Lecture (6)

Electricity and Magnetism I

First Stage
Department of Physics
College of Science
Al-Muthanna University
2018-2019

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Capacitance and Dielectrics

A capacitor is a device which stores electric charge. Capacitors vary in shape and size, but the basic configuration is two conductors carrying equal but opposite charges (as in figure).

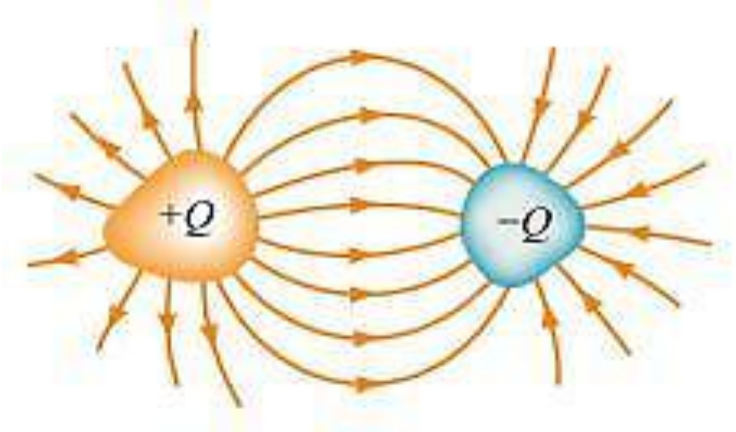


Figure basic configuration of Capacitor

Capacitor have many important applications in electronics. Some examples include storing electric potential energy, delaying voltage changes when coupled with resistor, filtering out unwanted frequency signals, forming resonant circuits and making frequency dependent and independent voltage dividers when combined with resistors.

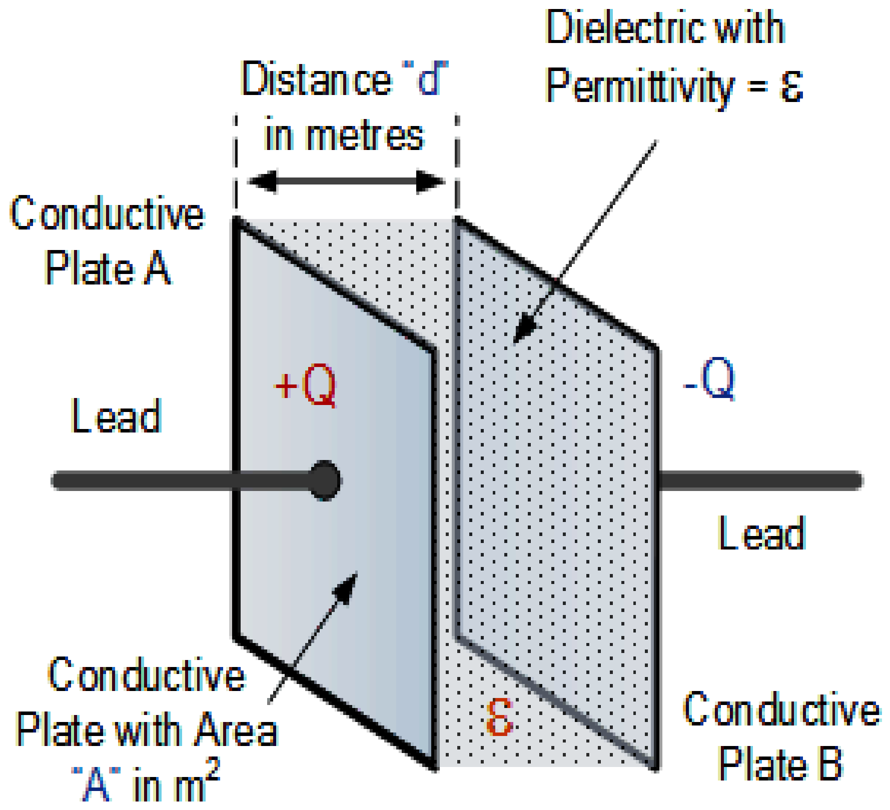
In the uncharged state, the charge on either one of the conductors in the capacitor is zero. During the charging process, a charge Q is moved from one conductor to the other one, giving one conductor a charge Q+, and the other one a charge Q- and a potential difference ΔV is created. Note that whether charged or uncharged, the net charge on the capacitor as a whole is zero.

