Lecture (2)

Electricity and Magnetism I

First Stage
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Electric Field

- An electric field $\vec{E}$ exist at a point in space if there is a force $\vec{F}$ of electric origin on a test charge q at rest at that point. The electric field is defined as:

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}}{\boldsymbol{q}} \qquad \boldsymbol{or} \qquad \vec{\boldsymbol{F}} = \boldsymbol{q}\,\vec{\boldsymbol{E}} \Longrightarrow \qquad \vec{\boldsymbol{E}} = \boldsymbol{k}\frac{\boldsymbol{q}}{\boldsymbol{r}^2}$$

- The intensity of electric field at a point is equal to the force per unit charge placed at that point. The electric field intensity $\vec{E}$ is expressed in (Newton/Coulomb) ($N.C^{-1}$) or using the fundamental units ($m.kg.s^{-2}.C^{-1}$).

- The force on a positively charged object is in the same direction as the electric field at that point, While the force on a negative test charge is in the opposite direction as the electric field at the point

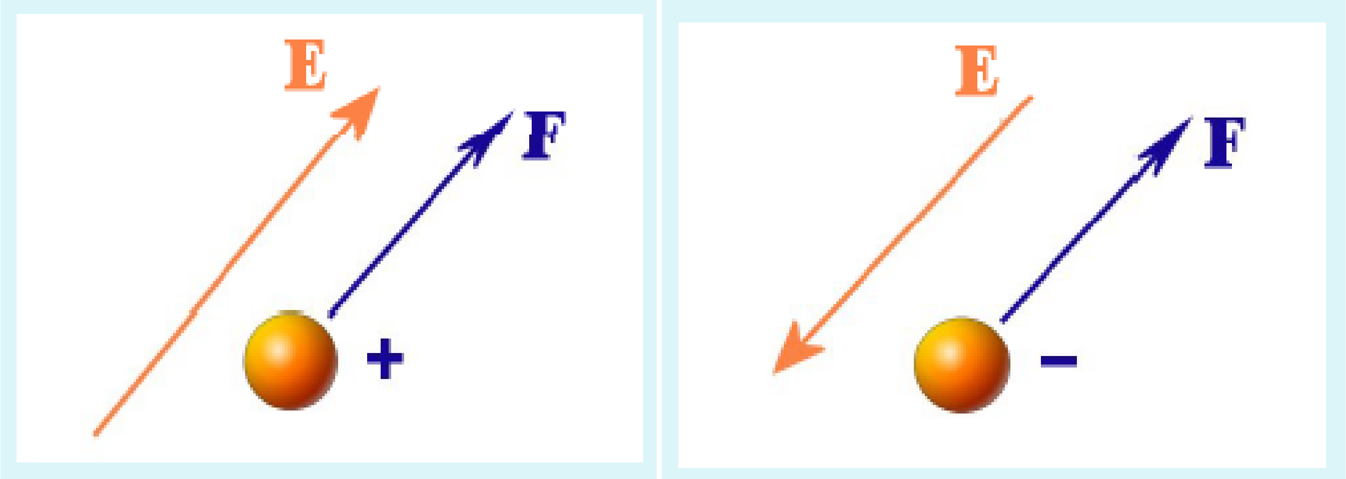
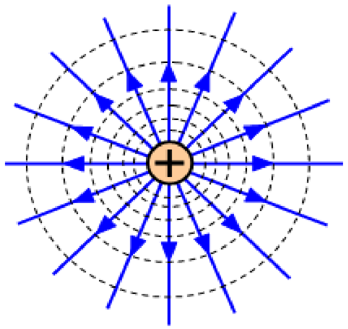
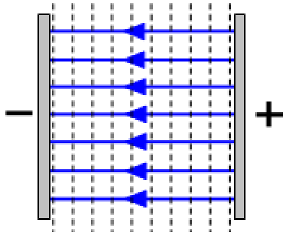
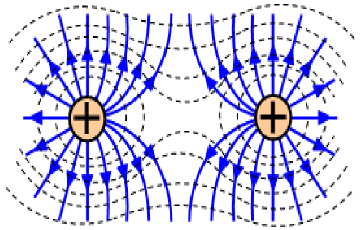


