Case 1 when $f(x,y) = \underline{e^{ax+by}}$ a, b constants

$F(a,b) = 0$ ((ثانياً

$F(a,b) \neq 0$ ((أولاً

أولاً)) إذا كان $F(a,b) \neq 0$ يكون الحل الخاص :-

$$\boxed{Z_p = \frac{1}{F(D_x, D_y)} e^{ax+by} = \frac{1}{F(a,b)} e^{ax+by}}$$

Ex.2:- Solve $(D_x^2 - D_x D_y - 6D_y^2) Z = e^{2x-3y}$

Solution:-

Find general solution !! $f(x,y) = 0$

$$m^2 - m - 6 = 0 \Rightarrow (m-3)(m+2) = 0$$

$$m_1 = 3 \ , \ m_2 = -2$$

$\boxed{Z_g = \phi_1(y+3x) + \phi_2(y-2x)}$ general solution

Find Particular solution !! $f(x,y) \neq 0$

$$Z = \frac{1}{(D_x - 3D_y)(D_x + 2D_y)} e^{2x-3y}$$

$a = 2$, $b = -3$

$$F(x,y) = (D_x - 3D_y)(D_x + 2D_y) \Rightarrow F(2,-3) = (2+9)(2-6)$$
$$F(2,-3) = -44 \neq 0$$

then $$Z_p = \frac{1}{F(a,b)} e^{ax+by}$$

$$\boxed{Z_p = \frac{1}{-44} e^{2x-3y}}$$ Particular solution

$$Z = Z_g + Z_p$$

$$\boxed{Z = \phi_1(y+3x) + \phi_2(y-2x) - \frac{1}{44} e^{2x-3y}}$$

general solution

ثانياً)) وإذا كان $F(a,b)=0$ يكون الحل الخاص :-

$$Z = \frac{1}{G(a,b)} \cdot \frac{1}{(D_x - \frac{a}{b} D_y)^r} e^{ax+by}$$

where $G(a,b) \neq 0$

* r هي قوى القوس الذي يساوي صفر.

$$\boxed{Z_p = \frac{1}{G(a,b)} \cdot \frac{x^r}{r!} e^{ax+by}}$$

Ex.3:- Solve $(D_x^2 - D_x D_y - 6D_y^2)Z = e^{3x+y}$

Solution:-

Find general solution !! $\quad f(x,y) = 0$

$$m^2 - m - 6 = 0 \Rightarrow (m-3)(m+2) = 0$$

$$m_1 = 3 \ , \ m_2 = -2$$

$$\boxed{Z_g = \phi_1(y+3x) + \phi_2(y-2x)}$$ general solution

Find Particular solution !! $\quad f(x,y) \neq 0$

$$Z = \frac{1}{(D_x - 3D_y)(D_x + 2D_y)} e^{3x+y}$$

$a = 3 \ , \ b = 1$

$$F(D_x, D_y) = (D_x - 3D_y)(D_x + 2D_y)$$

$$F(a,b) = F(3,1) = \underbrace{(3-3)}_{r=1}\overbrace{(3+2)}^{G(a,b)=5}$$

$$Z_p = \frac{1}{G(a,b)} \cdot \frac{x^r}{r!} e^{ax+by}$$

$$\boxed{Z_p = \frac{1}{5} \cdot x \cdot e^{3a+y}}$$ Particular solution

$$Z = Z_g + Z_p$$

$$\boxed{Z = \phi_1(y+3x) + \phi_2(y-2x) + \frac{1}{5} \cdot x \cdot e^{3a+y}}$$

general solution

Case 2 when $f(x,y) = \sin(ax+by)$ or $\cos(ax+by)$ a, b constants

أولاً)) $F(-a^2,-ab,-b^2) \neq 0$

ثانياً)) $F(-a^2,-ab,-b^2) = 0$

*) هذه الحاله تحل على رتبه ثانية فقط.

