 1 1]; b=[13;4;9];
n=length(b); x=zeros(n,1); P=[0;0;0]; e=10^-5; err=1;
while err>e
  for j=1:n
    if j==1
x(1)=(b(1)-a(1,2:n)*P(2:n))/a(1,1);
    elseif j==n
x(n) =(b(n)-a(n,1:n-1)*x(1:n-1))/a(n,n);
    else
x(j) = (b(j)-a(j,1:j-1)*x(1:j-1)-a(j,j+1:n)*
                    P(j+1:n))/a(j,j);
    end
  end
err= norm(x-P);
P= x;
```


تمرين الخامس //مناقشة// للفصل الاول

x	2	9	16	23
f(x)	45	48	50	53

Find f(5) by using Lagrange Interpolating

$$n+1 = 4 \Rightarrow n = 3$$

$$f(x_3) = P_3(x) = \sum_{i=0}^{3} L_i(x) f(x_i)$$

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$$L_0(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x-x_3)}=\frac{(x-9)(x-16)(x-23)}{(2-9)(2-16)(2-23)}$$

$$L_1(x)=\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}=\frac{(x-2)(x-16)(x-23)}{(9-2)(9-16)(9-23)}$$

$$L_2(x)=\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x-x_3)}=\frac{(x-2)(x-9)(x-23)}{(16-2)(16-9)(16-23)}$$

$$L_3(x)=\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_3-x_0)(x_3-x_1)(x-x_2)}=\frac{(x-2)(x-9)(x-16)}{(23-2)(23-9)(23-16)}$$

$$f(x)=L_0 f(x_0)+L_1(x) f(x_1)+L_2(x) f(x_2)+L_3 f(x_3)$$

تكميل
=

الطريقة الثانية

$x_{i+1}-x_i=h \rightarrow$ where $h=7$

$n+1=4 \Rightarrow n=3$

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$
2	45			
		3		
9	48		-1	
		2		2
16	50			
		3		
23	53			

$$f(x)\simeq f(x_0)+\sum_{k=0}^{3}\frac{\Delta^k f}{k!\,h^k}(x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

$$\simeq f(x_0)+\frac{\Delta f_0}{h}(x-x_0)+\frac{\Delta^2 f_0}{2!\,h^2}(x-x_0)(x-x_1)+\frac{\Delta^3 f_0}{3!\,h^3}(x-x_0)(x-x_1)(x-x_2)$$

وكن يشتقها غير موجودة (٢١) المشتقه

Sub: معادلة لوبلاس
Date: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f(x,y)$

نيوتن جوس

$$f'(1.5) = 0.6666$$

* If $f(x)$ is given :-
for Let $\Delta x = h$

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \dots \quad (*)$$

$$f(x-h) = f(x) - h f'(x) + \frac{h^2}{2!} f''(x) - \frac{h^3}{3!} f'''(x) + \dots \quad (\#)$$

From (*), we have

① $f'(x) = \frac{f(x+h)-f(x)}{h} + o(h)$ (forward difference formula)

From (#), we have

② $f'(x) = \frac{f(x)-f(x-h)}{h} + o(h)$ (backward difference formula)

Substract (#) from (*) we have

③ $f'(x) = \frac{f(x+h)-f(x-h)}{2h} + o(h^2)$ (Central difference formula)

$$f''(x) = \frac{f(x+h)-2f(x)+f(x-h)}{h^2} + o(h^2)$$

Ex: — Find $f'(2)$, where $f(x) = x^2 \dots$
$h = 0.1$

① Approximate ② exact ③ Compare the exact the Approximate.

Solution: ① forward difference formula

$$f'(x) = \frac{f(x+h)-f(x)}{h}$$

$$f'(2) = \frac{f(2+0.1)-f(2)}{0.1} \Rightarrow f'(2) = \frac{f(2.1)-f(2)}{0.1}$$

Sub:

Date:

$$= \frac{2.1e^{2.1} - 2e^{2}}{0.1} = 23.7084462$$

(2) Backward difference formula

$$f'(x) = \frac{f(x) - f(x-h)}{h}$$

$$f'(2) = \frac{f(2) - f(2-0.1)}{0.1} = \frac{f(2) - f(1.9)}{0.1}$$

$$= 20.749276$$

(3) Centeral difference formula

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

h^2 = order

$$f'(2) = \frac{f(2+0.1) - f(2-0.1)}{2*0.1} = \frac{f(2.1) - f(1.9)}{2*0.1}$$

$$f'(2) = 22.2287869$$

$$\therefore f(x) = xe^{x} \Rightarrow f'(x) = xe^{x} + e^{x} = (x+1)e^{x}$$

$$f'(2) = (2+1)e^{2} = 3e^{2}$$

$$f'(2) = 22.167168$$

خطأ

وعلينا المقارنة بين الثلاث

$$E_F = |22.167168 - 23.7084462|$$

$$E_B = |22.167168 - 20.749276| =$$

$$E_C = |22.167168 - 22.2287869|$$

تمرين

$$\boxed{h = 0.001}$$ H.W

Sub: مختبر عملي

Date: ٢٩/٤/٢٠١٨

((مختبر يوم الاربعاء الرياضيات ...))

س/ اكتب برنامج لايجاد f(1.5) من الجدول الاتي باستخدام طريقة لاكرانج

x	1	2	3	4	5
$f(x)$	2	5	17	26	37

