Electricity and Magnetics II

Lecture No.(14)- Semester 2

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Suppose an AC generator with $V(t) = (150\text{ V})\sin(100t)$ is connected to a series RLC circuit with $R = 40.0\ \Omega$, $L = 80.0$ mH, and $C = 50.0\ \mu$F, as shown in Figure 12.9.1.

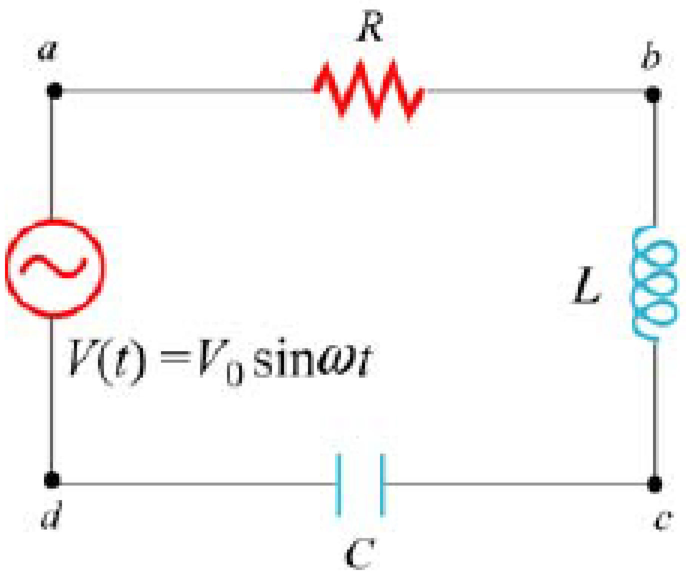


Figure 12.9.1 *RLC* series circuit

(a) Calculate V_{R0}, V_{L0} and V_{C0}, the maximum of the voltage drops across each circuit element.

(b) Calculate the maximum potential difference across the inductor and the capacitor between points b and d shown in Figure 12.9.1.

Solutions:

(a) The inductive reactance, capacitive reactance and the impedance of the circuit are given by

$$X_C = \frac{1}{\omega C} = \frac{1}{(100\ \text{rad/s})\left(50.0\times 10^{-6}\ \text{F}\right)} = 200\ \Omega \tag{12.9.7}$$

$$X_L = \omega L = (100\ \text{rad/s})\left(80.0\times 10^{-3}\ \text{H}\right) = 8.00\ \Omega \tag{12.9.8}$$

and

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{(40.0\ \Omega)^2+(8.00\ \Omega-200\ \Omega)^2}=196\ \Omega \qquad (12.9.9)$$

respectively. Therefore, the corresponding maximum current amplitude is

$$I_0=\frac{V_0}{Z}=\frac{150\ \text{V}}{196\ \Omega}=0.765\ \text{A} \qquad (12.9.10)$$

The maximum voltage across the resistance would be just the product of maximum current and the resistance:

$$V_{R0}=I_0R=(0.765\ \text{A})(40.0\ \Omega)=30.6\ \text{V} \qquad (12.9.11)$$

Similarly, the maximum voltage across the inductor is

$$V_{L0}=I_0X_L=(0.765\ \text{A})(8.00\ \Omega)=6.12\ \text{V} \qquad (12.9.12)$$

and the maximum voltage across the capacitor is

$$V_{C0} = I_0 X_C = (0.765\ \text{A})(200\ \Omega) = 153\ \text{V} \tag{12.9.13}$$

Note that the maximum input voltage V_0 is related to V_{R0}, V_{L0} and V_{C0} by

$$V_0 = \sqrt{V_{R0}{}^2 + (V_{L0} - V_{C0})^2} \tag{12.9.14}$$

(b) From b to d, the maximum voltage would be the difference between V_{L0} and V_{C0}:

$$|V_{bd}| = |\vec{V}_{L0} + \vec{V}_{C0}| = |V_{L0} - V_{C0}| = |6.12\ \text{V} - 153\ \text{V}| = 147\ \text{V} \tag{12.9.15}$$

A sinusoidal voltage $V(t) = (200\,\text{V})\sin\omega t$ is applied to a series RLC circuit with $L = 10.0$ mH, $C = 100$ nF and $R = 20.0\ \Omega$. Find the following quantities:

(a) the resonant frequency,

(b) the amplitude of the current at resonance,

(c) the quality factor Q of the circuit, and

(d) the amplitude of the voltage across the inductor at the resonant frequency.

