Lecture (3)

Electricity and Magnetism I

First Stage
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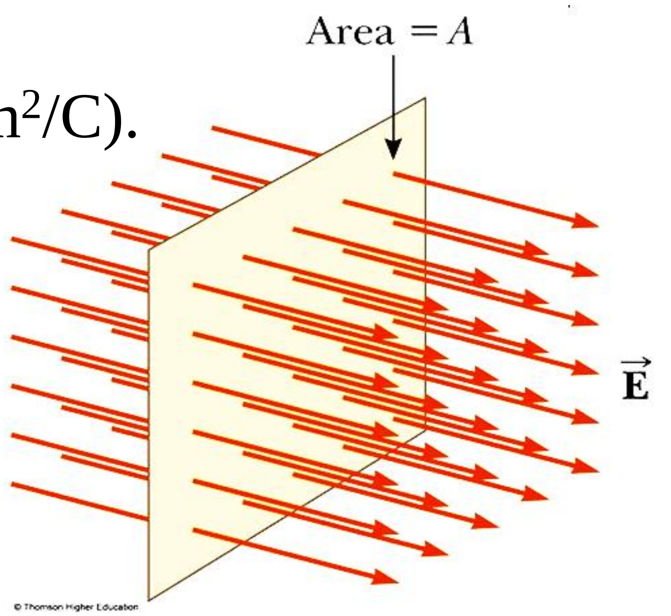
Dr. Ahmed Almurshedi
Ph.D., M.Sc., B.Sc.
Medical Physics

Electric Flux

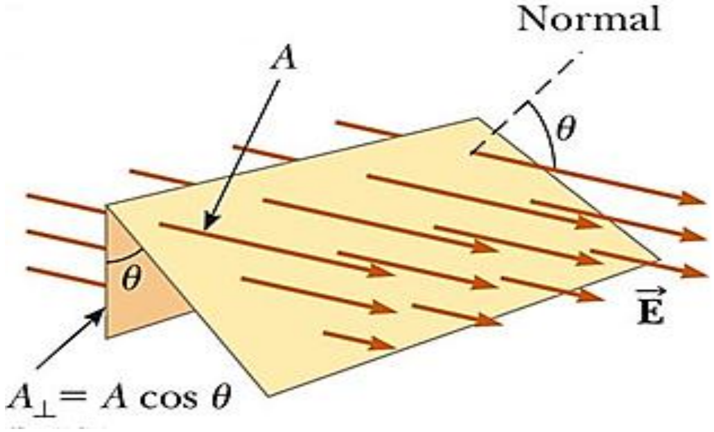
- The total number of lines penetrating the surface is proportional to the product **E.A**. This product of the magnitude of the electric field E and surface area **A** perpendicular to the field is called the electric flux φ_E see figure down.

$$\boldsymbol{\varphi_E = E.A}$$



- From the SI units of **E** and **A**, φ_E. is scalar quantity has units of (N.m^2/C).
- If the surface under consideration is **not perpendicular** to the field, the flux through it must be less than that given by Equation:

$$\boldsymbol{\varphi_E = E.A}$$



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- We can understand this by considering the number of lines that equal to the number that cross the area $\acute{A}$, which is a projection of area **A** onto a plane oriented perpendicular to the field.
- From above figure, we see that the two areas are related by: $\boldsymbol{\acute{A} = A\ \cos\theta}$
- Because the flux through **A** equals the flux through $\acute{A}$, we conclude that the flux through **A** is:

$$\boldsymbol{\varphi_E = E\,\acute{A} = E\,A\ \cos\theta}$$

- The flux through a surface of fixed area **A** has a maximum value EA. When the surface is perpendicular to the field (the normal to the surface is parallel to the field, or, $\theta=0^o$ and the flux is zero. When the surface is parallel to the field, (the normal to the surface is perpendicular or $\theta= 90^o$).

