Lecture (4)

Electricity and Magnetism I

First Stage
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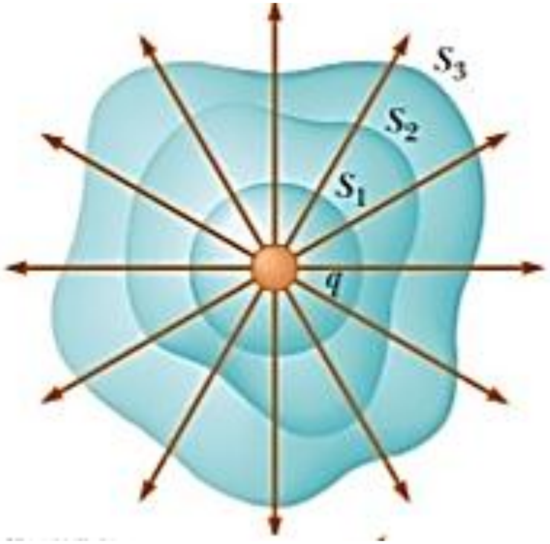
Gauss's Law

The electric flux through any closed surface is equal to the net charge inside the surface divided by $\epsilon_\circ$ that is:



$$\varphi_E = \oint E \, . \, dA = \frac{q}{\epsilon_\circ}$$

- Where q denotes the charge inside the surface and the circle the integral sign indicates that the integration is over a closed surface.
- Consider a positive point charge q surrounded by two closed surface S1 is spherical, whereas S2 and S3 is irregular (above figure). Coulomb's Law tell us that the magnitude of the electric field is constant everywhere on the spherical surface and given as:

$$\vec{E} = K \frac{|q|}{r^2}$$

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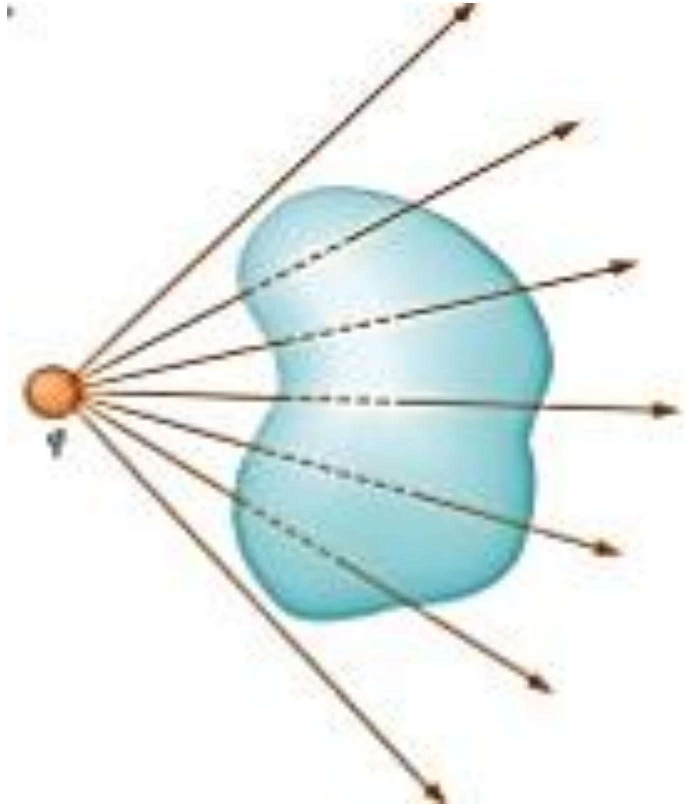
Since the electric field direction is radial, we can evaluate the flux through **S1** as:

$$\varphi_E = \oint E\, dA = EA \qquad \Longrightarrow \qquad \varphi_E = \frac{1}{4\pi\epsilon_\circ}\frac{q}{r^2}\left(4\pi r^2\right) = \frac{q}{\epsilon_\circ}$$

The figure below shows the number of the field lines crossing **S1** is the same as that lines crossing **S2**, that is the flux through the two surfaces are equal and independent of their shape.