Solution:

(a) The resonant frequency for the circuit is given by

$$f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{\left(10.0\times10^{-3}\ \mathrm{H}\right)\left(100\times10^{-9}\ \mathrm{F}\right)}} = 5033\,\mathrm{Hz} \quad (12.9.16)$$

(b) At resonance, the current is

$$I_0 = \frac{V_0}{R} = \frac{200\ \mathrm{V}}{20.0\ \Omega} = 10.0\,\mathrm{A} \quad (12.9.17)$$

(c) The quality factor Q of the circuit is given by

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi\left(5033\ \mathrm{s}^{-1}\right)\left(10.0\times10^{-3}\ \mathrm{H}\right)}{\left(20.0\ \Omega\right)} = 15.8 \quad (12.9.18)$$

(d) At resonance, the amplitude of the voltage across the inductor is

$$V_{L0} = I_0 X_L = I_0\omega_0 L = (10.0\ \mathrm{A})\,2\pi\left(5033\ \mathrm{s}^{-1}\right)\left(10.0\times10^{-3}\ \mathrm{H}\right) = 3.16\times10^{3}\,\mathrm{V} \quad (12.9.19)$$

EXAMPLE 7 ***LC* circuit.** A 1200-pF capacitor is fully charged by a 500-V dc power supply. It is disconnected from the power supply and is connected, at $t = 0$, to a 75-mH inductor. Determine: (*a*) the initial charge on the capacitor; (*b*) the maximum current; (*c*) the frequency f and period T of oscillation; and (*d*) the total energy oscillating in the system.

SOLUTION (*a*) The 500-V power supply, before being disconnected, charged the capacitor to a charge of

$$Q_0 = CV = (1.2 \times 10^{-9}\,\mathrm{F})(500\,\mathrm{V}) = 6.0 \times 10^{-7}\,\mathrm{C}.$$

(*b*) The maximum current, $I_{\max}$, is (see Eqs. 14 and 15)

$$I_{\max} = \omega Q_0 = \frac{Q_0}{\sqrt{LC}} = \frac{(6.0 \times 10^{-7}\,\mathrm{C})}{\sqrt{(0.075\,\mathrm{H})(1.2 \times 10^{-9}\,\mathrm{F})}} = 63\,\mathrm{mA}.$$

(*c*) Equation 14 gives us the frequency:

$$f = \frac{\omega}{2\pi} = \frac{1}{(2\pi\sqrt{LC})} = 17\,\mathrm{kHz},$$

and the period T is

$$T = \frac{1}{f} = 6.0 \times 10^{-5}\ \mathrm{s}.$$

(d) Finally the total energy (Eq. 16) is

$$U = \frac{Q_0^2}{2C} = \frac{(6.0 \times 10^{-7}\ \mathrm{C})^2}{2(1.2 \times 10^{-9}\ \mathrm{F})} = 1.5 \times 10^{-4}\ \mathrm{J}.$$

EXAMPLE 8 **Damped oscillations.** At $t = 0$, a 40-mH inductor is placed in series with a resistance $R = 3.0\,\Omega$ and a charged capacitor $C = 4.8\,\mu\text{F}$. (*a*) Show that this circuit will oscillate. (*b*) Determine the frequency. (*c*) What is the time required for the charge amplitude to drop to half its starting value? (*d*) What value of R will make the circuit nonoscillating?

SOLUTION (*a*) In order to oscillate, the circuit must be underdamped, so we must have $R^2 < 4L/C$. Since $R^2 = 9.0\,\Omega^2$ and $4L/C = 4(0.040\,\text{H})/(4.8 \times 10^{-6}\,\text{F}) = 3.3 \times 10^4\,\Omega^2$, this relation is satisfied, so the circuit will oscillate.

(*b*) We use Eq. 18:

$$f' = \frac{\omega'}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = 3.6 \times 10^2\,\text{Hz}.$$

(*c*) From Eq. 19, the amplitude will be half when

$$e^{-\frac{R}{2L}t} = \tfrac{1}{2}$$

or

$$t = \frac{2L}{R}\ln 2 = 18\,\text{ms}.$$

(*d*) To make the circuit critically damped or overdamped, we must use the criterion $R^2 \geq 4L/C = 3.3 \times 10^4\,\Omega^2$. Hence we must have $R \geq 180\,\Omega$.

EXAMPLE 9 **Reactance of a coil.** A coil has a resistance $R = 1.00\,\Omega$ and an inductance of 0.300 H. Determine the current in the coil if (*a*) 120-V dc is applied to it, (*b*) 120-V ac (rms) at 60.0 Hz is applied.