```
clc
clear
x=[1;2;3;4;5] ; y=[2;5;10;17;26];
w=length(x);
n=w-1;
L=zeros(w,w);
for k=1:w
v=1;
for j=1:w
if k~=j
v=conv(v,poly(x(j)))/(x(k)-x(j));
end
end
L(k,:)=v;
end
c=y'*L
polyval(c,1.5)
```

- Chapter 6 -

Numerical Integration
التكامل العددي

To evalute $\int_a^b f(x)\,dx$, we divide the interval $[a,b]$ into n subintervals, which have the same length i.e $h = \frac{b-a}{n}$

$x_0 = a$, $x_n = b$, $x_i = x_0 + ih$, $i = 1,2,3,\dots,n-1$

Let $f(x) \simeq P_n(x)$

$$\therefore \int_a^b f(x)\,dx = \int_a^b P_n(x)\,dx \quad \text{--- (1)}$$

we approximate $P_n(x)$ by newton forward-difference interpolating

$$P_n(x) = f_0 + (x-x_0)\frac{\Delta f_0}{1!h} + (x-x_0)(x-x_1)\frac{\Delta^2 f_0}{2!h^2} + \dots$$

$$+ (x-x_0)(x-x_1)\dots(x-x_{n-1})\frac{\Delta^n f_0}{n!h^n}$$

$$\therefore \int_a^b f(x)\,dx = \int_{x_0}^{x_n} \left(f_0 + (x-x_0)\frac{\Delta f_0}{1!h} + (x-x_0)(x-x_1)\frac{\Delta^2 f_0}{2!h^2} + \dots \right) dx \quad \text{--- (2)}$$

Let $\frac{x-x_0}{h} = q \rightarrow x - x_0 = hq$

$\rightarrow x - x_1 = h(q-1)$

$x - x_2 = h(q-2)$

$\vdots$

$(x - x_{n-1}) = h(q-(n-1))$

$x - x_n = h(q-n)$ and

$dx = h\,dq$

if $x = x_0 \longrightarrow q = 0$

if $x = x_n \longrightarrow q = n$

substitute the last relations into ② we have

$$\int_a^b f(x)\,dx = \int_0^n \left(f_0 + q\Delta f_0 + \frac{q(q-1)}{2}\Delta^2 f_0 + \frac{q(q-1)(q-2)}{6}\Delta^3 f_0 + \frac{q(q-1)(q-2)(q-3)}{24}\Delta^4 f_0 + \dots\right) h\,dq$$

$$= h \int_0^n \left(f_0 + q\Delta f_0 + \frac{q(q-1)}{2}\Delta^2 f_0 + \dots\right) dq \quad \text{--- ③}$$

The equation ③ is called Newton–cost formula $\boxed{\text{H.W}}$ Use Newton Back ward – difference interpolating formula to obtain Newton – cost formula

① Trapezoidal formula — قاعدة شبه المنحرف

let $n=1$, $[a,b] = [x_0, x_1]$ بالكامل
from ③ we have

$$\int_a^b f(x)\,dx = h \int_0^1 \left(f_0 + q\Delta f_0 + \frac{q(q-1)}{2}\Delta^2 f_0 + \frac{q(q-1)(q-2)}{6}\Delta^3 f_0 + \dots\right) dq$$

$$= h\left[f_0 q + \frac{q^2}{2}\Delta f_0 + \frac{1}{2}\left(\frac{q^3}{3} - \frac{q^2}{2}\right)\Delta^2 f_0 + \dots\right]_0^1$$

$$= h\left(f_0 + \frac{1}{2}\Delta f_0 - \frac{1}{12}\Delta^2 f_0 + \dots\right)$$

$$= h\left(f_0 + \frac{1}{2}(f_1 - f_0)\right) - \frac{h}{12}\Delta^2 f_0 \qquad \Delta f_0 = f_1 - f_0$$

$$\int_a^b f(x)\,dx = \frac{h}{2}(f_0 + f_1) - \frac{h}{12}\Delta^2 f_0$$

$$\Rightarrow \int_a^b f(x)\,dx \simeq \frac{h}{2}(f_0 + f_1) \quad \text{--- ④}$$

$$E = \frac{-h}{12}\Delta^2 f_0$$

(7)

Sub:

Date:

$$f''(\xi) = \frac{1}{h^2}\left(\Delta^2 f_0 + (q-1)\Delta^3 f_0 + \dots\right)$$

$$f''(\xi) = \frac{1}{h^2}\Delta^2 f_0 \rightarrow E = \frac{-h^3}{12} f''(\xi) \; , \; \xi \in [x_0, x_n] \quad (5)$$

If $[a,b] = [x_0, x_n]$

$$\Rightarrow [x_0, x_1], [x_1, x_2], \dots, [x_{n-1}, x_n]$$

$$\int_a^b f(x)\,dx = \int_{x_0}^{x_n} f(x)\,dx = \int_{x_0}^{x_1} f(x)\,dx + \int_{x_1}^{x_2} f(x)\,dx + \dots + \int_{x_{n-1}}^{x_n} f(x)\,dx$$

From (4), we have

$$\int_a^b f(x)\,dx = \frac{h}{2}[f_0 + f_1] + \frac{h}{2}[f_1 + f_2] + \dots + \frac{h}{2}[f_{n-1} + f_n]$$