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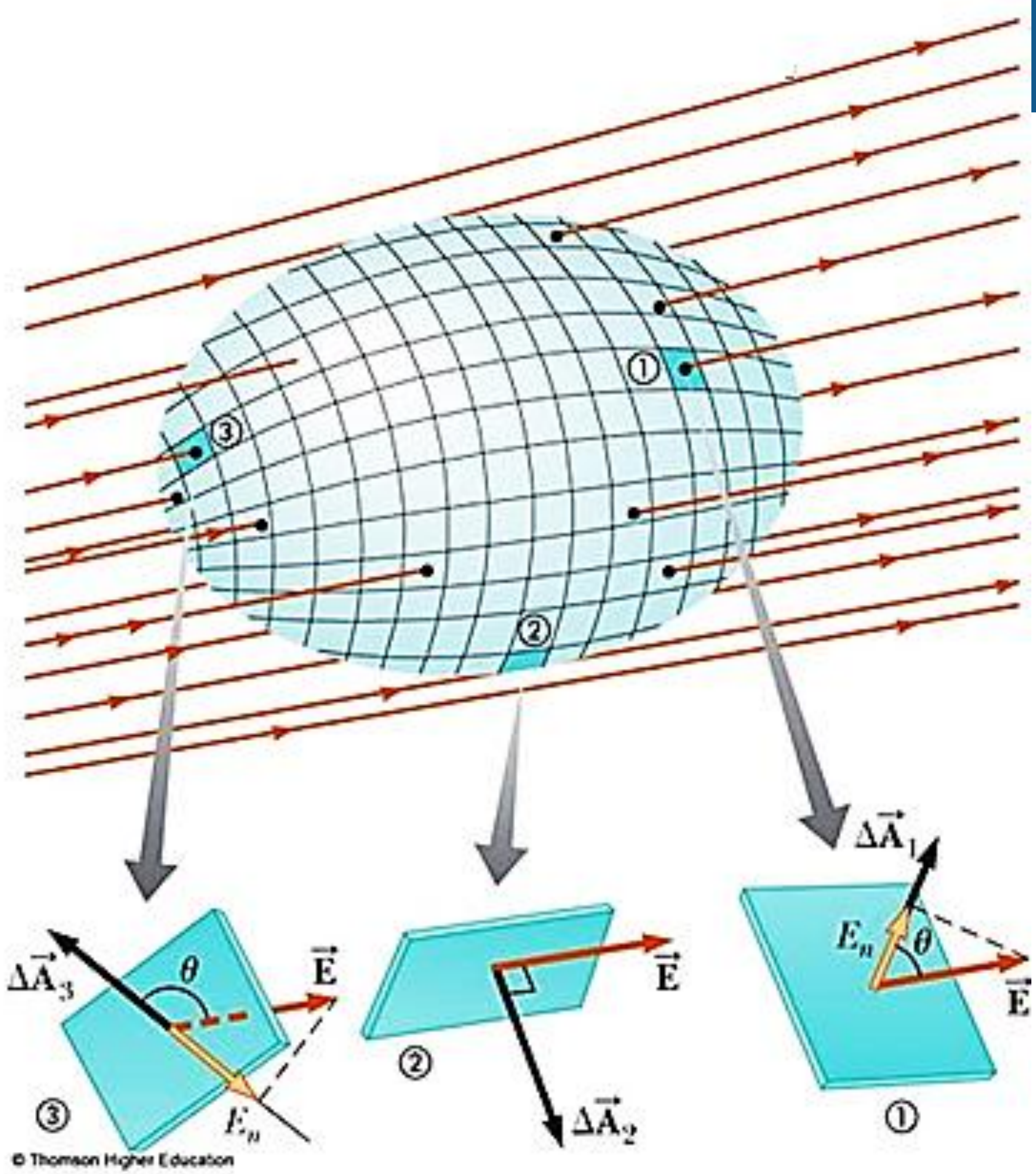
- Consider the closed surface in Figure below. The vectors A_i point in different directions for the various surface elements, but at each point they are normal to the surface and, by convention, always point outward. At the element (1), the field lines are crossing the surface from the inside to the outside and $\theta \leq 90°$; hence, the $\Delta E_{\varphi} = E\Delta A_1$ through this element is positive. For element (2), the field lines graze the surface (perpendicular to the vector ΔA_2; thus, $\theta = 90°$ and the flux is zero. For elements (3), where the field lines are crossing the surface from outside to inside $180° > \theta > 90°$ and the flux negative because cosθ is negative.