SOLUTION (*a*) With dc, we have no X_L so we simply apply Ohm's law:

$$I = \frac{V}{R} = \frac{120\text{ V}}{1.00\,\Omega} = 120\text{ A}.$$

(*b*) The inductive reactance is

$$X_L = 2\pi f L = (6.283)(60.0\text{ s}^{-1})(0.300\text{ H}) = 113\,\Omega.$$

In comparison to this, the resistance can be ignored. Thus,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{120\text{ V}}{113\,\Omega} = 1.06\text{ A}.$$

EXAMPLE 11 ***LRC* circuit.** Suppose $R = 25.0\,\Omega$, $L = 30.0\,\text{mH}$, and $C = 12.0\,\mu\text{F}$ in Fig. 19, and they are connected in series to a 90.0-V ac (rms) 500-Hz source. Calculate (*a*) the current in the circuit, (*b*) the voltmeter readings (rms) across each element, (*c*) the phase angle ϕ, and (*d*) the power dissipated in the circuit.

SOLUTION (*a*) First, we find the reactance of the inductor and capacitor at $f = 500\,\text{Hz} = 500\,\text{s}^{-1}$:

$$X_L = 2\pi f L = 94.2\,\Omega, \qquad X_C = \frac{1}{2\pi f C} = 26.5\,\Omega.$$

Then the total impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(25.0\,\Omega)^2 + (94.2\,\Omega - 26.5\,\Omega)^2} = 72.2\,\Omega.$$

From the impedance version of Ohm's law, Eq. 27,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{90.0\,\text{V}}{72.2\,\Omega} = 1.25\,\text{A}.$$

(*b*) The rms voltage across each element is

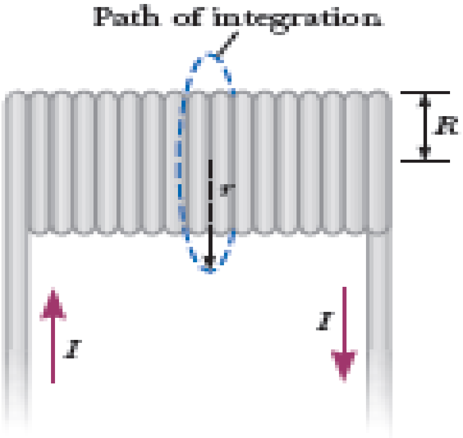
$$\begin{aligned}
(V_R)_{\text{rms}} &= I_{\text{rms}} R = (1.25\,\text{A})(25.0\,\Omega) = 31.2\,\text{V} \\
(V_L)_{\text{rms}} &= I_{\text{rms}} X_L = (1.25\,\text{A})(94.2\,\Omega) = 118\,\text{V} \\
(V_C)_{\text{rms}} &= I_{\text{rms}} X_C = (1.25\,\text{A})(26.5\,\Omega) = 33.1\,\text{V}.
\end{aligned}$$

Example /

A coil consists of 200 turns of wire. Each turn is a square of side *d* =18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.80 s, what is the magnitude of the induced emf in the coil while the field is changing?

A long solenoid of radius R has n turns of wire per unit length and carries a timevarying current that varies sinusoidally as I 5 Imax cos vt, where Imax is the maximum current and v is the angular frequency of the alternating current source (Fig.).

(A) Determine the magnitude of the induced electric field outside the solenoid at a distance r . R from its long central axis.



Evaluate the right side of Equation 31.9, noting that the magnetic field $\vec{\mathbf{B}}$ inside the solenoid is perpendicular to the circle bounded by the path of integration:

$$(1) \quad -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi R^2) = -\pi R^2 \frac{dB}{dt}$$

Evaluate the magnetic field inside the solenoid from

$$(2) \quad B = \mu_0 n I = \mu_0 n I_{\max} \cos \omega t$$

$$(3) \quad -\frac{d\Phi_B}{dt} = -\pi R^2 \mu_0 n I_{\max} \frac{d}{dt}(\cos \omega t) = \pi R^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$(4) \quad \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E(2\pi r)$$

$$E(2\pi r) = \pi R^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E = \boxed{\frac{\mu_0 n I_{\max} \omega R^2}{2r} \sin \omega t} \quad (\text{for } r > R)$$

(B) What is the magnitude of the induced electric field inside the solenoid, a distance r from its axis?

$$(5) \quad -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi r^2) = -\pi r^2 \frac{dB}{dt}$$

$$(6) \quad -\frac{d\Phi_B}{dt} = -\pi r^2 \mu_0 n I_{\max} \frac{d}{dt}(\cos \omega t) = \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E(2\pi r) = \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E = \boxed{\frac{\mu_0 n I_{\max} \omega}{2} r \sin \omega t} \quad (\text{for } r < R)$$

Consider a uniformly wound solenoid having N turns and length ℓ. Assume ℓ is much longer than the radius of the windings and the core of the solenoid is air.