$$\int_a^b f(x)\,dx = \frac{h}{2}[f_0 + 2f_1 + 2f_2 + \dots + 2f_{n-1} + f_n] \quad (6)$$

From (5), we have

$$E_T = \frac{-nh^3}{12} f''(\xi) \; , \; \xi \in [x_0, x_n] \stackrel{\text{أو}}{=} [a,b] \quad (7)$$

the equation (6) is called Trapezoidal formula and (7) the error of the method.

Ex: Find $\int_0^6 (2 + \sin(2\sqrt{x}))\,dx$ by using Trapezoidal formula and the error of the method where $n = 10$. $E_T = \frac{-nh^3}{12} f''(\xi)$

AL-WARAQ

$f_0 = f(x_0) = f(a) = f(1)$ / $f(x_1) = f(x_0+h)$
$= f(1+\frac{1}{2}) = f(\frac{3}{2})$

$f_1 = f(x_0+h) = f(1+\frac{1}{2})$ ---

(8)

Sub: $2f_2 = f(x_0+2h)$

Date:

Solution:- $a=1$, $b=6$, $f(x) = 2 + \sin(2\sqrt{x})$

$$h = \frac{b-a}{h} = \frac{6-1}{10} = \frac{1}{2}$$

هنا لا نستخدم $\Delta^4 f_0$

$$\int_a^b f(x)\,dx = \int_1^{6} (2 + \sin(2\sqrt{x}))\,dx = ??$$

نعوض بكل نقطة موجودة $f(\xi)$

$$= \frac{\frac{1}{2}}{2}\Big[f(1) + 2f(\tfrac{3}{2}) + 2f(2) + 2f(\tfrac{5}{2}) + 2f(3) + 2f(\tfrac{7}{2}) + 2f(4) + 2f(\tfrac{9}{2}) + 2f(5) + 2f(\tfrac{11}{2}) + f(6)\Big]$$

$$= 8.19385457$$

$f_9 = f(\frac{11}{2})$ ... $2f_{n-1}$

$$E_T = \left|\frac{-nh^3}{12} f''(\xi)\right| \Rightarrow f''(\xi) = \max(|f''(a)|, |f''(b)|)$$

$$f''(x) = \frac{d^2}{dx^2}(2 + \sin(2\sqrt{x})) = -\frac{\sin(2\sqrt{x})}{\sqrt{x}} - \frac{\cos(2\sqrt{x})}{2x\sqrt{x}}$$

$$f''(1) = -0.701224, \quad f''(6) = 0.15746257$$

$$\therefore f''(\xi) = \max(|-0.701224|, |0.15746257|) = \boxed{0.701224}$$

$$\therefore E_T = \left|\frac{-10(\frac{1}{2})^3}{12} * 0.701224\right| = E_T = 0.0730144$$

② Simpson's formula

let $n=2$, $[a,b] = [x_0, x_2]$

From ③, we have

$$\int_a^b f(x)\,dx = h\int_0^2 \Big[f_0 + q\Delta f_0 + \frac{q(q-1)}{2}\Delta^2 f_0 + \frac{q(q-1)(q-2)}{6}\Delta^3 f_0 + \frac{q(q-1)(q-2)(q-3)}{24}\Delta^4 f_0 + \dots\Big]$$

Δ^4 ...

$$= h\left(2f_0 + 2\Delta f_0 + \frac{1}{3}\Delta^2 f_0 - \frac{1}{90}\Delta^4 f_0 + \dots\right)$$

Sub:

Date:

$$\int_a^b f(x)\,dx = \frac{h}{3}(f_0 + 4f_1 + f_2) \quad \text{--- (8)}$$

$$E = \frac{-h^5}{90} f^{(4)}(\xi), \quad \xi \in [a,b] \quad \text{--- (9)}$$

let $[a,b] = [x_0, x_n]$ تقسيم

$[x_0, x_2], [x_2, x_4], \dots, [x_{n-2}, x_n]$

$$\therefore \int_a^b f(x)\,dx = \int_{x_0}^{x_n} f(x)\,dx$$

$$= \int_{x_0}^{x_2} f(x)\,dx + \int_{x_2}^{x_4} f(x)\,dx + \dots + \int_{x_{n-2}}^{x_n} f(x)\,dx$$

From (8), we have

$$\int_a^b f(x)\,dx = \frac{h}{3}(f_0 + 4f_1 + f_2) + \frac{h}{3}(f_2 + 4f_3 + f_4) + \dots + \frac{h}{3}(f_{n-2} + 4f_{n-1} + f_n)$$

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \dots + 2f_{n-2} + 4f_{n-1} + f_n\right] \quad \text{--- (10)}$$

formal (6) is called simpson's method

from (9), we have — error of method

$$E_s = \left|\frac{-h\,h^5}{180} f^{(4)}(\xi)\right|, \quad \xi \in [x_0, x_n] = [a,b] \quad \text{--- (11)}$$

EX:— find $\int_1^6 (2 + \sin(2\sqrt{x}))\,dx$ by using simpson's formula and E_s, where $n = 10$

Solution:—

$a = 1, \; b = 6, \; n = 10$

$h = \frac{b-a}{n} = \frac{1}{2}$

Sub:

Date:

