Lecture (5)

Electricity and Magnetism I

First Stage
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Electric Potential of a Point Charge

To obtain the electric potential due to a point charge, we use the equation below where r is the distance, since the electric field it produces is along the radius that is:

$$E = -\frac{dV}{dr} \qquad \textit{and} \qquad E = \frac{q}{4\pi\epsilon_\circ r^2}$$

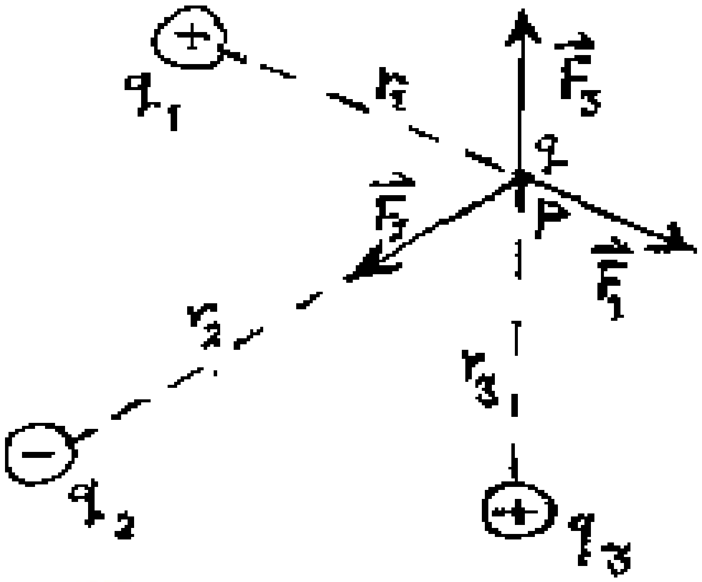
So,

$$\frac{1}{4\pi\epsilon_\circ}\frac{q}{r^2} = -\frac{dV}{dr}$$

Integrating, and assuming V=0 for r=∞ , we obtain:

$$\int_0^V dV = -\frac{q}{4\pi\epsilon_\circ}\int_\infty^r \frac{dr}{r^2}$$

The result of this integration is: $V = \frac{q}{4\pi\epsilon_\circ r}$



If we have several charges $q_1, q_2, q_3, \ldots,$ the electric potential at a point P is the scalar sum of their individual potentials, that is:

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$$V = \frac{q_1}{4\pi\epsilon_\circ r_1} + \frac{q_2}{4\pi\epsilon_\circ r_2} + \frac{q_3}{4\pi\epsilon_\circ r_3} + \cdots \ldots \ldots = \frac{1}{4\pi\epsilon_\circ} \sum_i \frac{q_i}{r_i}$$

If we place a charge $\acute{q}$ at a distance r from a charge q, the potential energy of the system is:

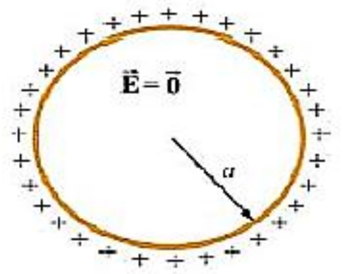
$$U_p = \acute{q}\, V \quad \Rightarrow \quad U_p = \frac{q\, \acute{q}}{4\pi\epsilon_\circ r} \quad \Rightarrow \quad U_p = \sum_{All\ Points} \frac{q\, \acute{q}}{4\pi\epsilon_\circ r}$$

This is the potential energy of a system of charge.

The electric potential V is positive or negative depending on the sign of the charge q.

Surfaces having the same electric potential at all points, V=constant are called equipotential surfaces. At each point of an equipotential surface, the direction of the electric field is perpendicular to the surface. The line of force are orthogonal to the equipotential surface.

Electric Potential Due to a Spherical Shell:



A spherical shell of radius a and charge Q.