①
③
②
$\Delta\vec{A}_3$
θ
$\vec{E}$
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③
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$\Delta\vec{A}_2$
$\Delta\vec{A}_1$
E_n
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①
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- The net flux through the surface is proportional to the net number of lines leaving the surface, where the net number means the number leaving the surface minus the number entering the surface.

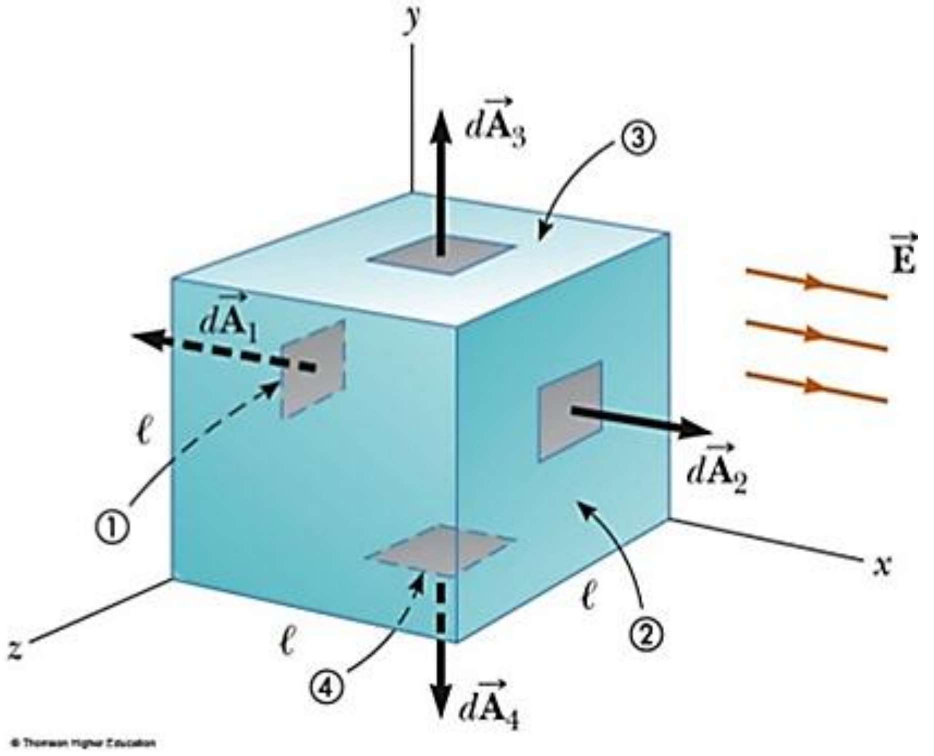
- If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative. Using the symbol $\oint$ to represent an integral over a closed surface, we can write the net flux φ_E through a closed surface.

$$\boldsymbol{\varphi_E = \oint E.dA = \oint E_n\, dA}$$

- Where E_n represents the component of the electric field normal to the surface. If the field is normal to the surface at each point and constant in magnitude.

Example 1: Flux through a Cube

Consider a uniform electric field **E** oriented in the x direction. Find the net electric flux through the surface of a cube of edge length ℓ, oriented as shown in figure down.



Sol.