(A) Find the inductance of the solenoid.

(B) Calculate the inductance of the solenoid if it contains 300 turns, its length is 25.0 cm, and its cross-sectional area is 4.00 cm^2.

(C) Calculate the self-induced emf in the solenoid if the current it carries decreases at the rate of 50.0 A/s.

$$\Phi_B = BA = \mu_0 niA = \mu_0 \frac{N}{\ell} iA \qquad L = \frac{N\Phi_B}{i} = \boxed{\mu_0 \frac{N^2}{\ell} A}$$

$$L = (4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}) \frac{300^2}{25.0 \times 10^{-2}\ \mathrm{m}} (4.00 \times 10^{-4}\ \mathrm{m^2})$$

$$= 1.81 \times 10^{-4}\ \mathrm{T \cdot m^2/A} = \boxed{0.181\ \mathrm{mH}}$$

$$\mathcal{E}_L = -L\frac{di}{dt} = -(1.81 \times 10^{-4}\ \mathrm{H})(-50.0\ \mathrm{A/s})$$

$$= \boxed{9.05\ \mathrm{mV}}$$

$$L = \mu_0 \frac{(n\ell)^2}{\ell} A = \mu_0 n^2 A\ell = \mu_0 n^2 V$$

Suppose the circuit elements have the following values: $\mathcal{E} = 12.0\ \mathrm{V}$, $R = 6.00\ \Omega$, and $L = 30.0\ \mathrm{mH}$.

(A) Find the time constant of the circuit.

(B) Switch S_2 is at position a, and switch S_1 is thrown closed at $t = 0$. Calculate the current in the circuit at $t = 2.00$ ms.

$$\tau = \frac{L}{R} = \frac{30.0 \times 10^{-3}\ \mathrm{H}}{6.00\ \Omega} = \boxed{5.00\ \mathrm{ms}}$$

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \frac{12.0\ \mathrm{V}}{6.00\ \Omega}\left(1 - e^{-2.00\ \mathrm{ms}/5.00\ \mathrm{ms}}\right) = 2.00\ \mathrm{A}\left(1 - e^{-0.400}\right)$$

$$= \boxed{0.659\ \mathrm{A}}$$

In a purely inductive AC circuit, $L = 25.0$ mH and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

$$X_L = \omega L = 2\pi f L = 2\pi(60.0\ \text{Hz})(25.0 \times 10^{-3}\ \text{H})$$
$$= \boxed{9.42\ \Omega}$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{150\ \text{V}}{9.42\ \Omega} = \boxed{15.9\ \text{A}}$$

$$X_L = 2\pi(6.00 \times 10^3\ \text{Hz})(25.0 \times 10^{-3}\ \text{H}) = 942\ \Omega$$

$$I_{\text{rms}} = \frac{150\ \text{V}}{942\ \Omega} = 0.159\ \text{A}$$

An 8.00-μF capacitor is connected to the terminals of a 60.0-Hz AC source whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\text{ Hz})(8.00 \times 10^{-6}\text{ F})} = \boxed{332\ \Omega}$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{150\text{ V}}{332\ \Omega} = \boxed{0.452\text{ A}}$$

What if the frequency is doubled? What happens to the rms current in the circuit?

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(120\text{ Hz})(8.00 \times 10^{-6}\text{ F})} = 166\ \Omega$$

$$I_{\text{rms}} = \frac{150\text{ V}}{166\ \Omega} = 0.904\text{ A}$$

A series *RLC* circuit has $R = 425\ \Omega$, $L = 1.25$ H, and $C = 3.50\ \mu$F. It is connected to an AC source with $f = 60.0$ Hz and $\Delta V_{max} = 150$ V.