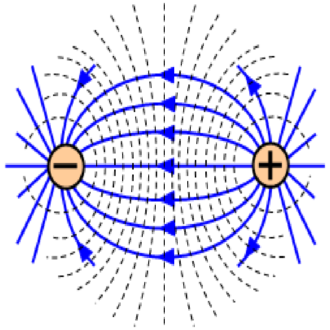
Figure the direction of Electric Field

- An electric field may be represented by lines of force, which are the lines, that at each point, are tangent to the direction of the electric field at that point.
- The electric field is radially outward from a positive charge and radially in toward a negative point charge.
- If we apply an electric field to a region where positive and negative particles or ions are present, the field will tend to move the positive and negative charged bodies in opposite directions, result charge separation, an effect called **polarization**.





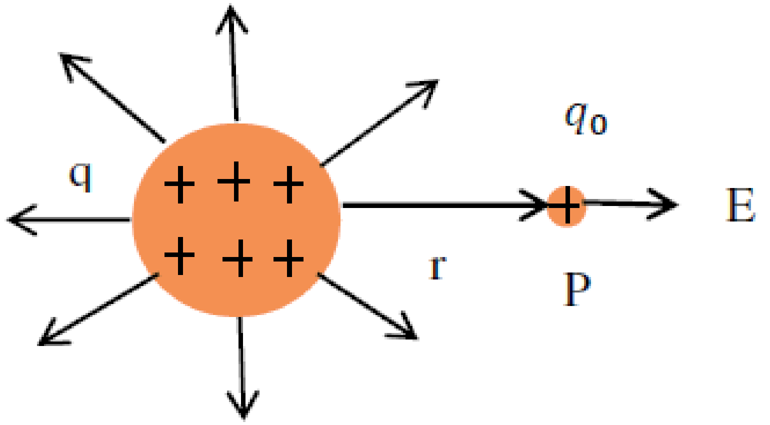




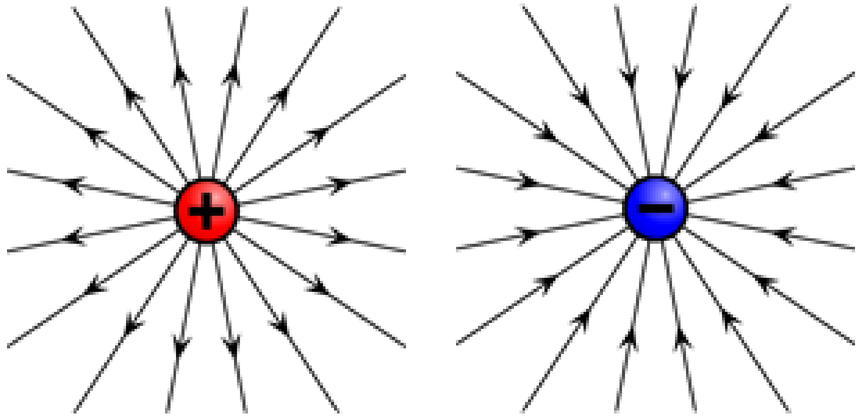
Electric Field of point charges

- Let us calculate the electric field at a point (**P**) arising from an isolated point charge **q**, at distance **r** from it. To do so we assume existence of a test charge $\mathbf{q_o}$ at point (**P**), see the figure. From coulomb's Law the force upon the test charge is:

$$\vec{F} = \frac{q\, q_o}{4\,\pi\,\epsilon_\circ\, r^2}\,\hat{r} \qquad \vec{E} = \frac{q}{4\,\pi\,\epsilon_\circ\, r^2}\,\hat{r}$$