The net flux is the sum of the fluxes through all faces of the cube. First, note that the flux through four of the faces (**3, 4** and the unnumbered ones) is zero because **E** is perpendicular to **dA** on these faces. The net flux through faces **1 and 2** is:

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$$\varphi_E = \oint E.dA_1 + \oint E.dA_2$$

$$\varphi_E = \int EdA\cos 180 + \int EdA\cos 0$$

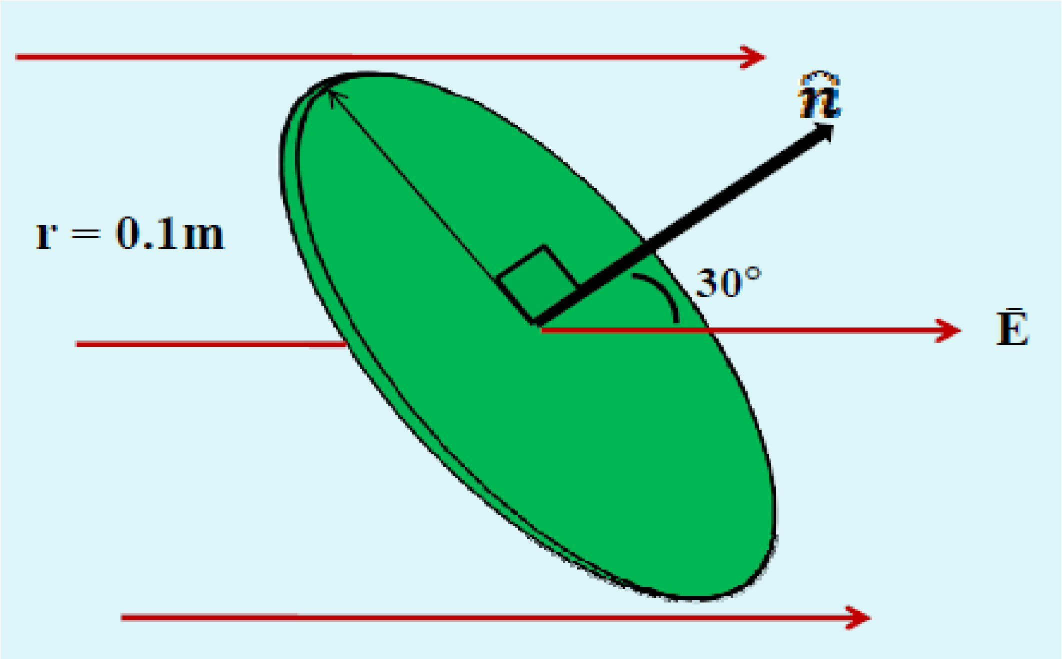
$$\varphi_E = -El^2 + El^2 = 0$$

Therefore, the net flux over all six faces is:

$$\varphi_E = 0$$

Example 2: Electric flux through a disk

Disk of radius 0.10 m with $\vec{n}$ at 30 degrees to E, with a magnitude of $2.0\ \times 10^3$ N/C. What is the flux?



Sol.

$$A = \pi r^2 = 0.0314\ m^2$$

$$\varphi_E = E\ A \cos\theta = E\ A \cos 30$$

$$\varphi_E = \left(2 \times \frac{10^3 N}{C}\right)(0..0314\ m^2)(0.866) = 54\ N\ m^2/C$$

Example 3: An electric field of 500 V/m makes an angle of 30° with the surface vector, which has a magnitude of 0.5 m^2. Find the electric flux that passes through the surface.

Sol.

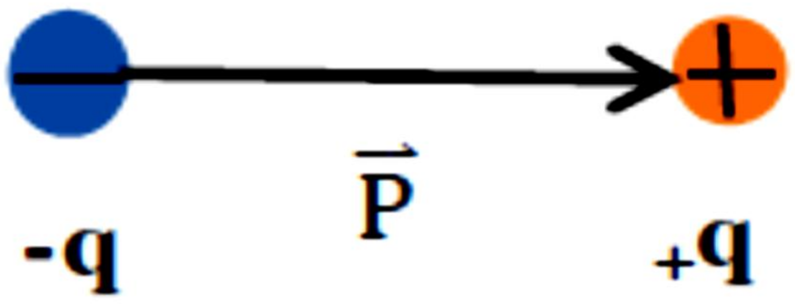
$$\boldsymbol{\varphi_E = E.A = E\,A\cos\theta}$$