Consider a metallic spherical shell of radius a and charge Q as showing in the figure below:

a) Find the electric potential everywhere b) Calculate the potential energy of the system

Sol.

a) In the previous lecture (Example 2) we calculated the electric field :

Outside the Spherical Shell $(\boldsymbol{r > a})$ $\boldsymbol{E = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}}$

Inside the Spherical Shell $(\boldsymbol{r < a})$ $\boldsymbol{E=0}$

The electric potential maybe calculated using:

$$\boldsymbol{E = -\frac{dV}{dr} \quad \Longrightarrow \quad V = \int E\, dr}$$

For $r > a$ we have:

$$\boldsymbol{V(r) - V(\infty) = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}\ dr}$$

$$\boldsymbol{V(r) - V(\infty) = K\frac{Q}{r}}$$

Where we have chosen $V(\infty) = 0$ as our reference point.

For $r < a$ we have:

$$\boldsymbol{V(r) - V(\infty) = -\int_{\infty}^{a} E(r > a)\, dr - \int_{a}^{r} E(r < a)\ dr}$$

$$\boldsymbol{V(r) - V(\infty) = -\int_{\infty}^{a} \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}\ dr = \frac{1}{4\pi\epsilon_o}\frac{Q}{a} = K\frac{Q}{a}}$$

b) Suppose the charge accumulated on the sphere at some instant is q. the amount of work that must be done by an external agent to bring charge dq from infinity and deposit it on the sphere is:

$$dW_{ext} = V\, dq$$

$$W_{ext} = \int_0^Q \frac{1}{4\pi\epsilon_o} \frac{q}{a}\, dq = \frac{Q^2}{8\pi\epsilon_o a}$$

But

$$W_{ext} = U$$

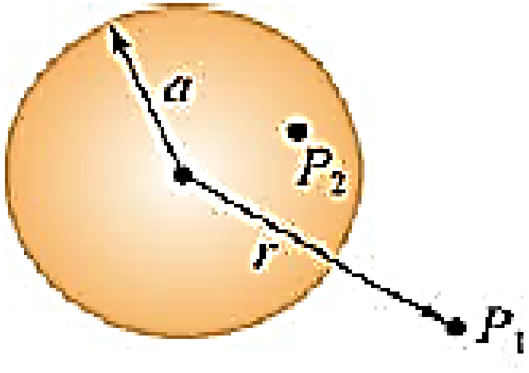
$$U = \frac{1}{2} QV$$

Electric Potential of a Uniformly Charged Sphere

An insulted solid sphere of radius a, has a uniform charge density ρ. Compute the electric potential everywhere.

Sol.

Using Gauss law as in the previous lecture (Example 6) we calculated the electric field due to the charge distribution is:



$$E = \frac{Q}{4\pi\epsilon_o r^2} \qquad r > a$$

$$E = \frac{Q\,r}{4\pi\epsilon_o a^3} \qquad r < a$$

The electric potential at P_1 outside the sphere is:

$$\boldsymbol{V_1(r) - V(\infty) = -\int_{\infty}^{r} \frac{Q}{4\pi\epsilon_o r^2}\, dr = \frac{1}{4\pi\epsilon_o}\frac{Q}{r} = K\frac{Q}{r}}$$

The electric potential at P_2 inside the sphere is:

$$\boldsymbol{V_2(r) - V(\infty) = -\int_{\infty}^{a} E(r > a)\, dr - \int_{a}^{r} E(r < a)}$$

$$\boldsymbol{V_2(r) - V(\infty) = -\int_{\infty}^{a} \frac{Q}{4\pi\epsilon_o r^2}\, dr - \int_{a}^{r} \frac{Q\, r}{4\pi\epsilon_o a^3}\, dr}$$

$$\boldsymbol{V_2(r) - V(\infty) = \frac{1}{4\pi\epsilon_o}\frac{Q}{a} - \frac{1}{4\pi\epsilon_o}\frac{Q}{a^3}\frac{1}{2}\left(r^2 - a^2\right) = \frac{1}{8\pi\epsilon_o}\frac{Q}{a}\left(3 - \frac{r^2}{a^2}\right)}$$

$$\boldsymbol{V_2(r) - V(\infty) = K\frac{Q}{2a}\left(3 - \frac{r^2}{a^2}\right)}$$