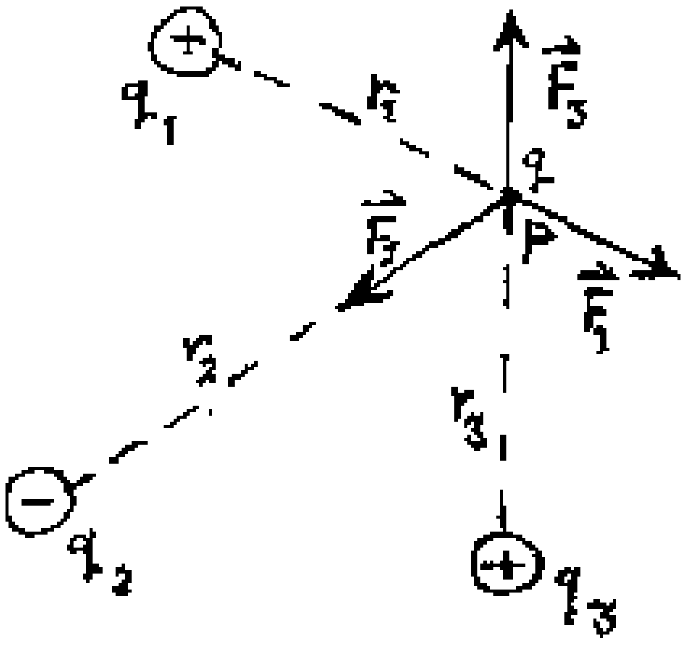
- If the charge **q** is negative the direction of **E** will be reversed.
- Figure below indicates the electric field at points near a positive and negative charge.



The lines of force of the electric field of a positive and a negative charges are straight lines passing through the charge.

- **The electric field at point due to a group of point charges** is the vector sum of the electric fields at that point due to each charge individually (as in the figure below).
- If we have n charges the net electric field E is:

$$\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2} + \cdots + \overrightarrow{E_n}$$



- Where **E_1, E_2 and E_n** are the electric fields due to the charges **q_1, q_2 and q_n** respectively. The magnitude of the electric field is obtained as:

$$\vec{E} = K\frac{|q|}{r^2} \quad \Longrightarrow \quad \vec{E} = \sum_i K\frac{q_i}{{r^2}_i}$$

- If the charge distribution is continues: $\vec{E} = K\int \frac{dq}{r^2}\,\overrightarrow{u_r}$

- Where $\overrightarrow{u_r}$ points from the charge increment (dq) to the field point.

Charge Density

- When dealing with continuous charge distribution it is convenient to use the concept of charge density.

- If the charge is distribution a long a line we define the linear charge density **λ** as:

$$\boldsymbol{\lambda = \frac{dq}{dl}}$$

- If the charge is distribution over a surface we define the surface charge density **σ** as:

$$\boldsymbol{\sigma = \frac{dq}{dA}}$$

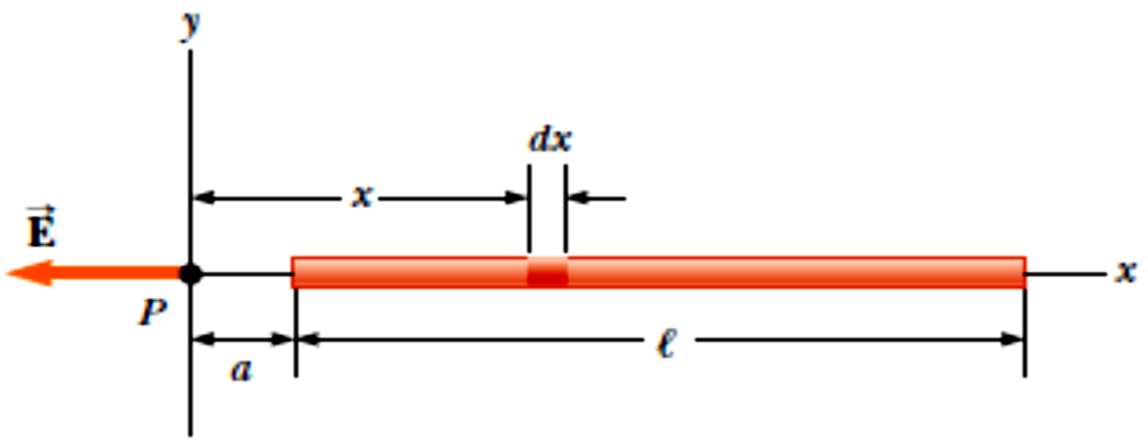
- If the charge is distribution with a volume we define the volume charge density ρ as:

$$\boldsymbol{\rho = \frac{dq}{dV}}$$

- **Some Measures of Electric Charge**

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

Electric Field due to charged Rod



A rod of length (l) has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.