$$\boldsymbol{\varphi_E = \left(500\,\frac{V}{m}\right)(0.5\,m^2)\,cos\,30^{o}}$$

$$\boldsymbol{\varphi_E = 217\,V\,m}$$

Electric Dipole

- An electric dipole consists of a pair of **+q** and **−q** charge particles, which are separated by a distance **d**. The magnitude of the dipole moment of this electric dipole is **p=q d**.
- The direction of the electric dipole moment is along the direction from the negative charge to the positive charge. Electric field generated by the pair of charge particles at a distance **r**, with **r >>d**, is called dipole electric field as shown in the figure below.



1. Electric field due to an electric dipole at an axial point.

The electric field at p due the positive charge is given by:

$$\boldsymbol{E_{+}} = \frac{\mathbf{1}}{\boldsymbol{4\pi\epsilon_{o}}} \frac{\boldsymbol{q}}{(\boldsymbol{r}-\boldsymbol{d})^{\mathbf{2}}}$$

The electric field at p due the negative charge is given by:

$$\boldsymbol{E_{-}} = \frac{\mathbf{1}}{\boldsymbol{4\pi\epsilon_{o}}} \frac{-\boldsymbol{q}}{(\boldsymbol{r}+\boldsymbol{d})^{\mathbf{2}}}$$

$E_1 > E_2$ therefore the magnitude of electric field at point p is:

$$\boldsymbol{E} = \boldsymbol{E_1} + \boldsymbol{E_2}$$

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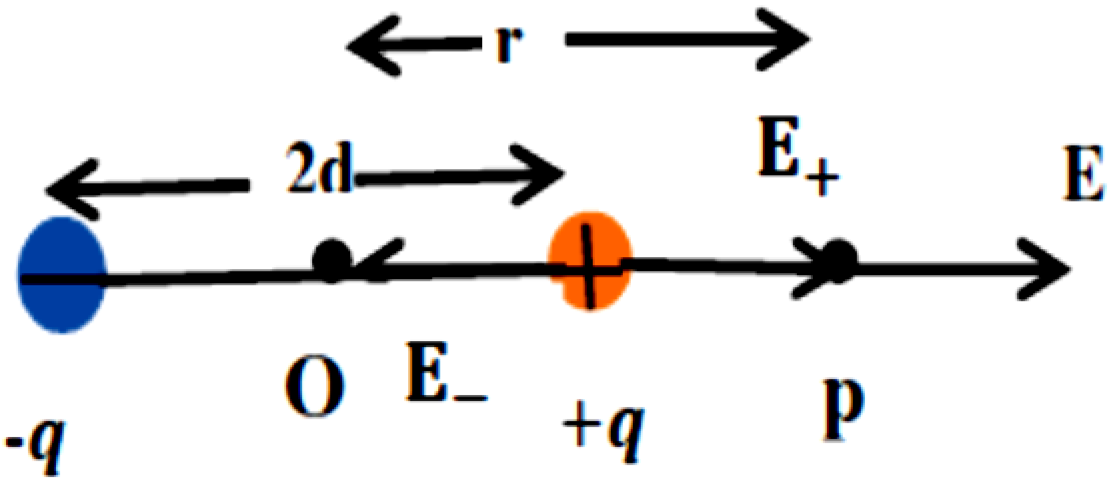
$$E = \frac{1}{4\pi\epsilon_o}\left(\frac{q}{(r-d)^2} + \frac{-q}{(r+d)^2}\right)$$