If the charge exists outside a closed surface, the electric field line entering the surface must leave that surface. Hence, the electric flux through that the surface is zero. We must be able to choose a hypothetical closed surface (Gaussian surface) such that the electric field over its surface is constant.

This can be achieved if the following remarks are satisfied:

1. The charge distribution must have a high degree of symmetry.

2. The Gaussian surface should have the same symmetry as that of the charge distribution.

3. The point at which E is to be evaluated should lie on the Gaussian surface.

4. If E is parallel to the surface or zero at every point then

$$\boldsymbol{\varphi = \oint E.dA = 0}$$

5. If E is perpendicular to the surface at every point, and since E is constant then

$$\boldsymbol{\varphi = \oint E.dA = E\ A = \frac{q}{\epsilon_\circ}}$$

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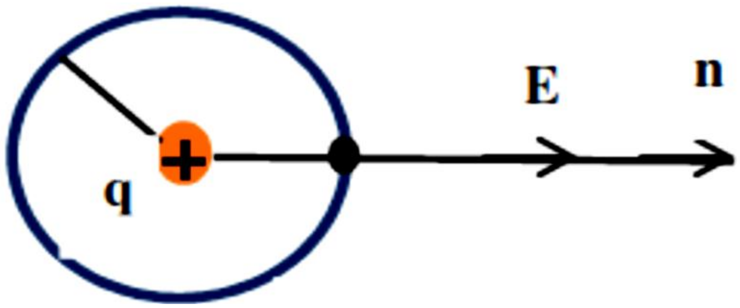
Example 1: Find the electric field at distance **r** from a point charge **q**.

Sol.

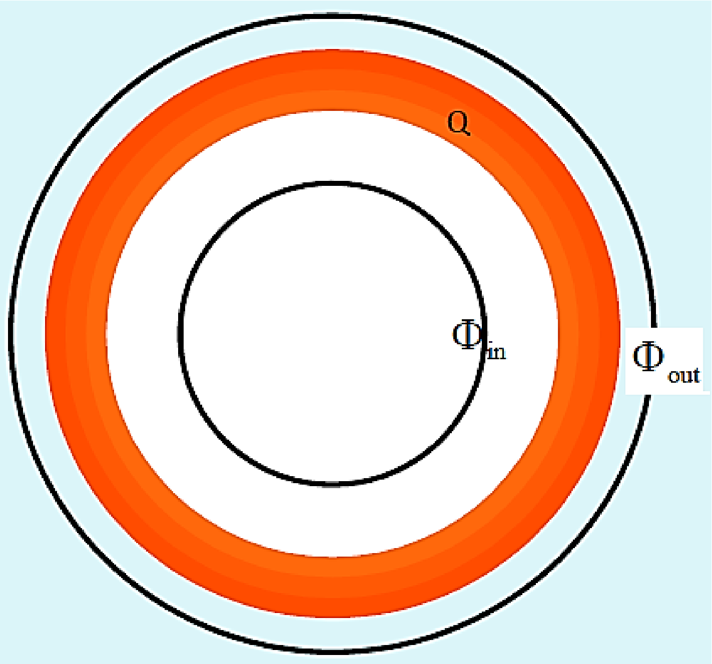
Choose the Gaussian surface as sphere of radius **r** see figure below.



$$\varphi_E = \oint E \cdot dA = \frac{q}{\epsilon_\circ}$$

We clear that **E** and **dA** are parallel and **E** is constant over the surface.

Example 2: What's the field around a charged spherical shell?