Energy Relation in an electric field

The total energy of a charged particle or ion of mass m and charge q moving in an electric field is:

$$U = U_k + U_P = \frac{1}{2}mv^2 + qV$$

When the ions moves from position P_1 (where the electric potential is V_1) to position P_2 (where the potential is V_2), and by combining above equation with the principle of conservation of energy gives:

$$\frac{1}{2}mv_1^2 + qV_1 = \frac{1}{2}mv_2^2 + qV_2$$

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = q(V_1 - V_2)$$

Note from above equation that appositively charged particle (q>0) gains kinetic energy when moving from a larger to smaller potential ($V_1 > V_2$), while a negatively charged particles (q<0),to gain energy, has to move from a lower to a higher potential ($V_1 < V_2$).

If we choose the zero of electric potential at P_2 ($V_2 = 0$) and arrange our experiment so that at P_1 the ions have zero velocity $v_1 = 0$.

So above equation becomes:

$$\frac{1}{2}mv^2 = qV$$

An expression that gives the kinetic energy acquired by a charged particle when it waves through an electric potential difference V. This is, for example, the principle applied in electrostatic accelerators.

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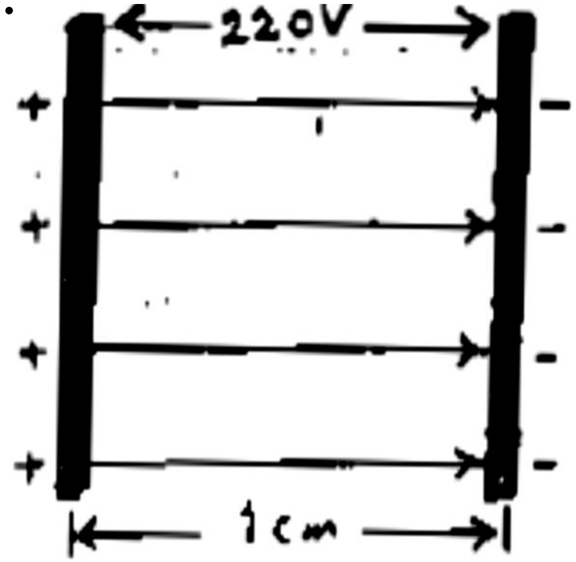
Problem 1: Find the electric field between two large parallel plates 1 cm apart, with a potential difference of 220 V.

Sol.

The electric field is uniform as in figure below, the potential difference is:



$$\mathbf{220\,V = V_b - V_a = \int_a^b \vec{E}\,\overrightarrow{dr} = E\,r}$$

$$\mathbf{E = \frac{220\,V}{0.01\,m} = 22000\,V/m}$$

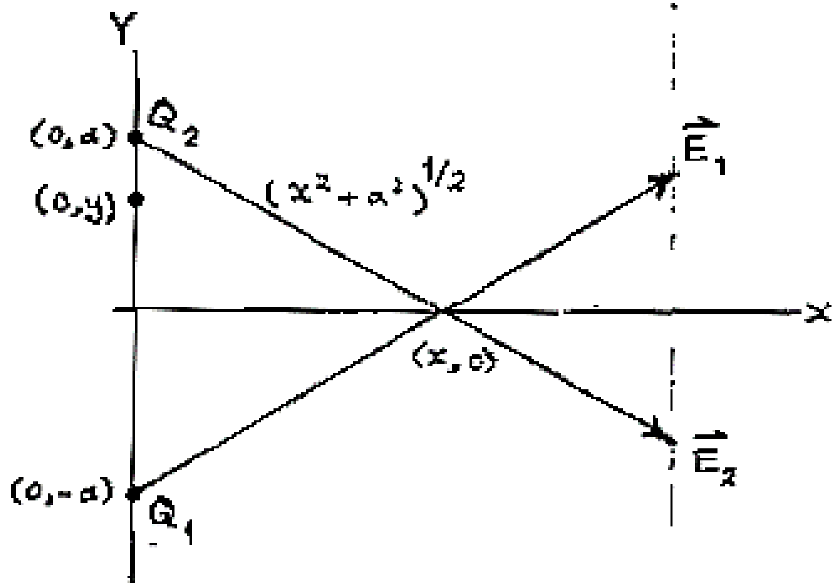
Problem 2: Find the electric potential on the x-axis at a point (x,0) when two positive charges of magnitude $Q = 10^{-9}\ C$ are on the y-axis at (0, a) and (0, -a) where a=1 cm.

