Electricity and Magnetics II

Lecture No.( )- Semester 2

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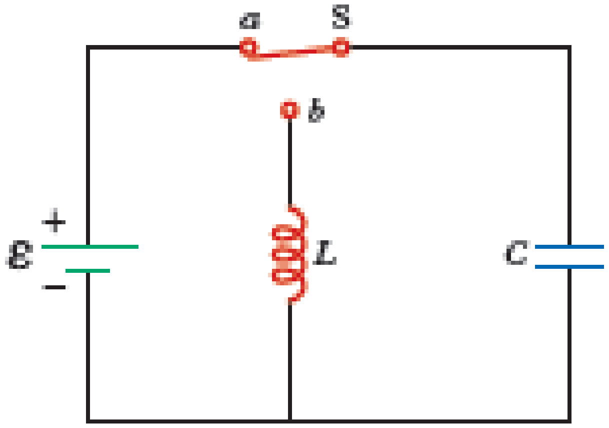
Exp/ the battery has an emf of 12.0 V, the inductance is 2.81 mH, and the capacitance is 9.00 pF. The switch has been set to position a for a long time so that the capacitor is charged. The switch is then thrown to position b, removing the battery from the circuit and connecting the capacitor directly across the inductor.

(A) Find the frequency of oscillation of the circuit.
(B) What are the maximum values of charge on the capacitor and current in the circuit?



$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi[(2.81 \times 10^{-3}\ \mathrm{H})(9.00 \times 10^{-12}\ \mathrm{F})]^{1/2}} = 1.00 \times 10^{6}\ \mathrm{Hz}$$

Find the initial charge on the capacitor, which equals the maximum charge:

$$Q_{max} = C\Delta V = (9.00 \times 10^{-12}\ \mathrm{F})(12.0\ \mathrm{V}) = 1.08 \times 10^{-10}\ \mathrm{C}$$

$$I_{max} = \omega Q_{max} = 2\pi f Q_{max} = (2\pi \times 10^{6}\ \mathrm{s}^{-1})(1.08 \times 10^{-10}\ \mathrm{C})$$

$$= 6.79 \times 10^{-4}\ \mathrm{A}$$

the resistance of the resistor represents all the resistance in the circuit. Suppose the switch is at position *a* so that the capacitor has an initial charge Q_{max}. The switch is now thrown to position *b*. At this instant, the total energy stored in the capacitor and inductor is $Q_{max}^2/2C$. This total energy, however, is no longer constant as it was in the *LC* circuit because the resistor causes transformation to internal energy. (We continue to ignore electromagnetic radiation from the circuit in this discussion.) Because the rate of energy transformation to internal energy within a resistor is i^2R,

$$\frac{dU}{dt} = -i^2R$$

where the negative sign signifies that the energy U of the circuit is decreasing in time. Substituting $U = U_E + U_B$ gives

$$\frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = -i^2R \tag{32.28}$$

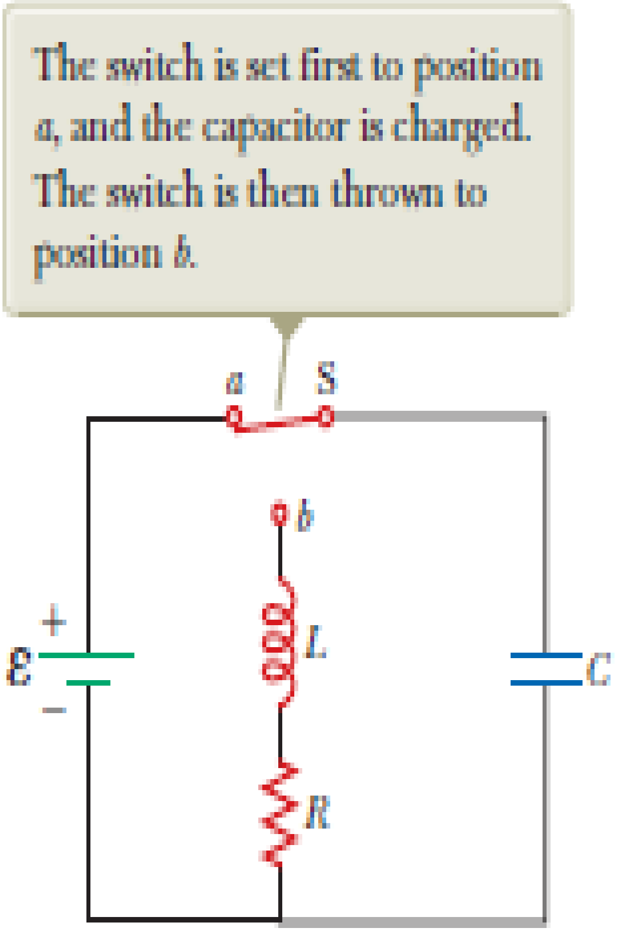


Figure 32.15 A series *RLC* circuit.