- Lets assume the rod is lying along the x axis, dx is the length of one small segment, and dq is the charge on the segment. Because the rod has a charge per unit length λ, the charge dq on the small segment is:

$$dq = \lambda\, dx$$

- The magnitude of the electric field at P due to one segment of the rod having a charge dq:

$$dE = k\,\frac{dq}{x^2} = k\frac{\lambda\, dx}{x^2}$$

$$\vec{E} = k \lim_{\Delta q_i} \sum_i \frac{\Delta q_i}{r_i^2}\,\hat{r}_i = k\int \frac{dq}{r^2}\,\hat{r}$$

- The total field at P using above equation:

$$\boldsymbol{E = \int_{a}^{l+a} k\,\lambda\,\frac{dx}{x^2}}$$

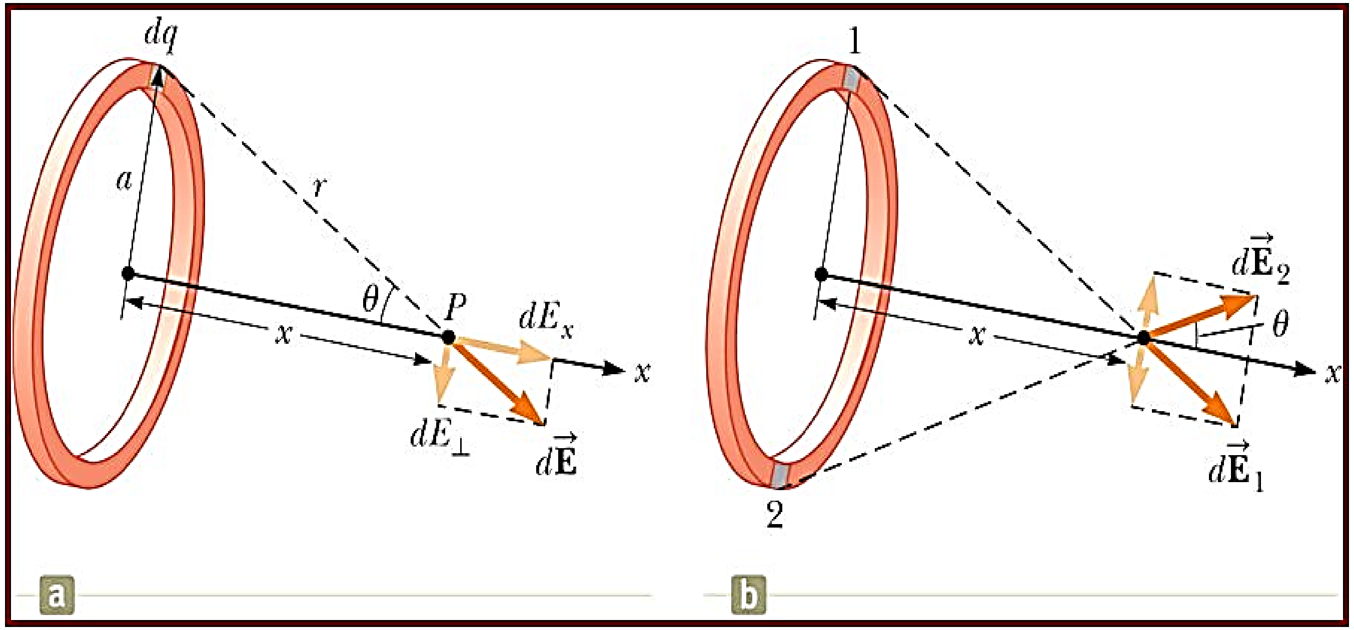
- Noting that k and $\lambda = Q/l$ are constants and can be removed from the integral, evaluate the integral:

$$\boldsymbol{E = k\,\lambda\int_{a}^{l+a}\frac{dx}{x^2} = k\,\lambda\left[-\frac{1}{x}\right]_{a}^{l+a}}$$

$$\boldsymbol{E = k\,\frac{Q}{l}\left(\frac{1}{a}-\frac{1}{l+a}\right) = \frac{k\,Q}{a(l+a)}}$$

The Electric Field of a Uniform Ring of Charge

- A ring of radius ***a*** carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring as in the above figure.