$$\int_1^6 (2+\sin(2\sqrt{x})) = \frac{\frac{1}{2}}{3}\left[f(1)+4f(\tfrac{3}{2})+2f(2)+4f(\tfrac{5}{2})+2f(3)+4f(4)+4f(\tfrac{7}{2})+2f(5)+4f(\tfrac{11}{2})+f(6)\right]$$

$$\int_1^6 (2+\sin(2\sqrt{x})) = 8.183015$$

$E_S = 0.005154$

الاربعاء ٢٠١٥/٤/٨

Note : ① we can use simpson's rule if n is even

② we can determine the value (n) if the error of trapezoidal is given (E_T)

$$E_T = \left|\frac{-nh^3}{12} f''(\xi)\right| , \quad \xi \in [a,b] \qquad h = \frac{b-a}{n}$$

$$= \frac{n \cdot \frac{(b-a)^3}{n^3}}{12} \cdot |f''(\xi)| = \frac{(b-a)^3}{12n^2} |f''(\xi)|$$

$$n \geq \sqrt{\frac{(b-a)^3}{12E_T} \cdot |f''(\xi)|}$$

where $f''(\xi) = \max(|f''(a)|, |f''(b)|)$

③ we can determine the value (n) if the error of simpson's rule given (E_S)

$$E_S = \left|\frac{-nh^5}{180} f^{(4)}(\xi)\right|$$

$$n \geq \sqrt[4]{\frac{(b-a)^5}{180 * E_S} \cdot |f^{(4)}(\xi)|}$$

where $f^{(4)}(\xi) = \max(|f^{(4)}(a)|, |f^{(4)}(b)|)$

Sub:

Date:

Ex: Determine the value h required to approximate

$$\int_0^2 \frac{1}{x+4}\,dx \quad \text{within } \epsilon = 10^{-5}$$

① Use Trapezoidal rule.

② Use Simpson's rule.

Solution: ①

$a=0 \;,\; b=2 \;,\; f(x) = \frac{1}{x+4}$

$f''(x) = \frac{2}{(x+4)^3}$

$f''(a) = \frac{2}{64} = \frac{1}{32}$

$f''(b) = \frac{2}{216} = \frac{1}{108}$

$f''(\xi) = \max(|f''(a)|, |f''(b)|) = \frac{1}{32}$

نعا نختار العدد الي يعطي اكبر [illegible]

$n \geq \sqrt{\frac{(b-a)^3}{12 * E_T} \cdot f''(\xi)} \rightarrow n \geq \sqrt{\frac{(2)^3}{12*10^{-5}} \cdot \frac{1}{32}} = 45.6435464$

$\therefore \boxed{n \simeq 46} \rightarrow \boxed{h = \frac{b-a}{n}} = \frac{2}{46} = \frac{1}{23}$

②

$n \geq \sqrt[4]{\frac{(b-a)^5}{180 * E_S} \cdot f^{(4)}(\xi)}$

$f^{(4)}(x) = \frac{24}{(x+4)^5} \Rightarrow f^{(4)}(0) = \frac{3}{128} \;,\; f^{(4)}(2) = \frac{1}{324}$

$f''(\xi) = \max(|f^{(4)}(a)|, |f^{(4)}(b)|)$

$= \max\left(\frac{3}{128}, \frac{1}{324}\right) = \frac{3}{128}$

$n \geq \sqrt[4]{\frac{2^5}{180*10^{-5}} \cdot \frac{3}{128}} = 4.51801$

$n \geq 4.51801 \rightarrow \boxed{n \simeq 6} \rightarrow h = \frac{1}{3}$

التقريب للعدد 6 ولا 5 لأن [illegible]
بأخذ الأعداد الزوجية [illegible]
[illegible]

AL-WARAQ

(3) Romberg Integration

To begin the presentation of Romberg Integration scheme recall Trapezoidal rule for approximating the integral $\int_a^b f(x)\,dx$ using n subintervals is

$x_i = a + ih$

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right] - \frac{nh^3}{12} f''(\xi)$$

$$= \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(a+ih)\right] - \frac{nh^3}{12} f''(\xi)$$

where $\xi \in [a,b]$, $h = \frac{b-a}{n}$

we first obtain Trapezoidal rule approximations with $n_1 = 1$, $n_2 = 2$, $n_3 = 4$, $\ldots$, and $n_m = 2^{m-1}$

where m is positive integer

The value of the step h_k corresponding to n_k are

$$h_k = \frac{b-a}{n_k} = \frac{b-a}{2^{k-1}}$$

with this notation the trapezoidal rule becomes

$$\int_a^b f(x)\,dx = \frac{h_k}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n_k-1} f(a+ih_k)\right] - \frac{n_k h_k^3}{12} f''(\xi)$$

$$= \frac{h_k}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{2^{k-1}-1} f(a+ih_k)\right] - \frac{2^{k-1} h_k^3}{12} f''(\xi)$$