(1) calculate this by superposition of the potentials due to each charges.

(2) calculate this by direct integration of $\int \vec{E}.\overrightarrow{dr}$. evaluate the result for x=a.

(3) find the work done by the electric field of two charges when a third charge of equal magnitude is moved from the point (2a, 0) to (a, 0).

Sol.



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(1) the potentials at point (a, 0) is:

$$V = \frac{Q}{4\pi \epsilon_\circ (x^2 + a^2)^{\frac{1}{2}}} + \frac{Q}{4\pi \epsilon_\circ (x^2 + a^2)^{\frac{1}{2}}}$$

$$V = \frac{2Q}{4\pi \epsilon_\circ (x^2 + a^2)^{\frac{1}{2}}}$$

(2) the field $\vec{E}$ at x has only x component

$$E_x = E_{1x} + E_{2x} = 2|E_1| \cos\theta$$

$$E_x = 2\ \frac{Q}{4\pi \epsilon_\circ (x^2+a^2)}\ \frac{x}{(x^2+a^2)^{\frac{1}{2}}}$$

$$E_x = \frac{2Qx}{4\pi \epsilon_\circ (x^2 + a^2)^{\frac{3}{2}}}$$

Thus we have,

$$V = \int_x^\infty E_x \, dx = \frac{2Q}{4\pi \epsilon_\circ} \int_x^\infty \frac{x dx}{(x^2 + a^2)^{\frac{3}{2}}}$$

$$V = \frac{2Q}{4\pi \epsilon_\circ} \left[\frac{-1}{(x^2 + a^2)^{\frac{1}{2}}} \right]_x^\infty$$

$$V = \frac{2Q}{4\pi \epsilon_\circ} \frac{1}{(x^2 + a^2)^{\frac{1}{2}}} \qquad as\ in\ (1)$$

$at \quad x = a = 1\ cm = 10^{-2}\ m$

$$V = \frac{2 \times 10^{-9}\ C}{4\pi\ (8.85 \times 10^{-12}\ C^2 . N^{-1} . m^{-2})\ [2(0.01\ m)^2]^{\frac{1}{2}}} = 1270\ V$$

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(3) $W = \int_a^b \vec{F}.\overrightarrow{dr} = \int_a^b Q\,\vec{E}.\overrightarrow{dr} = Q(V_b - V_a)$

Where

$$V_b = \frac{2Q}{4\pi\,\epsilon_\circ\,((2a)^2+a^2)^{\frac{1}{2}}} = \frac{2Q}{4\pi\,\epsilon_\circ\,a\,(5)^{\frac{1}{2}}}$$

$$V_a = \frac{2Q}{4\pi\,\epsilon_\circ\,(a^2+a^2)^{\frac{1}{2}}} = \frac{2Q}{4\pi\,\epsilon_\circ\,a\,(2)^{\frac{1}{2}}}$$

$$W = \frac{2Q^2}{4\pi\,\epsilon_\circ\,a}\left[\frac{1}{(5)^{\frac{1}{2}}} - \frac{1}{(2)^{\frac{1}{2}}}\right]$$

$$W = \frac{(10^{-9}\,C)^2}{2\pi\,(8.85\ \times 10^{-12}\,C^2.N^{-1}.m^{-2})\,(0.01\,m)} = -4.68\times 10^{-7}J$$

The work done is negative because the displacement is opposite to the direction of the electric force.