- The parallel component of an electric field contribution from a segment of charge dq on the ring:

$$dE_x = dE\ \cos\theta \qquad\qquad r^2 = a^2 + x^2$$

$$dE_x = k\ \frac{dq}{r^2}\cos\theta = k\ \frac{dq}{a^2 + x^2}\cos\theta$$

$$But \qquad \cos\theta = \frac{x}{r} = \frac{x}{\sqrt{a^2 + x^2}}$$

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$$dE_x = k\,\frac{dq}{a^2+x^2}\left[\frac{x}{\sqrt{a^2+x^2}}\right] = \frac{k\,x}{(a^2+x^2)^{3/2}}\,dq$$

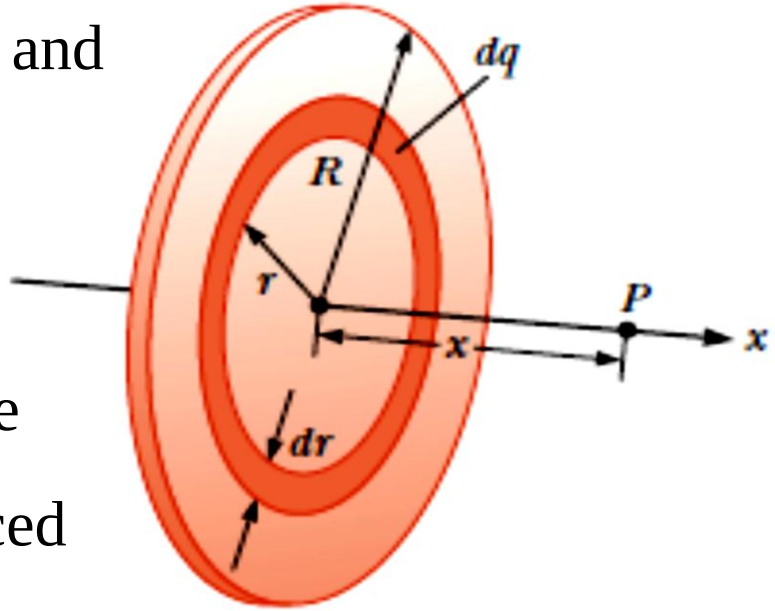
All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Integrate over the circumference of the ring to obtain the total field at P:

$$E_x = \int \frac{k\,x}{(a^2+x^2)^{3/2}}\,dq = \frac{k\,x}{(a^2+x^2)^{3/2}}\int dq \quad \Rightarrow \quad E = \frac{k\,x}{(a^2+x^2)^{3/2}}\,Q$$

The Electric Field of a Uniformly Charged Disk

A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk.

The amount of charge dq on the surface area of a ring of radius r and width dr as in figure.



$$dq = \sigma\, dA = \sigma(2\pi r\, dr) = 2\pi\sigma r\, dr$$

Use this result in the equation of E_x given in the previous example (**The Electric Field of a Uniform Ring of Charge**) (with a replaced by r and Q replaced by dq) to find the field due to the ring:

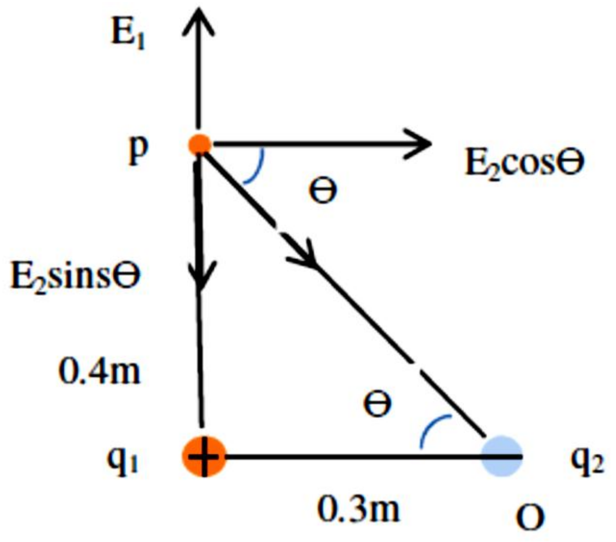
$$dE_x = \frac{k\,x}{(r^2 + x^2)^{3/2}}\, dq \quad \Rightarrow \quad dE_x = \frac{k\,x}{(r^2 + x^2)^{3/2}}\,(2\pi\sigma r\, dr)$$

To obtain the total field at P, integrate this expiration over the limits r=0 to r=R, noting that x is a constant in this situation:

$$E_x = k\,x\pi\sigma \int_0^R \frac{2r\,dr}{(r^2+x^2)^{3/2}} \qquad \Rightarrow \qquad E_x = k\,x\pi\sigma \int_0^R \left(r^2+x^2\right)^{-3/2} d(r^2)$$

$$E_x = k\,x\pi\sigma \left[\frac{\left(r^2+x^2\right)^{-1/2}}{-1/2}\right]_0^R = 2\pi k\sigma\left[1 - \frac{x}{(R^2+x^2)^{1/2}}\right]$$

Example1: Two point charges of 7μC, -5 μC are located as shown in figure below. Find the resultant of electric field at point P.