أولاً)) $F(-a^2,-ab,-b^2) \neq 0$:- يكون الحل الخاص

$$Z_P = \frac{1}{F(-a^2,-ab,-b^2)}\left(\sin(ax+by) \text{ or } \cos(ax+by)\right)$$

Ex. 4 :- Solve $(D_x^2 - D_x D_y - 6D_y^2)Z = \sin(2x-3y)$

Solution :-

Find general solution !! $f(x,y) = 0$

$$m^2 - m - 6 = 0 \Rightarrow (m-3)(m+2) = 0$$

$$m_1 = 3 \;,\; m_2 = -2$$

$\boxed{Z_g = \phi_1(y+3x) + \phi_2(y-2x)}$ general solution

$$(D_x^2-D_xD_y-6D_y^2)Z=\sin(2x-3y)$$

$$\Rightarrow Z=\frac{1}{D_x^2-D_xD_y-6D_y^2}\sin(2x-3y)$$

$a=2\ ,\ b=-3$

$$F(-a^2,-ab,-b^2)=F(-4,6,-9)=-4-6+54$$
$$=44\neq 0$$

then $\boxed{Z_p=\frac{1}{44}\sin(2x-3y)}$ Particular solution

$$Z=Z_g+Z_p=\phi_1(y+3x)+\phi_2(y-2x)+\frac{\sin(2x-3y)}{44}$$

general solution

ثانياً)) $F(-a^2,-ab,-b^2)=0$: نستخدم قاعدة أويلر لتعود الحالة الأولى.

$$\sin\theta=\frac{e^{i\theta}-e^{-i\theta}}{2i}\ ,\quad \cos\theta=\frac{e^{i\theta}+e^{-i\theta}}{2}$$

Ex.5 :- Find the particular solution

$$(D_x^2-4D_xD_y+3D_y^2)z=\cos(x+y)$$

Solution :-

$$z_p=\frac{1}{(D_x^2-4D_xD_y+3D_y^2)}\cos(x+y)$$

$$a=1 \;,\; b=1$$

$$F(-a^2,-ab,-b^2)=F(-1,-1,-1)=-1+4-3=0$$

then $\cos\theta=\dfrac{e^{i\theta}+e^{-i\theta}}{2}$, where $\theta=x+y$

$$\cos(x+y)=\frac{e^{xi+yi}+e^{-xi-yi}}{2}$$

$$z_p=\frac{1}{(D_x^2-4D_xD_y+3D_y^2)}\left[\frac{e^{xi+yi}+e^{-xi-yi}}{2}\right]$$

$$z_p=\frac{1}{2}\left[\frac{e^{xi+yi}}{(D_x^2-4D_xD_y+3D_y^2)}+\frac{e^{-xi-yi}}{(D_x^2-4D_xD_y+3D_y^2)}\right]$$

$$u_1 = \frac{e^{xi+yi}}{(D_x^2 - 4D_xD_y + 3D_y^2)} = \frac{e^{xi+yi}}{(D_x - 3D_y)(D_x - D_y)}$$

$a = i \quad , b = i \Rightarrow F(a,b) = (i-3i)(i-i)$
$= (-2i)(0)$

$G(a,b) = -2i$

$r = 1$

$$u_1 = \frac{1}{G(a,b)} \cdot \frac{x^r}{r!} e^{ax+by}$$

$$u_1 = \frac{1}{-2i} x e^{xi+yi}$$

$$u_2 = \frac{e^{-xi-yi}}{(D_x^2 - 4D_xD_y + 3D_y^2)} = \frac{e^{-xi-yi}}{(D_x - 3D_y)(D_x - D_y)}$$

$a = -i \quad , b = -i \Rightarrow F(a,b) = (-i+3i)(-i+i)$
$= (2i)(0)$

$G(a,b) = 2i$

$r = 1$

$$u_2 = \frac{1}{G(a,b)} \cdot \frac{x^r}{r!} e^{ax+by}$$

$$u_2 = \frac{1}{2i} x e^{-xi-yi}$$

$$Z_p = \frac{1}{2}\left[\underbrace{\frac{e^{xi+yi}}{(D_x^2 - 4D_x D_y + 3D_y^2)}}_{u_1} + \underbrace{\frac{e^{-xi-yi}}{(D_x^2 - 4D_x D_y + 3D_y^2)}}_{u_2}\right]$$

$$\boxed{Z_p = \frac{1}{2}\left[\frac{1}{-2i} x e^{xi+yi} + \frac{1}{2i} x e^{-xi-yi}\right]}$$

Particular solution

Ex. 6 :- Find the <u>particular solution</u>

$$(D_x^2 - 3D_x D_y + 2D_y^2) Z = e^{2x+3y} + e^{x+y} + \sin(x-2y)$$

Solution :-

$$Z = \frac{e^{2x+3y}}{D_x^2 - 3D_x D_y + 2D_y^2} + \frac{e^{x+y}}{D_x^2 - 3D_x D_y + 2D_y^2} + \frac{\sin(x-2y)}{D_x^2 - 3D_x D_y + 2D_y^2}$$