Consider spherical surface centered on charged shell

Outside:

$$\varphi_{out} = \oint \vec{E}.d\vec{A} = EA = E(4\pi r^2)$$

By applying Gauss law:

$$\varphi_{out} = \frac{Q}{\epsilon_\circ} \quad \Longrightarrow \quad E(4\pi r^2) = \frac{Q}{\epsilon_\circ} \quad \Longrightarrow \quad E = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}$$

Inside: charge within surface = 0

$$\varphi_{in} = 0 \quad \Longrightarrow \quad E = 0$$

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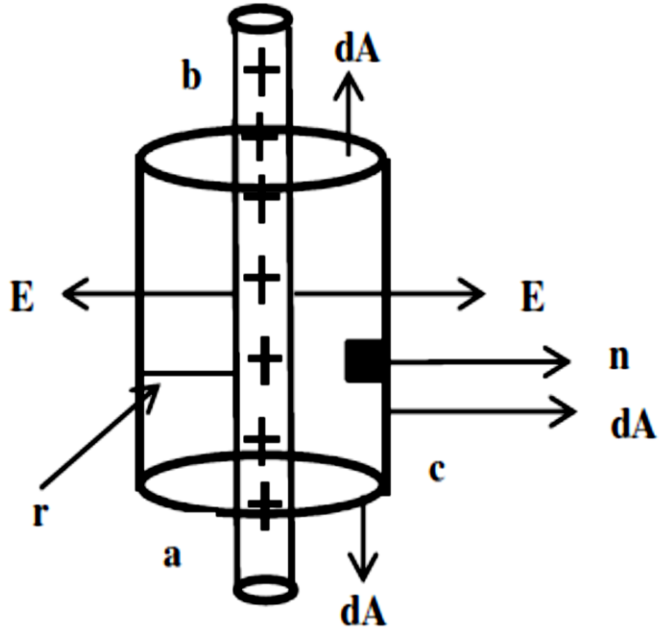
Example 3: Find the electric field at distance **r** from an infinite line charge of uniform density λ.

Sol.

Gaussian surface are select a circular cylinder of radius (**r**) with high (**h**) and coaxial with the line charge see figure below.

The cylinder has three surfaces; the integral in Gauss's Law has to b



$$\boldsymbol{\varphi_E = \oint E\,.\,dA = \frac{q}{\epsilon_\circ}}$$

$$\boldsymbol{\oint_a E.dA\cos 90 + \oint_b E.dA\cos 90 + \oint_c E.dA\cos 0 = \frac{q}{\epsilon_\circ}}$$

$$\boldsymbol{\varphi_E = \oint E\,.\,dA = \frac{q}{\epsilon_\circ}}$$

For the symmetry of the system, E is parallel to both bases furthermore it has a constant magnitude and directed radially outward at every point on the curved surface of the cylinder.

$$EA = E(2\pi rh) = \frac{q}{\epsilon_\circ}$$

$$q = \lambda\, h$$

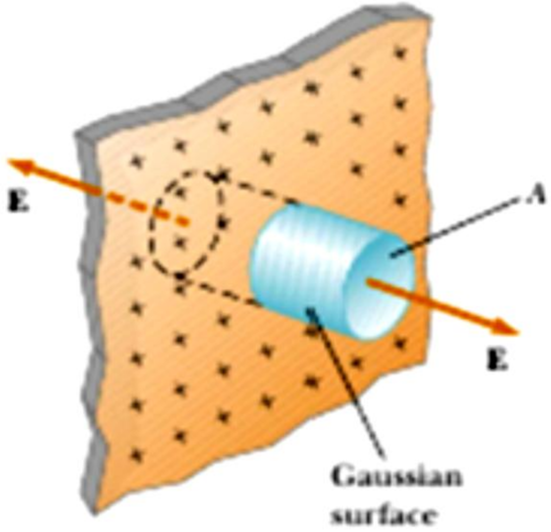
$$E = \frac{\lambda}{2\pi r\, \epsilon_\circ} \qquad \Longrightarrow \qquad E = 2K\frac{\lambda}{r}$$

If the wire is not too long it's the ends will be closed to any Gaussian surface. Since the electric field at closed to the ends is not uniform it will impossible to many the integral of Gauss's Law.