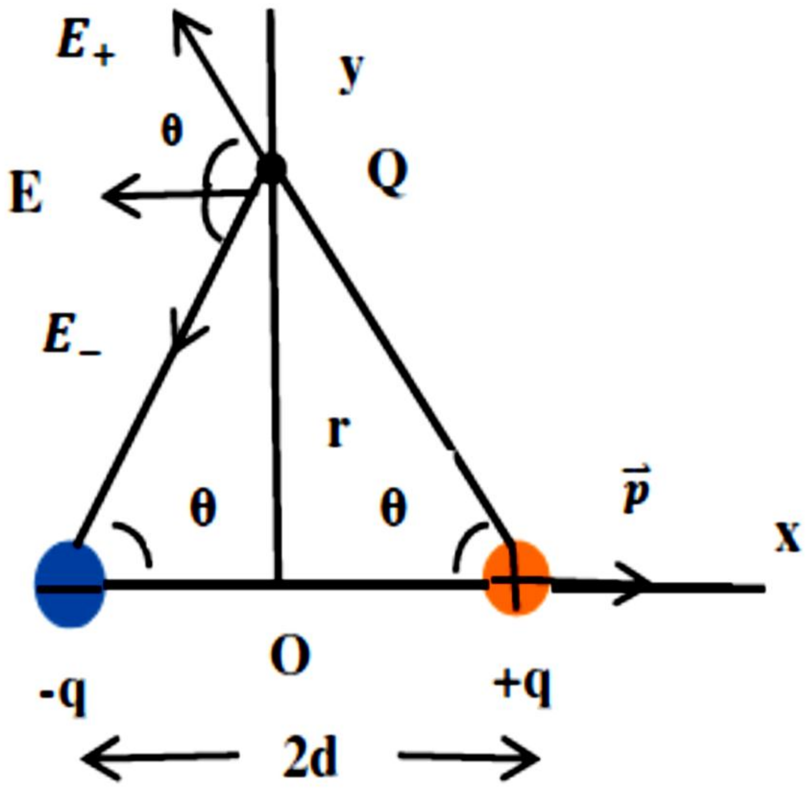
$$E = \frac{q}{4\pi\epsilon_o}\frac{4rd}{\left((r-d)^2\right)^2}$$

If p is far the dipole, then r>>d

$$E = \frac{q}{4\pi\epsilon_o}\frac{4rd}{r^4} \qquad p = 2qd \qquad \Longrightarrow \qquad E = \frac{1}{4\pi\epsilon_o}\frac{2p}{r^3}$$

2. Electric field at a point on the equatorial line.

The magnitudes of the electric fields due to the two charges +q and –q are given by:



$$E_+ = \frac{1}{4\pi\epsilon_o}\frac{1}{r^2+d^2}$$

$$E_- = \frac{1}{4\pi\epsilon_o}\frac{1}{r^2+d^2}$$

We resolve E+ and E- into components along the x and y directions. The y components of E+ and E- cancel out. But the x components of E+ and E- add to yield the resultant field E see figure. $E = E_+\cos\theta + E_-\cos\theta = 2E\cos\theta$ $\quad E_+ = E_-$

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$$\cos\theta = \frac{d}{(d^2 + r^2)^{1/2}}$$

$$E = 2\frac{1}{4\pi\epsilon_o}\frac{q}{(d^2 + r^2)}\frac{d}{(d^2 + r^2)^{1/2}} = \frac{2qd}{4\pi\epsilon_o\,(d^2 + r^2)^{3/2}}$$

d is negligible compared to r.