The simplest example of a capacitor consist of two conducting plates of area A, which are parallel to each other, and separated by a distance d, as showing in figure.

Experiments show that the amount of charge Q stored in a capacitor is linearly proportional to, the electric potential difference between the plates:

$$\boldsymbol{Q = C\,|\Delta V|}$$

Where C is the positive proportionality constant called capacitance. Physically, capacitance is a measure of the capacity of storing electric charge for a given potential difference ΔV.



The SI unit of capacitance is the farad (F). 1 F = 1 farad = 1 coulomb/volt = 1 C/V. A typical capacitance is in the picofarad $(1\,pF = 10^{-12}\,F)$ to millifarad $(1\,mF = 10^{-3}\,F = 1000\,\mu F, 1\,\mu F = 10^{-6}\,F)$ range.

Calculation of Capacitance

Parallel Plate Capacitor

Consider two metallic plates of equal area A separated by a distance d. the top plate carries a charge +Q while the bottom plate carries a charge –Q. the charge of the plates can be accomplished by means of a battery which produces a potential difference. Find capacitance of the system.

To find the capacitance C, The electric field between the plates using Gauss's law:

$$\oiint \vec{E} . d\,\vec{A} = \frac{q_{enc}}{\varepsilon_o}$$

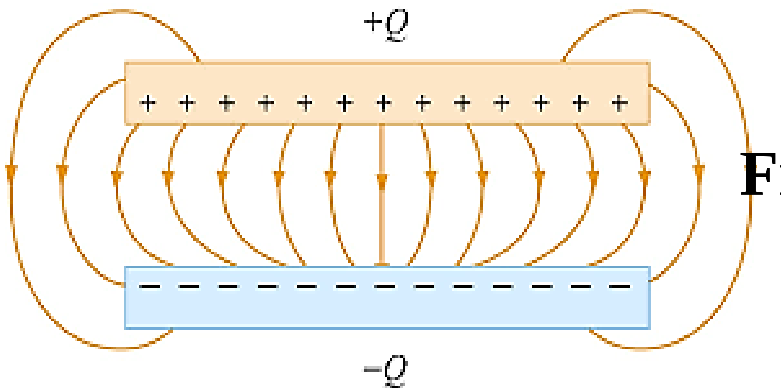


Figure the electric filed between the plates of parallel plate capacitor

By choosing a Gaussian "pillbox" with cap area $\acute{A}$ enclose the charge on the positive plate. The electric field in the region between the plates is:

$$\boldsymbol{E\acute{A}} = \frac{q_{enc}}{\varepsilon_o} = \frac{\sigma\,\acute{A}}{\varepsilon_o} \quad \Longrightarrow \quad \boldsymbol{E} = \frac{\sigma}{\varepsilon_o}$$

The potentials difference between plates is:

$$\Delta \boldsymbol{V} = \boldsymbol{V}_- - \boldsymbol{V}_+ = -\int_+^- \vec{\boldsymbol{E}}\,.\,d\vec{\boldsymbol{s}} = -\boldsymbol{Ed}$$