Example 4: Find the electric field **E** due to non-conductor infinite plane with uniform surface charge density **σ**.

Sol.

We select Gaussian surface as a small cylinder whose axis is perpendicular to the plane and whose ends have an area **A** see figure below.



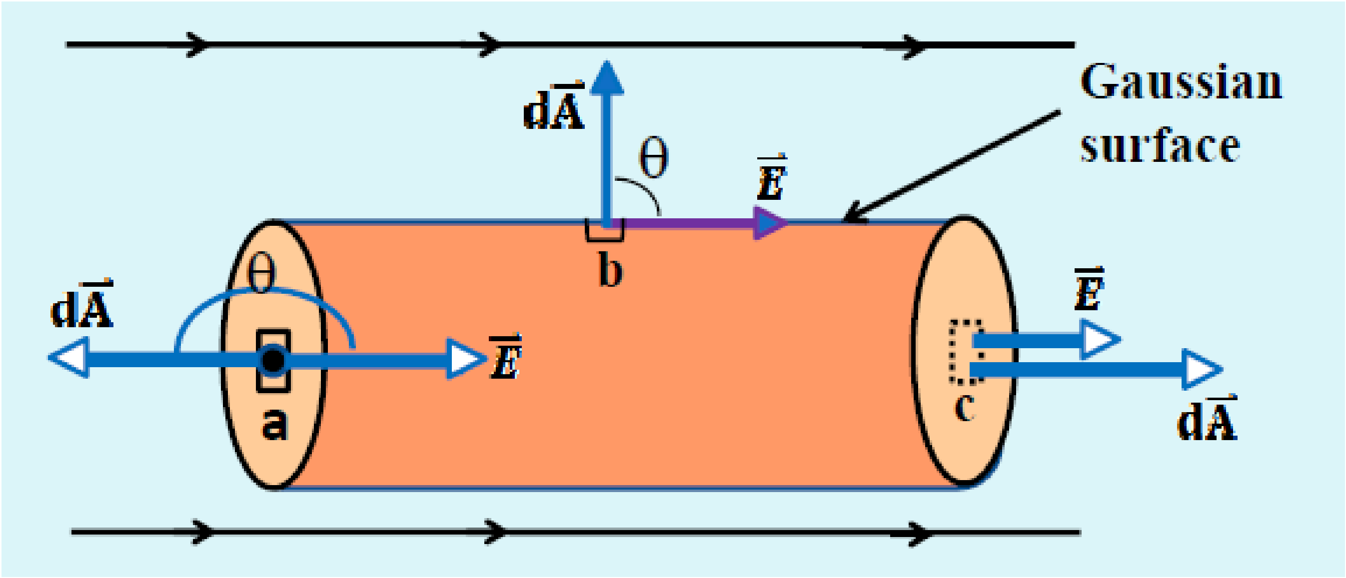
$$\oint_a \boldsymbol{E.dA\cos 0} + \oint_b \boldsymbol{E.dA\cos 90} + \oint_c \boldsymbol{E.dA\cos 0} = \frac{\boldsymbol{q}}{\epsilon_\circ}$$

$$\boldsymbol{EA + EA} = \frac{\boldsymbol{q}}{\epsilon_\circ}$$

$$\boldsymbol{q = \sigma A}$$

$$\boldsymbol{2EA} = \frac{\boldsymbol{\sigma A}}{\epsilon_\circ} \quad \Longrightarrow \quad \boldsymbol{E} = \frac{\boldsymbol{\sigma}}{\boldsymbol{2}\,\epsilon_\circ}$$

Example 5: Shows a Gaussian surface in the form of cylinder of radius **R** immersed in a uniform electric field E. With the cylinder axis parallel to the field. What is the flux $\boldsymbol{\varphi}$ of the electric field through this closed surface?