(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit. *continued*

$$\omega = 2\pi f = 2\pi(60.0\text{ Hz}) = 377\text{ s}^{-1}$$

$$X_L = \omega L = (377\text{ s}^{-1})(1.25\text{ H}) = \boxed{471\ \Omega}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\text{ s}^{-1})(3.50 \times 10^{-6}\text{ F})} = \boxed{758\ \Omega}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(425\ \Omega)^2 + (471\ \Omega - 758\ \Omega)^2} = \boxed{513\ \Omega}$$

(B) Find the maximum current in the circuit.

$$I_{max} = \frac{\Delta V_{max}}{Z} = \frac{150\text{ V}}{513\ \Omega} = \boxed{0.293\text{ A}}$$

(C) Find the phase angle between the current and voltage.

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{471\ \Omega - 758\ \Omega}{425\ \Omega}\right) = \boxed{-34.0^\circ}$$

(D) Find the maximum voltage across each element.

$$\Delta V_R = I_{\max} R = (0.293\text{ A})(425\ \Omega) = \boxed{124\text{ V}}$$

$$\Delta V_L = I_{\max} X_L = (0.293\text{ A})(471\ \Omega) = \boxed{138\text{ V}}$$

$$\Delta V_C = I_{\max} X_C = (0.293\text{ A})(758\ \Omega) = \boxed{222\text{ V}}$$

(E) What replacement value of L should an engineer analyzing the circuit choose such that the current leads the applied voltage by 30.0° rather than 34.0°? All other values in the circuit stay the same.

$$X_L = X_C + R\tan\phi$$

$$\omega L = \frac{1}{\omega C} + R\tan\phi$$

$$L = \frac{1}{\omega}\left(\frac{1}{\omega C} + R\tan\phi\right)$$

$$L = \frac{1}{(377\text{ s}^{-1})}\left[\frac{1}{(377\text{ s}^{-1})(3.50\times10^{-6}\text{ F})} + (425\ \Omega)\tan(-30.0°)\right]$$

$$L = \boxed{1.36\text{ H}}$$

$$\Delta v_R = (124\,\mathrm{V})\ \sin 377t$$

$$\Delta v_L = (138\,\mathrm{V})\ \cos 377t$$

$$\Delta v_C = (-222\,\mathrm{V})\ \cos 377t$$

Consider a series RLC circuit for which $R = 150\ \Omega$, $L = 20.0$ mH, $\Delta V_{rms} = 20.0$ V, and $\omega = 5\,000\ \text{s}^{-1}$. Determine the value of the capacitance for which the current is a maximum.

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow C = \frac{1}{\omega_0^2 L}$$

$$C = \frac{1}{(5.00 \times 10^3\ \text{s}^{-1})^2(20.0 \times 10^{-3}\ \text{H})} = \boxed{2.00\ \mu\text{F}}$$

An electricity-generating station needs to deliver energy at a rate of 20 MW to a city 1.0 km away. A common voltage for commercial power generators is 22 kV, but a step-up transformer is used to boost the voltage to 230 kV before transmission.

(A) If the resistance of the wires is 2.0 Ω and the energy costs are about 11¢/kWh, estimate the cost of the energy converted to internal energy in the wires during one day.

$$I_{rms} = \frac{P_{avg}}{\Delta V_{rms}} = \frac{20 \times 10^6 \text{ W}}{230 \times 10^3 \text{ V}} = 87 \text{ A}$$

$$P_{wires} = I_{rms}^2 R = (87 \text{ A})^2(2.0\ \Omega) = 15 \text{ kW}$$

$$T_{ET} = P_{wires}\,\Delta t = (15 \text{ kW})(24 \text{ h}) = \boxed{363 \text{ kWh}}$$

$$\text{Cost} = (363 \text{ kWh})(\$0.11/\text{kWh}) = \boxed{\$40}$$

$$I_{rms} = \frac{P_{avg}}{\Delta V_{rms}} = \frac{20 \times 10^6\,\mathrm{W}}{22 \times 10^3\,\mathrm{V}} = 909\,\mathrm{A}$$

$$P_{wires} = I_{rms}^2 R = (909\,\mathrm{A})^2(2.0\,\Omega) = 1.7 \times 10^3\,\mathrm{kW}$$

$$T_{ET} = P_{wires}\,\Delta t = (1.7 \times 10^3\,\mathrm{kW})(24\,\mathrm{h}) = 4.0 \times 10^4\,\mathrm{kWh}$$

$$\mathrm{Cost} = (4.0 \times 10^4\,\mathrm{kWh})(\$0.11/\mathrm{kWh}) = \boxed{\$4.4 \times 10^3}$$