Sol.

Let us calculate the magnitudes of the electric field due to each charge.

$$(OP)^2 = (0.4)^2 + (0.3)^2 \quad \Rightarrow \quad OP = 0.5\, m$$

$$\sin\theta = \frac{0.4}{0.5} = 0.8 \quad \Rightarrow \quad \theta = 53°$$

$$\overrightarrow{E_1} = 9 \times 10^9 \frac{7 \times 10^{-6}}{(0.4)^2} \hat{j}$$

$$\overrightarrow{E_2} = 9 \times 10^9 \frac{5 \times 10^{-6}}{(0.5)^2} \cos 53° \, \hat{i} - 9 \times 10^9 \frac{5 \times 10^{-6}}{(0.5)^2} \sin 53° \, \hat{j}$$

$$\vec{E}_P = \overrightarrow{E_1} + \overrightarrow{E_2}$$

$$\vec{E}_P = 9 \times 10^9 \frac{7 \times 10^{-6}}{(0.4)^2} \hat{j} + 9 \times 10^9 \frac{5 \times 10^{-6}}{(0.5)^2} \cos 53° \, \hat{i} - 9 \times 10^9 \frac{5 \times 10^{-6}}{(0.5)^2} \sin 53° \, \hat{j}$$

$$\vec{E}_P = \left(1.1 \times 10^5 \, \hat{i} + 2.5 \times 10^5 \, \hat{j}\right) N/C$$

Example 2: Find the electric field at a distance 1 cm from a positive point charge of magnitude $q = 10^{-10}\ C$.

Sol.

The force on a test charge $\acute{q}$ a distance r from q is: $\boldsymbol{F = K\frac{\acute{q}\,q}{r^2}}$

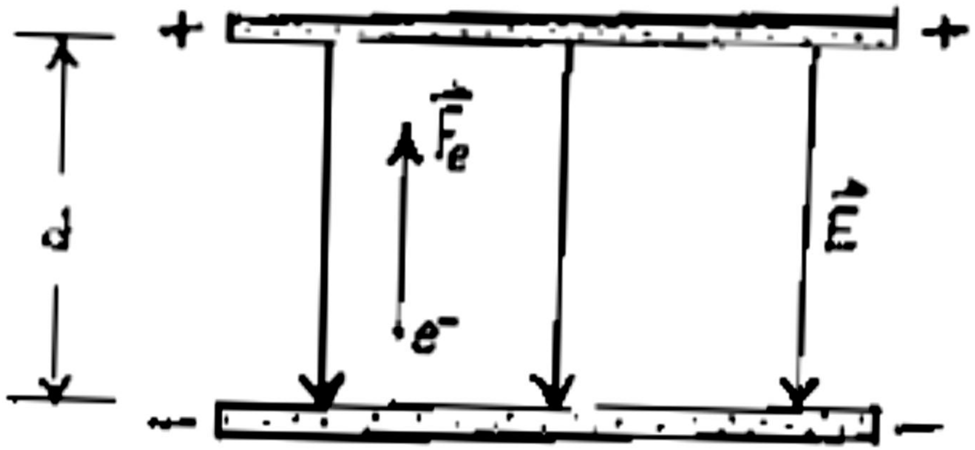
The electric field is the force per unit charge

$$\boldsymbol{E = \frac{F}{\acute{q}} = K\frac{q}{r^2} = 9 \times 10^9 (N.m^2/C^2)\ \frac{10^{-10}\ C}{(0.01\ m)^2} = 9000\ N/C}$$

The direction of the field is the radial direction outward from q.