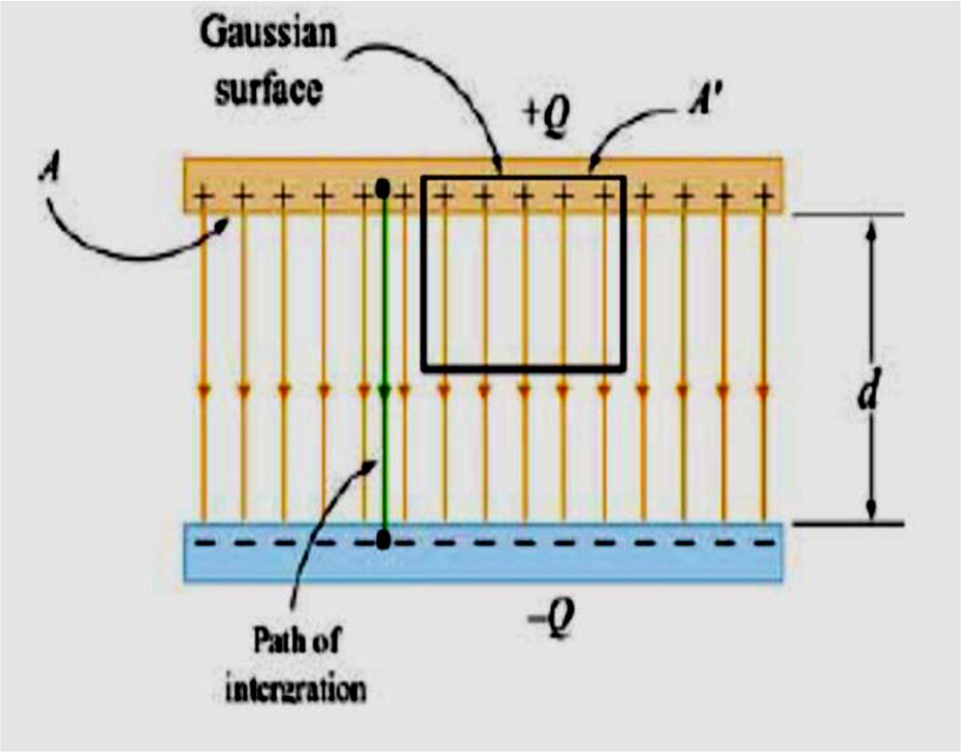


Figure Gaussian surface for calculating the electric field between the plates

Where we have taken the path of integration to be straight line from the positive plate to the negative plate following the field lines. Since the electric field lines are always directed from higher potentials to lower potential, -V<+V.

$$|\Delta V| = Ed \qquad \Longrightarrow \qquad |\Delta V| = \frac{\sigma d}{\varepsilon_o}$$

From the definition of capacitance, we have:

$$C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_o A}{d} \qquad (Parallel\ Plate)$$

The capacitance C increases linearly with the area A since for a given potential difference VΔ, a bigger plate can hold more charge. On the other hand, C is inversely proportional to d, the distance of separation because the smaller the value of d, the smaller the potential difference |VΔ| for a fixed Q.

Cylindrical Capacitor

Consider a solid cylindrical conductor of radius a surrounded by a coaxial cylindrical shell of inner radius b, as shown in Figure below. The length of both cylinders is L and we take this length to be much larger than b − a, the separation of the cylinders, so that edge effects can be neglected. The capacitor is charged so that the inner cylinder has charge +Q while the outer shell has a charge –Q. What is the capacitance.

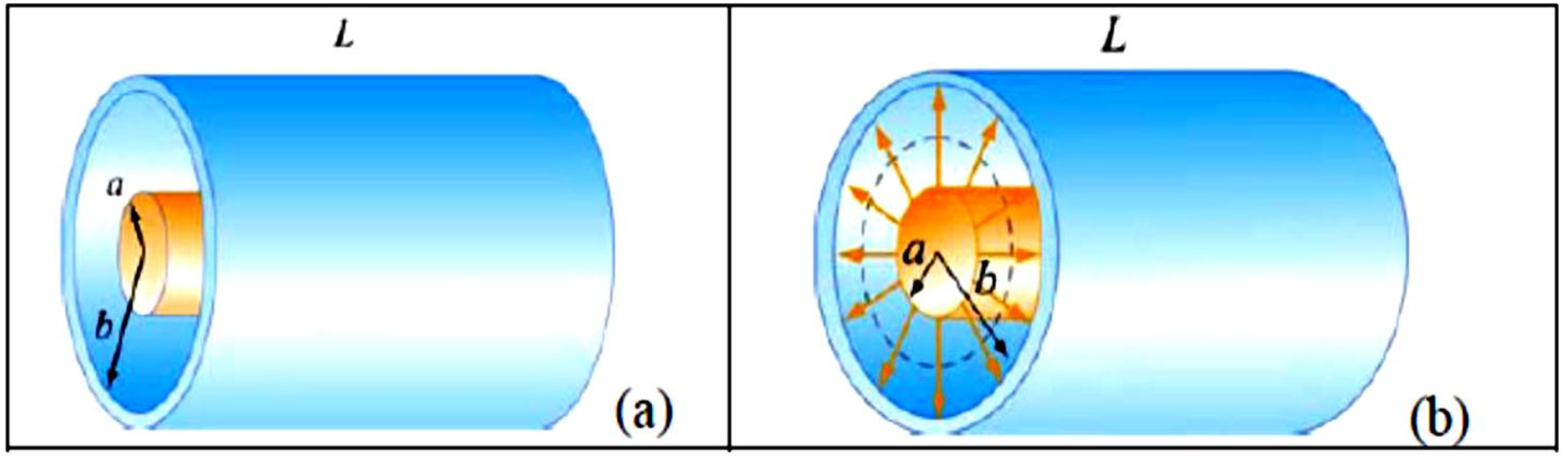


Figure (a) A cylindrical capacitor. (b) End view of the capacitor. The electric field is non-vanishing only in the region a < r < b.