We can find the flux $\boldsymbol{\varphi}$ through the Gaussian surface by integrating the scalar product $\vec{E}.d\vec{A}$ over that surface. We can do the integration by writing the flux as the sum of three terms integral over the left cap **a**, the cylindrical surface **b**, and the right cap **c**.

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$$\varphi = \oint \vec{E}.d\vec{A} = \int_a \vec{E}.d\vec{A} + \int_b \vec{E}.d\vec{A} + \int_c \vec{E}.d\vec{A}$$

For all points on the left cap, the angle θ between **Ē** and **dĀ** is **180**° and the magnitude E of the field is uniform. Thus, $\int_a \vec{E}.d\vec{A} = \int EdA\,(\cos 180) = -E\int dA = -EA$

$A = \pi R^2$, similarly for the right cap, where θ = 0 for all points,

$$\int_c \vec{E}.d\vec{A} = \int EdA\,(\cos 0) = E\int dA = EA$$

Finally, for the cylindrical surface, where the angle θ = 90 at all points,

$$\int_b \vec{E}.d\vec{A} = \int EdA\,(\cos 90) = 0$$

So, $$\varphi = -EA + 0 + EA = 0$$

The net flux is zero because the field lines that represent the electric field all pass entirely through that Gaussian surface, from the left to the right.

Example 6: An electric charge +Q is uniformly distributed throughout a non conducting solid sphere of radius a. Determine the electric field everywhere inside and outside the sphere.

Sol.

The charge distribution is spherically symmetric with the charge density given by:

$$\boldsymbol{\rho = \frac{Q}{V} = \frac{Q}{(4/3)\pi a^3}}$$

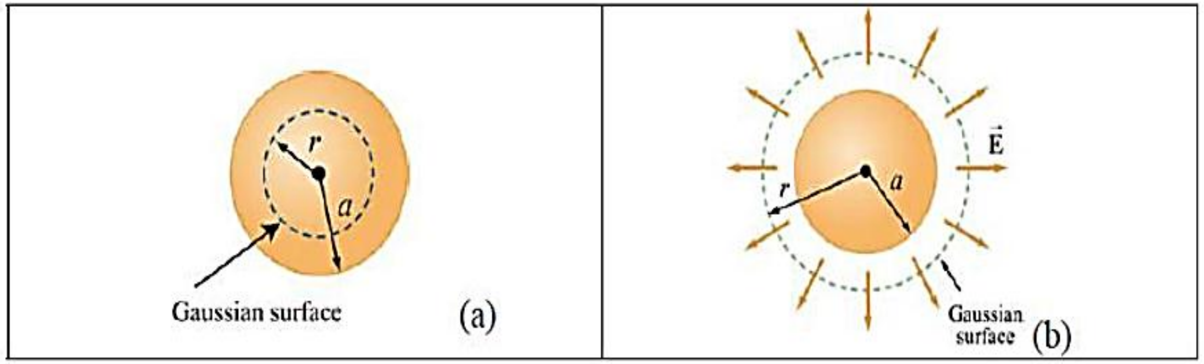
Where V is the volume of the sphere. In this case, the electric field E is radially symmetric and directed outward. The magnitude of the electric field is constant on spherical surface of radius r. the regions $r \leq a \; and \; r \geq a$ shall be studied separately.

Case 1: $r \leq a$

We choose our Gaussian surface to be sphere of radius $r \leq a$ as in the figure below a.

The flux of gaussean surface is:

$$\oint_s \boldsymbol{E} . \boldsymbol{dA} = \boldsymbol{EA} = \boldsymbol{E}(4\pi r^2)$$



Gaussian surface for uniformly charged solid sphere for (a) $r \leq a$ and (a) $r > a$

With uniform charged distribution, the charge enclose is:

$$q_{enc} = \int \rho \, dV = \rho V = \rho\left(\frac{4}{3}\pi r^3\right) = Q\frac{r^3}{a^3}$$