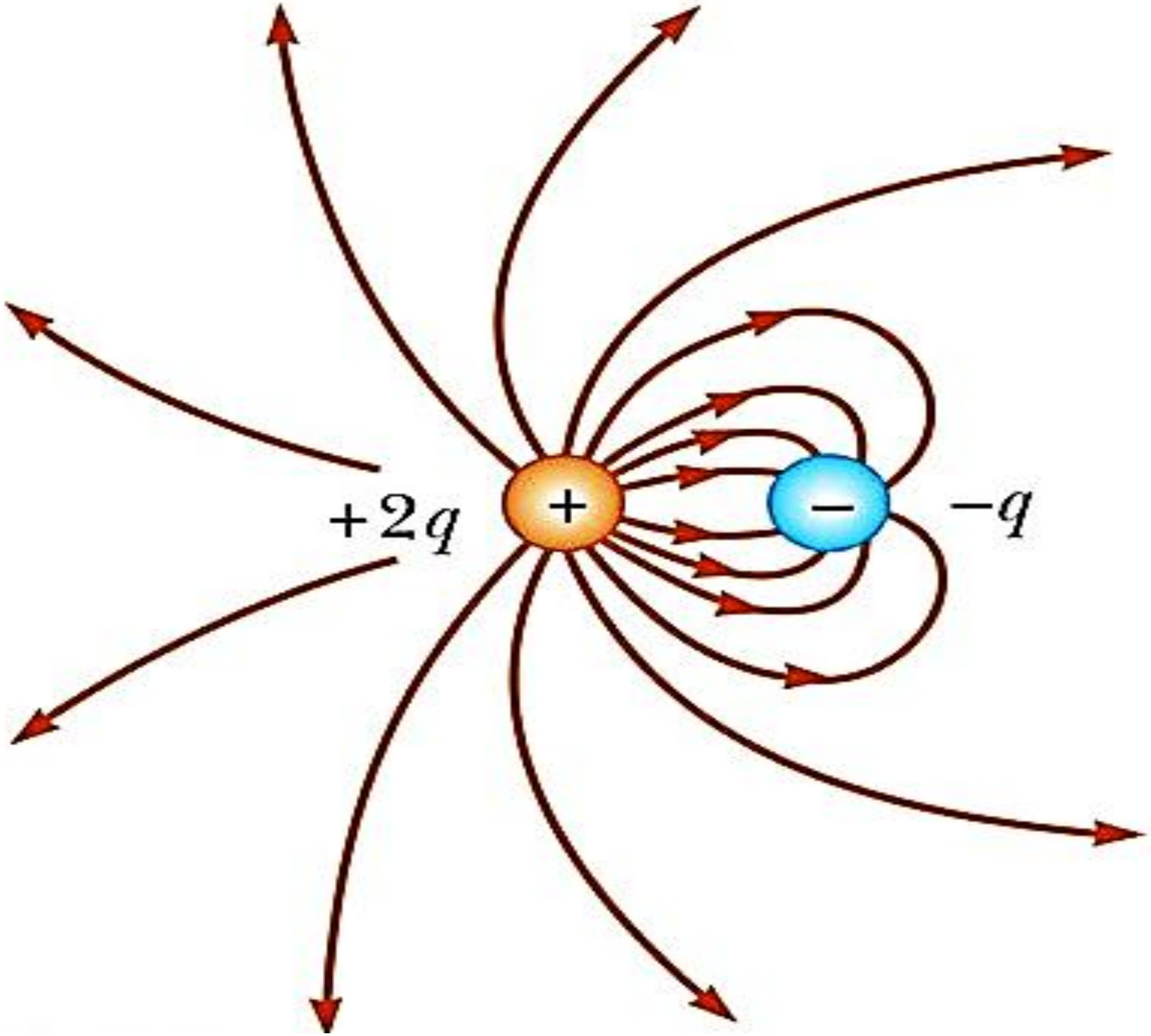
$$E = \frac{1}{4\pi\epsilon_o}\frac{p}{r^3}$$

Direction of E is opposite to that of dipole moment vector p.

Lines of Electric Force

The electric field in a region of space represented by imaginary lines known as electric field force introduce by Faraday. The electric field lines have the following properties:

1. The electric field vector E is tangent to the electric field line at each point.
2. The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.
3. The lines must be begin at positive charges and terminate at negative charges.
4. No two-field lines can crosses.



If the electric field lines associated with a positive charge **+2q** and a negative charge **-q**. In this case, the number of lines leaving **+ 2q** is twice the number terminating at **–q** see figure below.

H.W: What is the electric flux through a cylindrical surface? The electric field, E, is uniform and perpendicular to the surface. The cylinder has radius r and length L.