Example 3: A uniform electric field of magnitude E exist between two conducting plates by a distance d. An electron is released from rest at the negative plate and is attracted to the positive plate. (a) find its kinetic energy when it collides with the positive plate, in terms of E, d and the electric charge. (b) If E=10 N/C and d=1 cm find the velocity of impact.

Sol.



(a) $F_e = eE = ma \quad \Longrightarrow \quad a = \frac{e\,E}{m}$

For constant acceleration: $v^2 = v_o^2 + 2ax$

If the electron starts from rest ($v_o = 0$) and travels a distance d.

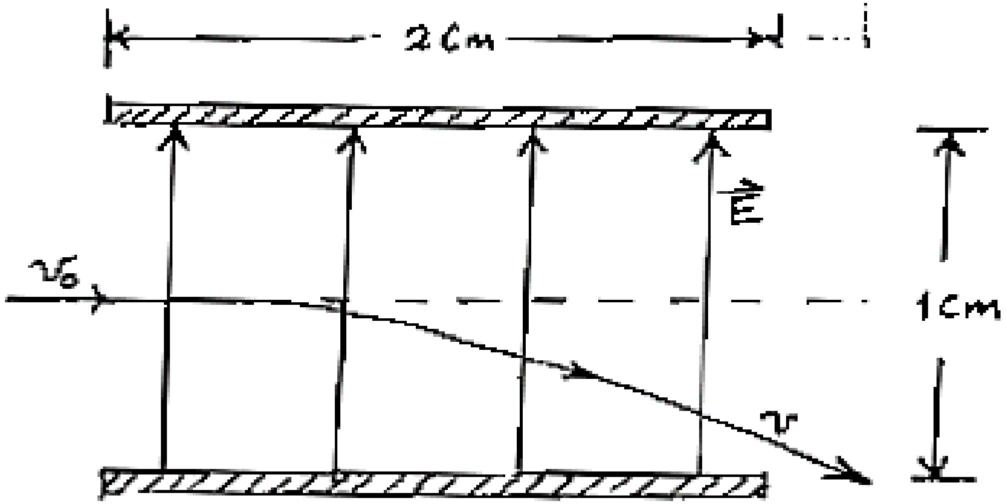
$$v^2 = 2ad = 2\,\frac{e\,E}{m}\,d$$

So, the kinetic energy is: $k = \frac{1}{2}mv^2 = e\,E\,d$

(b) its velocity is $\boldsymbol{v = \sqrt{\frac{2\,e\,E\,d}{m}}} \quad \Longrightarrow \quad \boldsymbol{v = \sqrt{\frac{2\,(1.6\times10^{-19}\,C)\,(10\,N/C)\,(0.01\,m)}{9.1\times10^{-31}\,kg}}}$

$$\boldsymbol{v = 1.88 \times 10^{5}\,m/s}$$

Example 4: an electron enters a region of space with a uniform field of magnitude 5000 N/C as shown in the figure down. What is the minimum velocity it must have to escape hitting one of the plates?



Sol.

The electric force on the electron is:

$$\boldsymbol{F = q\,E = -e\,E}$$

and is directed down as shown in the figure opposite to the field direction.

The equation of motion are:

$$\boldsymbol{F_x = 0 = ma_x \;\Rightarrow\; a_x = 0 \;\Rightarrow\; x = x_o + v_{ox}\,t}$$

$$F_y = -e\,E = ma_y \quad \Longrightarrow \quad a_y = \frac{-e\,E}{m}$$

$$y = y_o + v_{oy}\,t - \frac{e\,E}{2\,m}t^2$$

Since, $x_o = 0$, $y_o = 0$, $v_{ox} = v_o$, *and* $v_{oy} = 0$ $x = v_o\,t$

We have: $y = \frac{-e\,E}{2m}t^2 \quad \Longrightarrow \quad y = \frac{-e\,E}{2m}\left(\frac{x}{v_o}\right)^2$

To hit the plate as shown in the figure, x=2 cm when y=-0.5 cm. Solving the last expression for v_o.

$$v_o = \sqrt{\frac{-eE}{2m}\,\frac{x^2}{y}} \quad \Longrightarrow \quad v_o = \sqrt{\frac{(-1.6\times10^{-19}\,C)(5000\,N/C)(0.02\;m)^2}{2(9.1\times10^{-31}\,kg)(-0.005\,m)}} = 5.93\times10^6\,m/s$$