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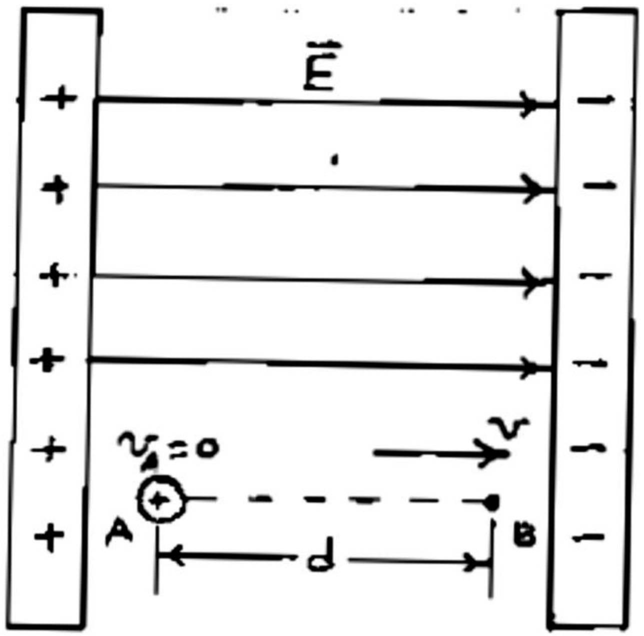
Problem 3: A proton is released from rest in a uniform electric field that has a magnitude of $8 \times 10^4\ V/m$ (as in the figure below). The proton undergoes a displacement of 0.50 m in the direction of $\vec{E}$. (A) find the change in electric potential between points A and B. (B) Find the change in the potential energy of the proton field system for this displacement. (C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.



Sol.

(A) Because the positively charged proton moves in the direction of the field, we expect it to move to a position of lower electric potential

$$\Delta V = -Ed = -\left(8 \times 10^4 \frac{V}{m}\right)(0.05\, m)$$

$$\Delta V = -4.0 \times 10^4 \; V$$

(B)

$$\Delta U = q\, \Delta V = e\, \Delta V = (1.6 \times 10^{-19}\, C)(-4 \times 10^4\, V)$$

$$\Delta U = -6.4 \times 10^{-15}\, J$$

The negative sign means the potential energy of the system decreases as the proton moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time, the system loses electric potential energy.

(C) The charge field system is isolated, so the mechanical energy of the system is conserved:

$$\Delta k + \Delta U = 0$$

$$\frac{1}{2}mv^2 + e\,\Delta V = 0$$

$$v = \sqrt{\frac{-(2e\,\Delta V)}{m}} = \sqrt{\frac{-2(1.6 \times 10^{-19}\,C)(-4 \times 10^{4}\,V)}{1.67 \times 10^{-27}\,kg}}$$

$$v = 2.8 \times 10^{6}\,m/s$$

Problem 4: A charge moves a distance of 2.0 cm in the direction of a uniform electric field whose magnitude is 215 N/C.As the charge moves, its electrical potential energy decreases by $6.9 \times 10^{-19}\, J$. Find the charge on the moving particle. What is the potential difference between the two locations?