To convert this equation into a form that allows us to compare the electrical oscillations with their mechanical analog, we first use $i = dq/dt$ and move all terms to the left-hand side to obtain

$$Li\frac{d^2q}{dt^2} + i^2R + \frac{q}{C}i = 0$$

Now divide through by i:

$$L\frac{d^2q}{dt^2} + iR + \frac{q}{C} = 0$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \qquad (32.29)$$

The *RLC* circuit is analogous to the damped harmonic oscillator discussed in Section 15.6 and illustrated in Figure 15.20. The equation of motion for a damped block–spring system is, from Equation 15.31,

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \tag{32.30}$$

When R is small, a situation that is analogous to light damping in the mechanical oscillator, the solution of Equation 32.29 is

$$q = Q_{\max}e^{-Rt/2L}\cos\omega_d t \tag{32.31}$$

where ω_d, the angular frequency at which the circuit oscillates, is given by

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2} \tag{32.32}$$

That is, the value of the charge on the capacitor undergoes a damped harmonic oscillation in analogy with a block–spring system moving in a viscous medium. Equation 32.32 shows that when $R << \sqrt{4L/C}$ (so that the second term in the

RLC Circuit		One-Dimensional Particle in Simple Harmonic Motion
Charge	$q \leftrightarrow x$	Position
Current	$i \leftrightarrow v_x$	Velocity
Potential difference	$\Delta V \leftrightarrow F_x$	Force
Resistance	$R \leftrightarrow b$	Viscous damping coefficient
Capacitance	$C \leftrightarrow 1/k$	(k = spring constant)
Inductance	$L \leftrightarrow m$	Mass
Current = time derivative of charge	$i = \frac{dq}{dt} \leftrightarrow v_x = \frac{dx}{dt}$	Velocity = time derivative of position
Rate of change of current = second time derivative of charge	$\frac{di}{dt} = \frac{d^2q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	Acceleration = second time derivative of position
Energy in inductor	$U_B = \frac{1}{2}Li^2 \leftrightarrow K = \frac{1}{2}mv^2$	Kinetic energy of moving object
Energy in capacitor	$U_E = \frac{1}{2}\frac{q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2$	Potential energy stored in a spring
Rate of energy loss due to resistance	$i^2R \leftrightarrow bv^2$	Rate of energy loss due to friction
RLC circuit	$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$	Damped object on a spring

Concepts and Principles

When the current in a loop of wire changes with time, an emf is induced in the loop according to Faraday's law. The **self-induced emf** is

$$\mathcal{E}_L = -L\frac{di}{dt} \quad (32.1)$$

where L is the **inductance** of the loop. Inductance is a measure of how much opposition a loop offers to a change in the current in the loop. Inductance has the SI unit of **henry** (H), where $1\ \text{H} = 1\ \text{V}\cdot\text{s/A}$.

The inductance of any coil is

$$L = \frac{N\Phi_B}{i} \quad (32.2)$$

where N is the total number of turns and Φ_B is the magnetic flux through the coil. The inductance of a device depends on its geometry. For example, the inductance of an air-core solenoid is

$$L = \mu_0 \frac{N^2}{\ell} A \quad (32.4)$$

where ℓ is the length of the solenoid and A is the cross-sectional area.

If a resistor and inductor are connected in series to a battery of emf $\mathcal{E}$ at time $t = 0$, the current in the circuit varies in time according to the expression

$$i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) \quad (32.7)$$

where $\tau = L/R$ is the **time constant** of the RL circuit. If we replace the battery in the circuit by a resistanceless wire, the current decays exponentially with time according to the expression

$$i = \frac{\mathcal{E}}{R}e^{-t/\tau} \quad (32.10)$$

where $\mathcal{E}/R$ is the initial current in the circuit.

The energy stored in the magnetic field of an inductor carrying a current i is

$$U_B = \tfrac{1}{2}Li^2 \quad (32.12)$$

This energy is the magnetic counterpart to the energy stored in the electric field of a charged capacitor.

The energy density at a point where the magnetic field is B is

$$u_B = \frac{B^2}{2\mu_0} \quad (32.14)$$

The **mutual inductance** of a system of two coils is

$$M_{12} = \frac{N_2\Phi_{12}}{i_1} = M_{21} = \frac{N_1\Phi_{21}}{i_2} = M \quad (32.15)$$

This mutual inductance allows us to relate the induced emf in a coil to the changing source current in a nearby coil using the relationships

$$\mathcal{E}_2 = -M_{12}\frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M_{21}\frac{di_2}{dt} \quad (32.16,\ 32.17)$$

In an LC circuit that has zero resistance and does not radiate electromagnetically (an idealization), the values of the charge on the capacitor and the current in the circuit vary sinusoidally in time at an angular frequency given by

$$\omega = \frac{1}{\sqrt{LC}} \quad (32.22)$$

The energy in an LC circuit continuously transfers between energy stored in the capacitor and energy stored in the inductor.

In an RLC circuit with small resistance, the charge on the capacitor varies with time according to

$$q = Q_{\text{max}}e^{-Rt/2L}\cos\omega_d t \quad (32.31)$$

where

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2} \quad (32.32)$$