From Gauss law: $\varphi_E = \frac{q_{enc}}{\epsilon_\circ}$

$$E(4\pi r^2) = \frac{\rho}{\epsilon_o}\left(\frac{4}{3}\pi r^3\right) \qquad E = \frac{Q\,r}{4\pi\epsilon_o a^3} \qquad r \leq a$$

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Case 2: $r \geq a$

In this case Gaussian surface is a sphere of radius $r \geq a$ as showing in the figure (b). Since the radius of Gaussian surface is greater than of the radius of sphere all the charge is enclosed in our Gaussian surface $q_{enc} = Q$ with the electric flux through the Gaussian surface given by $\varphi = E(4\pi r^2)$ and from Gauss law:

$$\boldsymbol{E(4\pi r^2) = \frac{Q}{\epsilon_o} \qquad \Longrightarrow \qquad E = \frac{Q}{4\pi\epsilon_o r^2} \qquad r > a}$$

Problem 1: What is the strength and direction of the electric field 3.74 cm on the left hand side of a 9.1 μC negative charge?

Sol.

$$\boldsymbol{E = \frac{k\,q}{d^2}}$$

$$\mathbf{E = (9.0 \times 10^{9}\ N \cdot m^{2} / C^{2}) (9.1 \times 10^{-6}\ C) / (3.74 \times 10^{-2}\ m)^{2}}$$

$$\mathbf{E = 5.8552 \times 10^{7}\ N / C}$$

Which way would a test charge (they are always positive) move when placed on the left hand side of a negative charge ? It would be attracted toward it and therefore to the right.

$$\mathbf{E = 5.9 \times 10^{7}\ N / C \text{ ; to the right}}$$

Problem 2: At what distance from a negative charge of 5.536 nC would the electric field strength be 1.90 x 10^5 N/C ?

Sol.

$$\boldsymbol{E = \frac{k\,q}{d^2} \quad \Longrightarrow \quad d = \sqrt{\frac{k\,q}{E}}}$$

$$\boldsymbol{d = \sqrt{\frac{9 \times 10^9\ \frac{N.m^2}{C^2}\ (5.536 \times 10^{-9}\ C)}{1.9 \times 10^5\ N/C}}}$$

$$\boldsymbol{d = 1.6194 \times 10^{-2}\ m}$$

$$\boldsymbol{d = 1.6\ cm}$$

Problem 3: There are three charges q1, q2, and q3 having charge 6 C, 5 C and 3 C enclosed in a surface. Find the total flux enclosed by the surface.

Sol.

Total charge Q: **Q = q1 + q2 + q3**

$$\mathbf{Q = 6\,C + 5\,C + 3\,C \quad \Longrightarrow \quad Q = 14\,C}$$

The total flux: $$\boldsymbol{\varphi = \frac{Q}{\epsilon_\circ} \Longrightarrow \varphi = \frac{14}{8.854\times10^{-12}} \Longrightarrow \varphi = 1.584\ N\,m^2\,/C}$$

Therefore, the total flux enclosed by the surface is $\varphi = 1.584\ N\,m^2\,/C$

Problem 4: Find the electric flux in a cylinder having length 5 cm, radius 2 cm having electric field intensity 2 N/C.

Sol.

$$\boldsymbol{\varphi = E\,A \quad \Longrightarrow \quad \varphi = E\,(2\,\pi\,r\,l)}$$

$$\boldsymbol{\varphi = 2\,N/C\,(2\,\pi\,\times\,0.02\,m\,\times\,0.05\,m)}$$

$$\boldsymbol{\varphi = 0.0125\,Nm^2/C^{-1}}$$

Therefore, the electric flux in a cylinder is 0.0125 Nm^2/C^{-1}.

Problem 5: Determine the electric flux for a Gaussian surface that contains 100 million electrons.

Sol.

$$\boldsymbol{\varphi = \frac{Q}{\epsilon_\circ}}$$

$$\boldsymbol{\varphi = \frac{100x10^{6}\,(1.6x10^{-19})}{8.85x10^{-12}} \quad \Longrightarrow \quad \varphi = 1.8\,N\,m^{2}/C}$$

Problem 6: A uniformly charged solid spherical insulator has a radius of 0.23 m. The total charge in the volume is 3.2 pC. Find the E-field at a position of 0.14 m from the center of the sphere.