$$R_{k,1} = \frac{h_k}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{2^{k-1}-1} f(a+ih_k)\right]$$

Sub:

Date:

$$\rightarrow R_{1,1} = \frac{h_1}{2}\left[f(a) + f(b) \right] \quad --- \quad (12)$$

$$R_{2,1} = \frac{h_2}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{1} f(a + i h_2) \right]$$

$$= \frac{b-a}{4}\left[f(a) + f(b) + 2 f(a + h_2) \right]$$

$$= \frac{1}{2}\left[R_{1,1} + h_1 f(a + h_2) \right]$$

$$R_{3,1} = \frac{1}{2}\left[R_{2,1} + h_2 \{ f(a + h_3) + f(a + 3h_3) \} \right]$$

In General

$$R_{k,1} = \frac{1}{2}\left[R_{k-1,1} + h_{k-1} \sum_{i=1}^{2^{k-2}} f(a + (2i-1) h_k) \right] \quad ---- \quad (13)$$

Equation (12) & (13) are called the first step of Romberg integration

تعديل الناتج من التكامل ...

Now by Richardson's extrapolation, we have ← التعديل الخارجي

$$R_{k,j} = R_{k,j-1} + \frac{R_{k,j-1} - R_{k-1,j-1}}{4^{j-1} - 1}$$

$$R_{k,j} = \frac{4^{j-1} R_{k,j-1} - R_{k-1,j-1}}{4^{j-1} - 1} \quad --- \quad (14)$$

$k = 2, 3, \ldots, n \quad , \quad j = 2, \ldots, n$

The results that are generated form these formula are shown table

Sub:

Date:

$$\begin{array}{cccccc}
R_{1,1} & & & & & \\
R_{2,1} & R_{2,2} & & & & \\
R_{3,1} & R_{3,2} & R_{3,3} & & & \\
R_{4,1} & R_{4,2} & R_{4,3} & R_{4,4} & & \\
\vdots & \vdots & \vdots & \vdots & \ddots & \\
R_{n,1} & R_{n,2} & R_{n,3} & R_{n,4} & \cdots & \boxed{R_{n,n}}
\end{array}$$

Ex: Use Romberg integration to compute $\int_0^1 x^2 e^{-x} dx$, $n=3$

Solution: $n=3$, $a=0$, $b=1$, $f(x) = x^2 e^{-x}$

from (12)

$$R_{1,1} = \frac{h_1}{2}[f(a)+f(b)] = \frac{1}{2}[f(0)+f(1)] = \frac{1}{2}[0+e^{-1}]$$

$$R_{1,1} = \frac{1}{2}e^{-1} = 0.1839397206$$

from (13)

$$R_{2,1} = \frac{1}{2}\left[R_{1,1} + h_1 * \sum_{i=1}^{1} f(a+(2i-1)h_2)\right]$$

$$= \frac{1}{2}[R_{1,1} + h_1 * f(a+h_2)] = \frac{1}{2}[R_{1,1} + 1 \cdot f(\tfrac{1}{2})] = 0.1677801928$$

$$R_{3,1} = \frac{1}{2}[R_{2,1} + h_2\{f(a+h_3) + f(a+3h_3)\}]$$

$$= \frac{1}{2}[R_{2,1} + \frac{1}{2}\{f(\tfrac{1}{4}) + f(\tfrac{3}{4})\}]$$

$h_1 = 1$, $h_2 = \frac{1}{2}$, $h_3 = \frac{1}{4}$

$$R_{3,1} = 0.1624884051$$

from (14), we obtain

$$R_{2,2} = \frac{4^{2-1}R_{2,1} - R_{1,1}}{4^{2-1}-1} = \frac{4R_{2,1} - R_{1,1}}{3} = 0.1624016835$$

Sub:

Date:

$$R_{3,2} = \frac{4R_{3,1} - R_{2,1}}{3} = 0.1607224759$$

$$R_{3,3} = \frac{4^{3-1} R_{3,2} - R_{2,2}}{4^{3-1} - 1} = \frac{16 R_{3,2} - R_{2,2}}{15}$$

$$R_{3,3} = 0.160615287$$

$$\therefore \int_0^1 x^2 e^{-x} \, dx = 0.160615287$$

The exact solution of $\int_0^1 x^2 e^{-x} \, dx = 0.160615287$

$T_{10} = 0.160037412$

$T_{10000000} = 0.160602794$

تحضير الخامس ٢٠١٥/٤/٢٩

نكتب برنامج لإيجاد $f(1.5)$ للجدول الآتي

x	1	2	3	4	5	6
$f(x)$	2	5	10	17	26	37

باستخدام طريقة نيوتن

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```
clc
x=[1;2;3;4;5;6]; y=[2;5;10;17;26;37];
n=length(x); d=zeros(n,n); d(:,1)=y';

for j=2:n
 for k=j:n
d(k,j)=(d(k,j-1)-d(k-1,j-1))/(x(k)-x(k-j+1));
end
end
P=d(n,n);
for k=(n-1):-1:1
P=conv(P,poly(x(k)));
m=length(P);
P(m)=P(m)+d(k,k);
end
P
polyval(P,1.5)
```