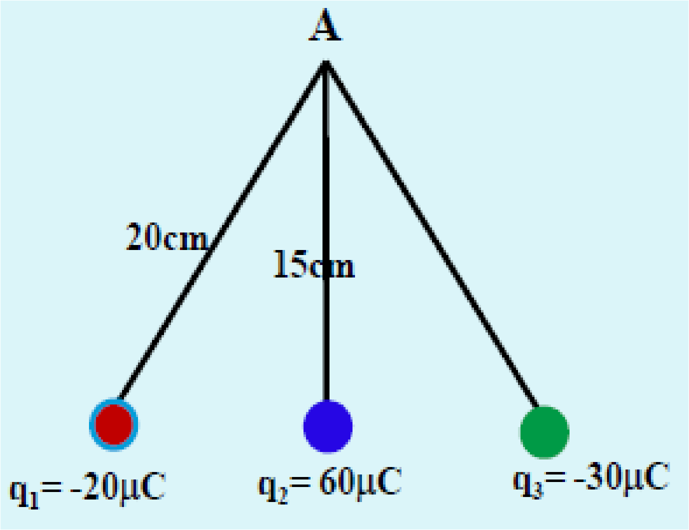
Sol.

$$\Delta U = -qEd \qquad \Longrightarrow \qquad q = -\frac{\Delta U}{Ed}$$

$$q = \frac{-6.9 \times 10^{-19}\, J}{(215\, N/C)(0.02\, m)} = 1.6 \times 10^{-19}\, C$$

$$\Delta V = -Ed = -\left(215 \frac{N}{C}\right)(0.02\, m) = -4.3\, V$$

Problem 5: Three point charges are arranged along a straight line as shown in Figure. If q1 = -20 μC, q2 = +60 μC and q3 = -30 μC, calculate the electric potential at point A.



Sol.

$$V_{Tot} = \sum_{i=1}^{3} V_i$$

$$V_1 = \frac{q_1}{4\pi\epsilon_o r_1} = \frac{(9\times10^9)(-20\times10^{-6}\ C)}{0.2\ m} = -9\times10^5\ V$$

$$V_2 = \frac{q_2}{4\pi\epsilon_o r_2} = \frac{(9\times10^9)(60\times10^{-6}\ C)}{0.15\ m} = 3.6\times10^6\ V$$

$$V_3 = \frac{q_3}{4\pi\epsilon_o r_3} = \frac{(9\times10^9)(-30\times10^{-6}\ C)}{0.2\ m} = -1.3\times10^6\ V$$

$$V_{Tot} = V_1 + V_2 + V_3$$

$$V_{Tot} = -9\times10^5\ V + 3.6\times10^6\ V - 1.3\times10^6\ V =$$

Problem 6: Two charges Q1= +3 nC and Q2 = -5 nC are separated by 8 cm. Calculate the electric potential at point A and B. The potential difference What work is done by the E-field if a +2 mC charge is moved from A to B?

Sol.

$$V_A = \frac{K\,Q_1}{r_1} + \frac{K\,Q_2}{r_2}$$

$$\frac{K\,Q_1}{r_1} = \frac{(9\times10^9\ Nm^2/C^2)(3\times10^{-9}\ C)}{0.06\ m} = 450\ V$$

$$\frac{K\,Q_2}{r_2} = \frac{(9\times10^9\ Nm^2/C^2)(-5\times10^{-9}\ C)}{0.02\ m} = -2250\ V$$

$$V_A = 450\ V - 2250\ V = -1800\ V$$

$$V_B = \frac{K\,Q_1}{r_1} + \frac{K\,Q_2}{r_2}$$

$$\frac{K\,Q_1}{r_1} = \frac{(9\times10^9\ Nm^2/C^2)(3\times10^{-9}\ C)}{0.02\ m} = 1350\ V$$

$$\frac{K\,Q_2}{r_2} = \frac{(9\times10^9\ Nm^2/C^2)(-5\times10^{-9}\ C)}{0.1\ m} = -450\ V$$

$$V_B = 1350\ V - 450\ V = 900\ V$$

$$V_{AB} = V_A - V_B = -1800\ V - 900\ V = -2700\ V$$

Note point B is at higher potential.

$$W_{AB} = q(V_A - V_B) = 2\times10^{-6}\ C\ (-2700\ V) = -5.4\times10^{-3}\ CV$$

$$W_{AB} = -5.4\times10^{-3}\ J = -5.4\ mJ$$