To calculate the capacitance, we first compute the electric field everywhere. Due to the cylindrical symmetry of the system, we choose our Gaussian surface to be a coaxial cylinder with length $(l < L)$ and radius r where a < r < b.

Using Gauss's law, we have:

$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_o} \quad \Rightarrow \quad EA = E(2\pi r l) = \frac{\lambda l}{\varepsilon_o} \quad \Rightarrow \quad E = \frac{\lambda}{2\pi r \varepsilon_o}$$

where λ= Q/L is the charge per unit length. The electric field is non-vanishing only in the region a < r < b. For a < r, the enclosed charge is q=0 since any net charge in a conductor must reside on its surface.

Similarly, for r > l , the enclosed charge is $q_{enc} = \lambda l - \lambda l = 0$ since the Gaussian surface encloses equal but opposite charges from both conductors. The potential difference is given by:

$$\boldsymbol{\Delta V = V_b - V_a = -\int_a^b E_r\ dr = -\frac{\lambda}{2\pi\varepsilon_o}\int_a^b \frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_o}\ln\left(\frac{b}{a}\right)}$$

where we have chosen the integration path to be along the direction of the electric field lines. The outer conductor with negative charge has a lower potential.

$$\boldsymbol{C = \frac{Q}{|\Delta V|} = \frac{\lambda L}{\lambda \ln(b/a)/2\pi\varepsilon_o} = \frac{2\pi\varepsilon_o L}{\ln(b/a)}}$$

we see that the capacitance C depends only on the geometrical factors, L, a and b.

Spherical Capacitor

As a third example, let's consider a spherical capacitor which consists of two concentric spherical shells of radii *a* and *b*, as shown in Figure below. The inner shell has a charge +Q uniformly distributed over its surface, and the outer shell an equal but opposite charge –Q. What is the capacitance of this configuration?

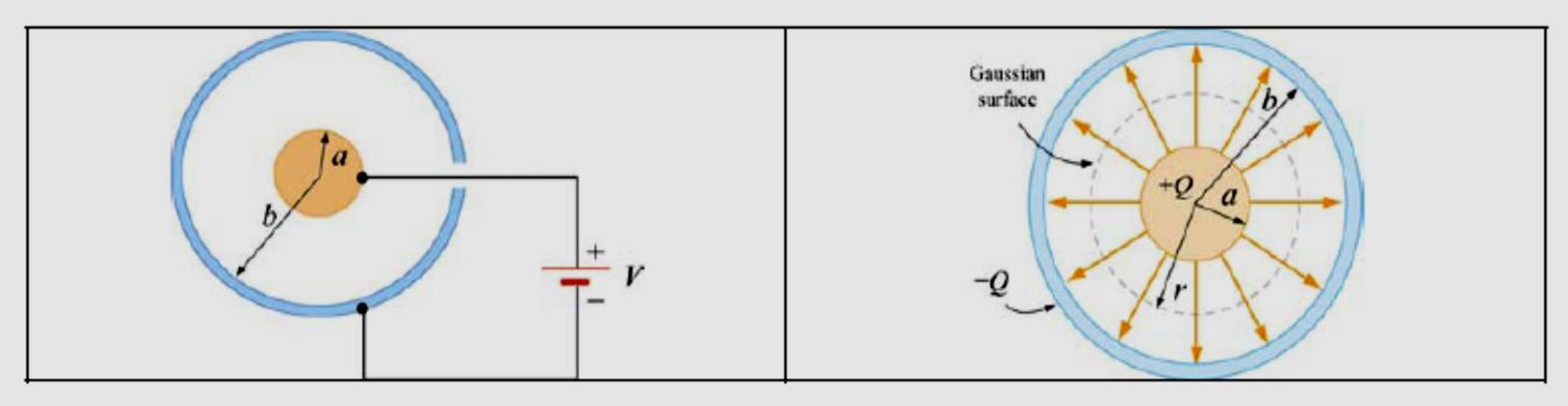


Figure (a) spherical capacitor with two concentric spherical shells of radius a &b. (b) Gaussian surface for calculating the electric field.