التنفيذ

P = 0 0 1 0 1

ans

3.2500

(7)

chapter 6
Numerical Solutions of ordinary differential equations

الحلول العددية للمعادلات التفاضلية الاعتيادية

ⓐ Solution of initial-value Problems

$y' = f(t,y)$, $a \le t \le b$

Consider the initial-value Problem of ordinary differential equation is given by

① $y(a)=\alpha$
② $y(a)=\alpha$, $y(b)=\beta$

$$\frac{dy}{dt} = y'(t) = f(t, y(t)) \quad , \quad a \le t \le b \;, \; y(a) = \alpha$$

Numerical methods: الطرائق العددية

① Euler's method:

$$y(t_{i+1}) = y(t_i) + h f(t_i, y_i) + \frac{h^2}{2} f'(\xi_i, y_i) + \cdots$$
$$= y(t_i) + h f(t_i, y_i) + \frac{h^2}{2} y''(\xi) \;, \; \xi \in [a,b]$$

Euler's method constract $w_i = y(t_i)$ for each $i = 0,1,2,\ldots,n$

thus, $w_0 = \alpha$

$$\left. w_{i+1} = w_i + h f(t_i, w_i) \right\} \text{--- ①} \quad i = 0,1,2,\ldots,n$$

تمثل صيغة اويلر لحساب القيم التقريبية لمعادلة التفاضلية الاعتيادية

Ex: Use Euler's method to approximate the solution of for initial-value Problem

$$y' = y - t^2 + 1 \;, \; 0 \le t \le 1$$

$y(0) = 0.5$ with $n = 5$

Solution:

[illegible] 0.2 [illegible] w_2 [illegible] 0.4

$a_0 = 0, b = 1, n = 5 \longrightarrow h = \frac{b-a}{n} = \frac{1}{5} = 0.2$

$t_i = a + ih \quad , i = 0,1,2,3,4,5 \quad , f(t,y) = y - t^2 + 1$

$t_i = 0.2i$

$w_0 = \alpha = 0.5$

$w_{i+1} = w_i + h f(t_i, w_i)$

$w_0 = 0.5$

$w_{i+1} = w_i + 0.2[w_i - t_i^2 + 1] = 1.2w_i - 0.2t_i^2 + 0.2$

$= 1.2w_i - 0.2(0.2i)^2 + 0.2$

$w_{i+1} = 1.2w_i - 0.008i^2 + 0.2$

$i=0$

$w_1 = 1.2w_0 - 0.008(0)^2 + 0.2$

$= 1.2 * 0.5 + 0.2$

$w_1 = 0.8 = w(0.2)$

$i=1$

$w_2 = 1.2w_1 - 0.008(1)^2 + 0.2$

$= 1.2 * 0.8 - 0.008 + 0.2$

$= 1.152 = w(0.4)$

$i=2$

$w_3 = 1.2w_2 - 0.008(2)^2 + 0.2$

$= 1.2 * 1.152 - 0.008 * 4 + 0.2$

$= 1.5504 = w(0.6)$

$i=3$

$w_4 = 1.2w_3 - 0.008(3)^2 + 0.2$

$= 1.9884 = w(0.8)$

$i=4$

$w_5 = w(1) = 2.4587$

the exact solution $\boxed{y(t) = (t+1)^2 - 0.5e^t}$

t_i	w_i	y_i	$\lvert y_i - w_i \rvert$
0	0.5	0.5	0
0.2	0.8	0.829286	0.029286
0.4			
0.6			

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② Higher-order Taylor's method

Taylor's method of order n is given by $w_0 = \alpha$,
$w_{i+1} = w_i + h\,T^{(n)}(t_i, w_i)$ for each $i = 0, 1, \dots, N-1$ ④
where

$$T^{(n)}(t_i, w_i) = f(t_i, w_i) + \frac{h}{2} f'(t_i, w_i) + \dots + \frac{h^{n-1}}{n!} f^{(n-1)}(t_i, w_i)$$

Note that Euler's method is Taylor's method of order one

تايلر من الرتبة الأولى هي اويلر

يعني هنا لو اردنا تايلر بـ (n) تأخذ لحد $(n-1)$ مشتقات

Example :-

Use Taylor method of order two and order four to approximate the solution for the initial value problem

$$y' = y - t^2 + 1, \quad 0 \le t \le 1, \quad y(0) = 0.5, \quad n = 5$$

Sol

$$a = 0, \; b = 1, \; h = 0.2, \; f(t, y) = y - t^2 + 1$$

① order two

$$f(t, y) = y - t^2 + 1$$

$$f'(t, y) = \frac{d}{dt}(y - t^2 + 1) = y' - 2t = y - t^2 + 1 - 2t$$

$$w_0 = 0.5$$

$$w_{i+1} = w_i + h\,T^{(2)}(t_i, w_i)$$

$$= w_i + 0.2\,T^{(2)}(t_i, w_i)$$

$$T^{(2)}(t_i, w_i) = f(t_i, w_i) + \frac{h}{2} f'(t_i, w_i)$$

$$= w_i - t_i^2 + 1 + \frac{0.2}{2}\left[w_i - t_i^2 + 1 - 2t_i\right]$$

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$\therefore w_{i+1} = w_i + 0.2\{w_i - t_i^2 + 1 + 0.1 w_i - 0.1 t_i^2 + 0.1 - 0.2 t_i\}$