Sol.

$$E = \left(\frac{q}{4\,\pi\,\epsilon_\circ\,R^3} \right) r$$

$$E = \left(\frac{3.2 \times 10^{-12}}{4\,\pi\,\epsilon_\circ\,(0.23)^3} \right) 0.14$$

$$E = 0.331\,N/C$$

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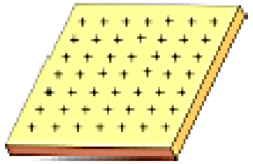
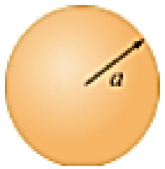
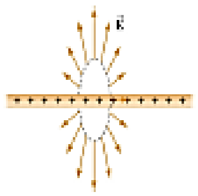
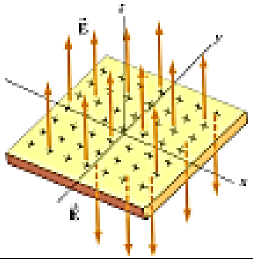
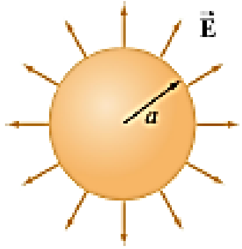
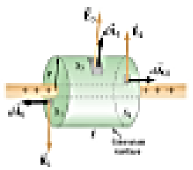
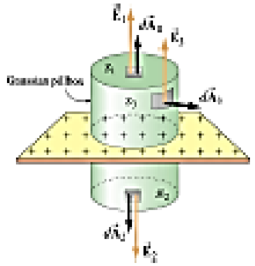
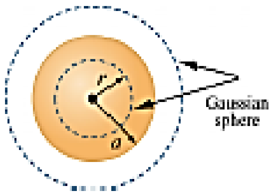
H.W

1) A uniformly charged disk of radius 35cm carries charges with a density of 7.9×10^3 C/m^2. Calculate the electric field on the axis of the disk at (a) 5cm, (b) 10cm, (c) 50cm, and (d) 200 cm from the center of the disk.

2) Two 2μC point charges are located on the x-axis. One is at x=1m, and the other is at x=-1m. (a) Determine the electric field on the y-axis at y=0.5m. (b) Calculate the electric force on a -3μC charge placed on the y-axis at y=0.5m.

3) A uniformly charged ring of radius 10cm has a total charge of 75μC. Find the electric field on the axis of the ring at (a) 1cm, (b) 5cm, (c) 30cm, and (d) 100 cm from the center of the ring.

System	Infinite line of charge	Infinite plane of charge	Uniformly charged solid sphere
Figure			
Identify the symmetry	Cylindrical	Planar	Spherical
Determine the direction of $\vec{E}$			
Divide the space into different regions	$r > 0$	$z > 0$ and $z < 0$	$r \le a$ and $r \ge a$
Choose Gaussian surface	 Coaxial cylinder	 Gaussian pillbox	 Concentric sphere
Calculate electric flux	$\Phi_E = E(2\pi r l)$	$\Phi_E = EA + EA = 2EA$	$\Phi_E = E(4\pi r^2)$
Calculate enclosed charge q_{in}	$q_{enc} = \lambda l$	$q_{enc} = \sigma A$	$q_{enc} = \begin{cases} Q(r/a)^3 & r \le a \\ Q & r \ge a \end{cases}$
Apply Gauss's law $\Phi_E = q_{in}/\varepsilon_0$ to find E	$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$	$E = \dfrac{\sigma}{2\varepsilon_0}$	$E = \begin{cases} \dfrac{Qr}{4\pi\varepsilon_0 a^3}, & r \le a \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}, & r \ge a \end{cases}$

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