The electric field is non vanishing only in the region $a<r<b$ using Gauss law

$$\oiint_s \vec{E}.d\vec{A} = E_r A = E_r(4\pi r^2) = \frac{Q}{\epsilon_o} \qquad \Longrightarrow \qquad E_r = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}$$

Therefore, the potential difference between the two conducting shells is:

$$\Delta V = V_b - V_a = -\int_a^b E_r\,dr = -\frac{Q}{4\pi\epsilon_o}\int_a^b \frac{dr}{r^2} = -\frac{Q}{4\pi\epsilon_o}\left(\frac{1}{a}-\frac{1}{b}\right) = -\frac{Q}{4\pi\epsilon_o}\left(\frac{b-a}{ab}\right)$$

$$C = \frac{Q}{|\Delta V|} = 4\pi\epsilon_o\left(\frac{ab}{b-a}\right)$$

Again the capacitor C depends only on the physical dimensions, a and b.

$$\lim_{b\longrightarrow\infty} C = \lim_{b\longrightarrow\infty} 4\pi\epsilon_o\left(\frac{a}{1-\frac{a}{b}}\right) = 4\pi\epsilon_o a$$

Thus, for a single isolated spherical conductor of radius R,

the capacitance is: $C = 4\pi\epsilon_o R$

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System	Capacitance
Isolated charged sphere of radius R	$C = 4\pi\varepsilon_0 R$
Parallel-plate capacitor of plate area A and plate separation d	$C = \varepsilon_0 \frac{A}{d}$
Cylindrical capacitor of length L, inner radius a and outer radius b	$C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$
Spherical capacitor with inner radius a and outer radius b	$C = 4\pi\varepsilon_0 \frac{ab}{(b-a)}$

Capacitor in Electric Circuits

A capacitor can be charged by connecting the plates to the terminals of a battery, which are maintained at a potential difference ΔV called the terminal voltage.