$w_{i+1} = 1.22 w_i - 0.22 t_i^2 - 0.04 t_i + 0.22$

$w_{i+1} = 1.22 w_i - 0.0088 i^2 - 0.008 i + 0.22 \qquad (t_i = 0.2 i)$

$w_1 = 1.22 * 0.5 - 0.0088(0)^2 - 0.008(0) + 0.22$

$= 1.22 * 0.5 + 0.22 = 0.83$

$w_2 = 1.2158$

$w_3 = 1.6520706$

$w_4 = 2.1323327$

$w_5 = 2.6486459$

الاربعاء ٢٢/٤/٢٠١٥

(3) Range - Kutta method of ordard four :-

Range - Kutta method of ordard four formula is given by

$w_0 = \alpha$ (initial condition) تحسب بالشرط الابتدائي

$w_{i+1} = w_i + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$, $i = 0, 1, \dots, n-1$

where

$K_1 = h f(t_i, w_i)$

$K_2 = h f(t_i + \frac{h}{2}, w_i + \frac{K_1}{2})$

$K_3 = h f(t_i + \frac{h}{2}, w_i + \frac{K_2}{2})$

$K_4 = h f(t_i + h, w_i + K_3)$

المراد $y' = f(t, y)$

Ex :- Use range - kutta method to approximate Solution to the following initial value Problem.

$y' = y - t^2 + 1$, $0 \le t \le 1$, $y(0) = 0.5$ with $n=5$

Solution :- $a = 0$, $b = 1$, $t_i = a + ih = ih$

$h = \frac{1}{5} = 0.2 \Longrightarrow t_i = 0.2i$

$(t_i + \frac{0.2}{2})^2$

$w_0 = y(0) = 0.5$

$w_{i+1} = w_i + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$K_1 = 0.2 * f(t_i, w_i)$

$= 0.2 * (w_i - t_i^2 + 1) = 0.2(w_i - (0.2i)^2 + 1)$

$K_2 = 0.2 * f(t_i + \frac{h}{2}, w_i + \frac{K_1}{2})$

$= 0.2 * (w_i + \frac{K_1}{2} - (t_i + \frac{h}{2})^2 + 1)$

$K_3 = 0.2 * f(t_i + \frac{h}{2}, w_i + \frac{K}{2})$

$= 0.2(w_i + \frac{K}{2} - (0.2i + 0.1)^2 + 1)$

$K_4 = 0.2 f(t_i + h, w_i + K_3)$

$= 0.2 * (w_i + K_3 - (0.2i + 0.2)^2 + 1)$

معادلة المرتبة الثانية [illegible]

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حل مسائل القيم الحدودية

(b) Solution: Boundary value Problems

Consider the linear second boundary-value Problem is given by

$$y''(x) = P(x)\,y'(x) + q(x)\,y(x) + r(x)\ ,\quad a \le x \le b$$

with Boundary conditions

$$y(a) = \alpha \ ,\quad y(b) = \beta$$

احول المشتقة [illegible] الى [illegible] finite different [illegible]

$$y''(x_i) = P(x_i)\,y'(x_i) + q(x_i)\,y(x_i) + r(x_i)$$

with boundary conditions

$$y(x_0) = \alpha \ ,\quad y(x_n) = \beta$$

From chapter 5, by using finite difference formulas

$$y''(x_i) = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \quad (\text{centeral - difference})$$

المعادلات [illegible] حولناها الى معادلات finite diff.

$$y'(x_i) = \frac{y_{i+1} - y_{i-1}}{2h} \quad (\text{centeral - difference})$$

$$\frac{y_{i+1} + y_{i-1} - 2y_i}{h^2} = \left[P(x_i)\left(\frac{y_{i+1} - y_{i-1}}{2h}\right) + q(x_i)\,y_i + r(x_i) \right] 2h^2$$

$$2y_{i+1} - 4y_i + 2y_{i-1} = h\,P(x_i)\,y_{i+1} - h\,P(x_i)\,y_{i-1} + 2h^2 q(x_i)\,y_i + 2h^2 r(x_i)$$

$$(2 - h\,P(x_i))\,y_{i+1} - (4 + 2h^2 q(x_i))\,y_i + (2 + h\,P(x_i))\,y_{i-1} = 2h^2 r(x_i)$$

$$(2 + h\,P(x_i))\,y_{i-1} - (4 + 2h^2 q(x_i))\,y_i + (2 - h\,P(x_i))\,y_{i+1} = 2h^2 r(x_i)$$

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$i=1$

$$(2+hP(x_1))y_0-(4+2h^2q(x_1))y_1+(2-hP(x_1))y_2=2h^2r(x_1)$$

$$-(4+2h^2q(x_1))y_1+(2-hP(x_1))y_2=2h^2r(x_1)-(2+hP(x_1))y_0 \quad (1)$$

$i=2$

$$(2+hP(x_2))y_1-(4+2h^2q(x_2))y_2+(2-hP(x_2))y_3=2h^2r(x_2) \quad (2)$$

$i=n-1$

$$(2+hP(x_{n-1}))y_{n-2}-(4+2h^2q(x_{n-1}))y_{n-1}+(2-hP(x_{n-1}))y_n=2h^2r(x_{n-1})$$

$$(2+hP(x_{n-1}))y_{n-2}-(4+2h^2q(x_{n-1}))y_{n-1}=2h^2r(x_{n-1})-(2-hP(x_{n-1}))y_n \quad (n-1)$$