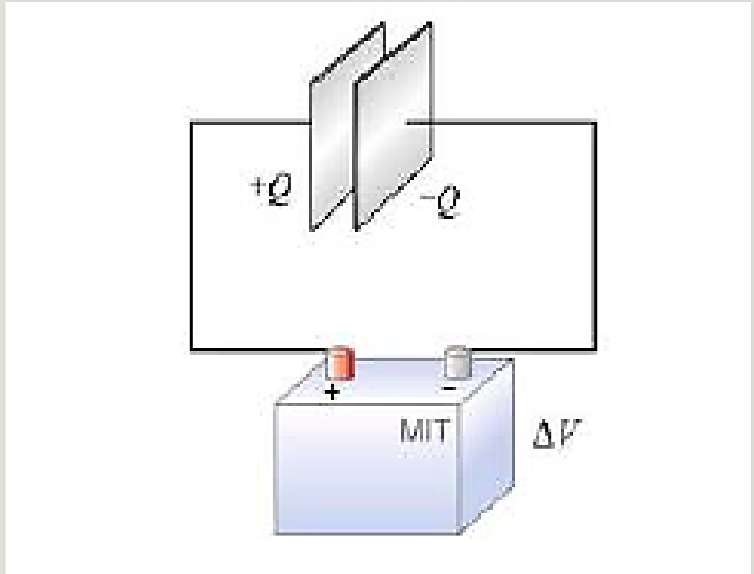

Figure charging a capacitor

Parallel Connection

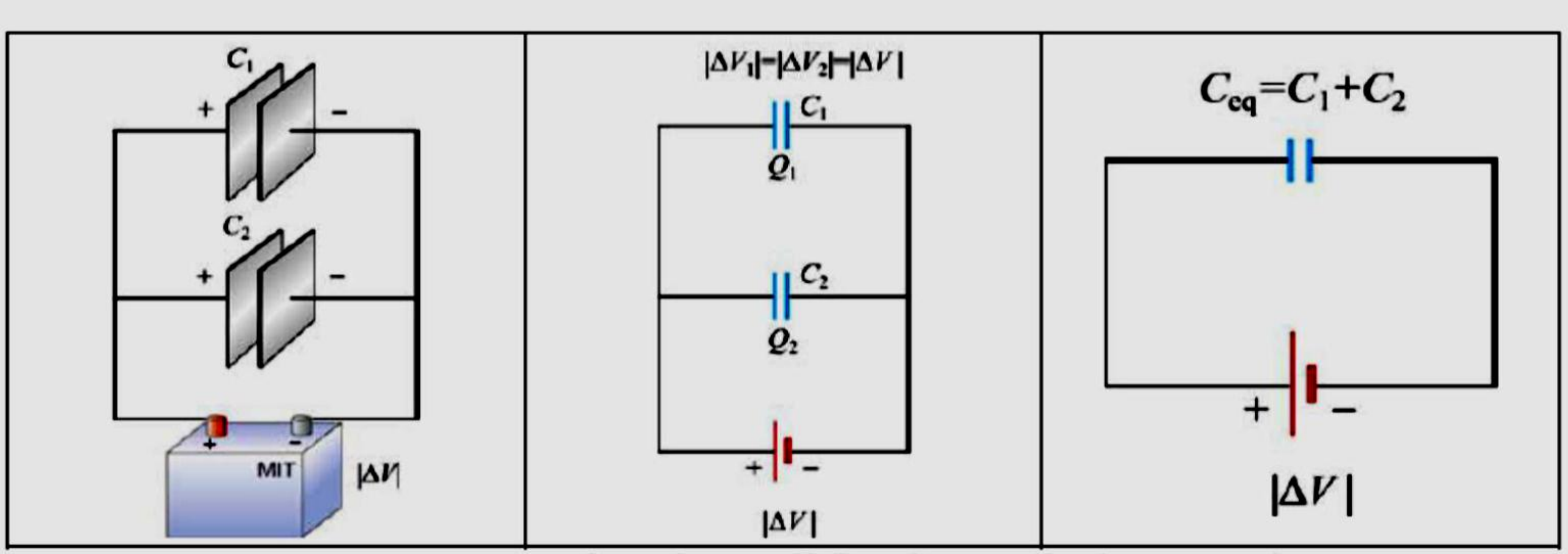

Figure Capacitors in parallel and an equivalent capacitor

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The left plates of both capacitors C1 and C2 are connected to the positive terminal of the battery and have the same electric potential as the positive terminal. Similarly, both right plates are negatively charged and have the same potential as the negative terminal. Thus, the potential difference |ΔV| is the same across each capacitor. This gives: $C_1 = \frac{Q_1}{|\Delta V|}$ $C_2 = \frac{Q_2}{|\Delta V|}$

These two capacitors can be replaced by a single equivalent capacitor Ceq with a total charge Q supplied by the battery. However, since Q is shared by the two capacitors, we must have

$$Q = Q_1 + Q_2 = C_1|\Delta V| + C_2|\Delta V| = (C_1 + C_2)|\Delta V|$$

The equivalent capacitor is given by: $C_{eq} = \frac{Q}{|\Delta V|} = C_1 + C_2$

Thus, capacitors that are connected in parallel add. The generalization to any number of capacitors is: $C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N = \sum_{i=1}^{N} C_i$ $(Parallel)$

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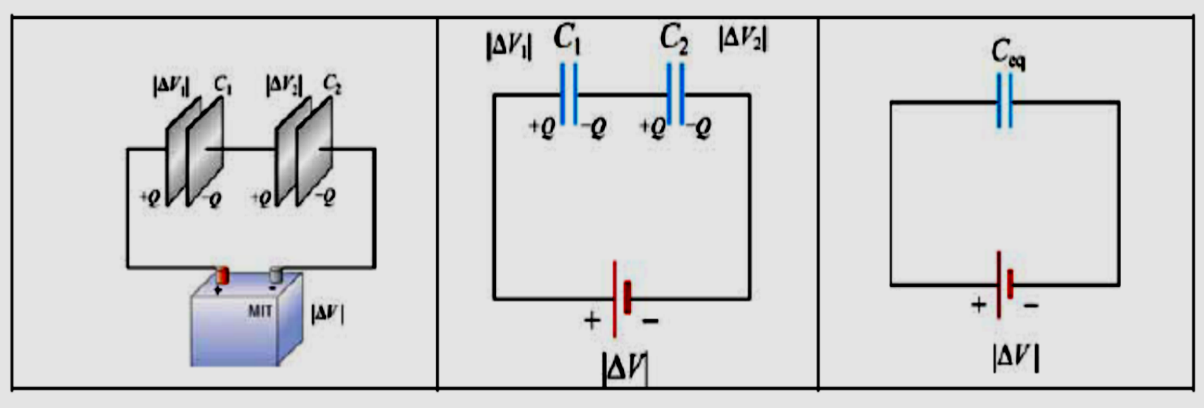
Series Connection



Suppose two initially uncharged capacitors C_1 *and* C_2 are connected in series, as showing in the figure below. A potential difference $|\Delta V|$ is then applied across both capacitors. The left plate of C_1 is connected to the positive terminal of the battery and becomes positively charged with a charge +Q, while the right plate of C_2 is connected to negative terminal and becomes negatively charged with charge –Q as electrons flow in. What about inner plates? The inner plates were initially uncharged. Now the outside plates each attract an equal and opposite charge. So, the right plate of C_1 will acquire a charge –Q and the left plate of C_2+Q.

The potential difference across capacitors C_1 *and* C_2 are: $|\Delta \boldsymbol{V}_{\mathbf{1}}| = \frac{\boldsymbol{Q}}{\boldsymbol{C}_{\mathbf{1}}}$ $|\Delta \boldsymbol{V}_{\mathbf{2}}| = \frac{\boldsymbol{Q}}{\boldsymbol{C}_{\mathbf{2}}}$

The total potential difference is: $|\Delta \boldsymbol{V}| = |\Delta \boldsymbol{V}_{\mathbf{1}}| + |\Delta \boldsymbol{V}_{\mathbf{2}}|$

The total potential difference across any number of capacitors in series connection is equal to the sum of potential differences across the individual capacitors. These two capacitors can be replaced by a single equivalent capacitor

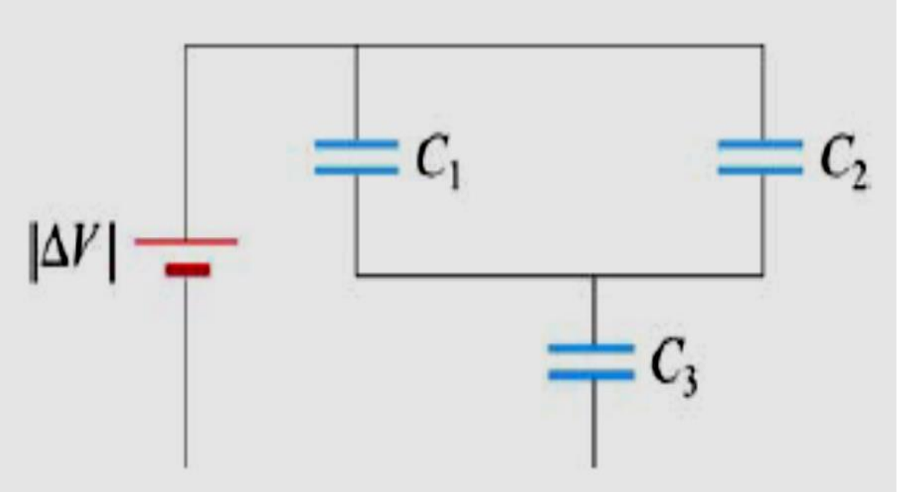
$$\boldsymbol{C}_{eq} = \frac{\boldsymbol{Q}}{|\Delta \boldsymbol{V}|}$$

Using the fact that the potentials add in series: $\frac{\boldsymbol{Q}}{\boldsymbol{C}_{eq}} = \frac{\boldsymbol{Q}}{\boldsymbol{C}_1} + \frac{\boldsymbol{Q}}{\boldsymbol{C}_2}$

and so the equivalent capacitance for two capacitors in series becomes: $\frac{\mathbf{1}}{\boldsymbol{C}_{eq}} = \frac{\mathbf{1}}{\boldsymbol{C}_1} + \frac{\mathbf{1}}{\boldsymbol{C}_2}$

$$\frac{\mathbf{1}}{\boldsymbol{C}_{eq}} = \frac{\mathbf{1}}{\boldsymbol{C}_1} + \frac{\mathbf{1}}{\boldsymbol{C}_2} + \cdots + \frac{\mathbf{1}}{\boldsymbol{C}_N} = \sum_{i=1}^{N} \frac{\mathbf{1}}{\boldsymbol{C}_i} \qquad (\boldsymbol{Series})$$

Example: Find the equivalent capacitance for the combination of capacitors shown in Figure below.



Sol.

Since C1 and C2 are connected in parallel, their equivalent capacitance C12 given by:

$$C_{12} = C_1 + C_2$$

Now the capacitance C12 is in series with C3. So the equivalent capacitance C123 is given by:

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$