The resulting system of equations is expressed in matrix form

$A\underline{y}=\underline{b}$

$$A=\begin{bmatrix} -(4+2h^2q(x_1)) & 2-hP(x_1) & 0 & 0 & 0 \\ 2+hP(x_2) & -(4+2h^2q(x_2)) & 2-hP(x_2) & 0 & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & & 2+hP(x_{n-1}) & -(4+2h^2q(x_{n-1})) \end{bmatrix}$$

$$y=\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-1} \end{bmatrix} ; \quad b=\begin{bmatrix} 2h^2r(x_1)-(2+hP(x_1))y_0 \\ 2h^2r(x_2) \\ \vdots \\ 2h^2r(x_{n-1})-(2-hP(x_{n-1}))y_n \end{bmatrix}$$

The system $A\underline{y}=\underline{b}$ can be solve by (any method "chapter 3")

Sub: ٢٠١٠/٤/٢٤
Date: الخميس

الابتدائية

اكتب برنامج لايجاد الحل التقريبي لمسائل القيم

$y' = y - t^2 + 1 \quad , \quad 0 \le t \le 1 \quad , \quad y(0) = 0.5$

١- باستخدام طريقة اويلر

٢- باستخدام طريقة رانج كوتا

قانون اويلر $w_0 = \alpha = 0.5$

$w_{i+1} = w_i + h f(t_i, w_i)$

solution:

اولاً نكتب البرنامج الفرعي

```
function Z = f(t,y)
  Z = y-t^2 +1
  end
```

```
clc
clear
a=0 ; b=1 ; n=5 ; w(1)=0.5 ; t(1)=a;
h=(b-a)/n ;
for i= 1:n
w(i+1) =w(i)+ h*f(t(i) ,w(i));
t(i+1) =a+i*h ;
end
w'
```

0.5000
0.8000
1.1520
1.5504
1.9885
2.4582

>> format long

نروح الى
m-file

ثم نحصل على قيم بعدد مراتب عشرية

0.500000000000000
0.800000000000000
1.152000000000000
1.550400000000000
1.988480000000000
2.458176000000000

Ex:- Use finite-difference method to approximate the solution to the following boundary value problem $y'' = 4(y-x)$, $0 \le x \le 1$, $y(0)=0$, $y(1)=2$ (1) with $h=\frac{1}{3}$ ($n=3$) (2) with $h=\frac{1}{4}$

$y_0 = y(0) = 0$
$y_n = y_3 = y(b) = y(1) = 2$
$x_i = a + ih$
$x_1 =$

Solution:-

(1) From centeral finite-difference

$$y'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = 4(y_i - x_i)$$

$y_i = y(x_i) = y(\frac{?}{3})$

$$y_{i+1} - 2y_i + y_{i-1} = h^2 4 y_i - 4h^2 x_i$$

$$y_{i-1} - (2+4h^2)y_i + y_{i+1} = -4h^2 x_i$$

$\boxed{i=1}$

$$y_0 - (2+4h^2)y_1 + y_2 = -4h^2 x_1$$

نقيم الـ (i) نأخذ كم
h بحيث هنا كمية $n=3$
ولتأخذ $i=3$
ونأخذ فقط $i=1$, $i=2$

$$-(2+4h^2)y_1 + y_2 = -4h^2 x_1 - y_0$$

$$-(2+4(\tfrac{1}{3})^2)y_1 + y_2 = -4 * (\tfrac{1}{3})^2 * \tfrac{1}{3} - 0$$

$(x_1 = 1*(\frac{1}{3}) = \frac{1}{3})$

$$-\frac{22}{9}y_1 + y_2 = \frac{-4}{27}$$

$$-66y_1 + 27y_2 = -4 \quad (1)$$

$\boxed{i=2}$

$$y_1 - (2+\tfrac{4}{9})y_2 + y_3 = \frac{-4}{9}x_2$$

$$x_2 = 2h = \frac{2}{3}$$

$= x_i = a + ih = \frac{1}{3}$ ($a=0$)

$\therefore x_i = ih$

$x_1 = \frac{1}{3}$, $x_2 = \frac{2}{3}$, $x_3 = 1$

$x_4 = \frac{4}{3}, \ldots$

$$y_1 - \frac{22}{9}y_2 + y_3 = \frac{-4}{9}\cdot\frac{2}{3}$$

$$y_1 - \frac{22}{9}y_2 = \frac{-8}{27} - 2$$

$$27y_1 - 66y_2 = -62 \quad (2)$$

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from ① we obtain

$$y_2 = \frac{-4 + 66 y_1}{27} \quad --- ③$$

Substitute ③ into ② we have

$$27 y_1 - 66\left(\frac{-4+66y_1}{27}\right) = -62$$

$$-1210 y_1 = -646 \Rightarrow y_1 = \frac{-646}{-1210} = 0.533884$$

$$y_2 = 1.1569016$$

x_i	y_i
0	0
$\frac{1}{3}$	0.533884
$\frac{2}{3}$	1.1569016
1	2