Electricity and Magnetics II

Lecture No.(2)- Semester 2

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8.5 Charged Particles in a Uniform Magnetic Field

If a particle of mass m moves in a circle of radius r at a constant speed v, what acts on the particle is a radial force of magnitude $F = mv^2 / r$ that always points toward the center and is perpendicular to the velocity of the particle.

In Section 8.2, we have also shown that the magnetic force $\vec{\mathbf{F}}_B$ always points in the direction perpendicular to the velocity $\vec{\mathbf{v}}$ of the charged particle and the magnetic field $\vec{\mathbf{B}}$. Since $\vec{\mathbf{F}}_B$ can do not work, it can only change the direction of $\vec{\mathbf{v}}$ but not its magnitude. What would happen if a charged particle moves through a uniform magnetic field $\vec{\mathbf{B}}$ with its initial velocity $\vec{\mathbf{v}}$ at a right angle to $\vec{\mathbf{B}}$? For simplicity, let the charge be $+q$ and the direction of $\vec{\mathbf{B}}$ be into the page. It turns out that $\vec{\mathbf{F}}_B$ will play the role of a centripetal force and the charged particle will move in a circular path in a counterclockwise direction, as shown in Figure 8.5.1.

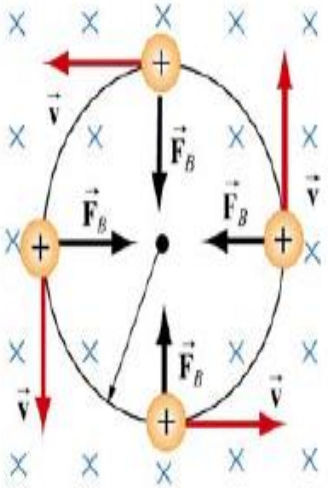


Figure 8.5.1 Path of a charge particle moving in a uniform $\vec{B}$ field with velocity $\vec{v}$ initially perpendicular to $\vec{B}$.

With

$$qvB = \frac{mv^2}{r} \tag{8.5.1}$$

the radius of the circle is found to be

$$\boxed{r = \frac{mv}{qB}} \tag{8.5.2}$$

The period T (time required for one complete revolution) is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB} \tag{8.5.3}$$

Similarly, the angular speed (cyclotron frequency) ω of the particle can be obtained as

$$\boxed{\omega = 2\pi f = \frac{v}{r} = \frac{qB}{m}} \tag{8.5.4}$$

If the initial velocity of the charged particle has a component parallel to the magnetic field $\vec{\mathbf{B}}$, instead of a circle, the resulting trajectory will be a helical path, as shown in Figure 8.5.2:

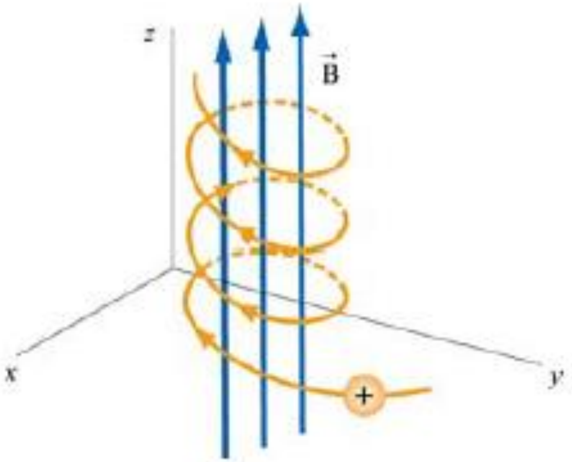


Figure 8.5.2 Helical path of a charged particle in an external magnetic field. The velocity of the particle has a non-zero component along the direction of $\vec{\mathbf{B}}$.

8.6 Applications

There are many applications involving charged particles moving through a uniform magnetic field.

8.6.1 Velocity Selector

In the presence of both electric field $\vec{\mathbf{E}}$ and magnetic field $\vec{\mathbf{B}}$, the total force on a charged particle is

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right) \tag{8.6.1}$$

This is known as the Lorentz force. By combining the two fields, particles which move with a certain velocity can be selected. This was the principle used by J. J. Thomson to measure the charge-to-mass ratio of the electrons. In Figure 8.6.1 the schematic diagram of Thomson's apparatus is depicted.

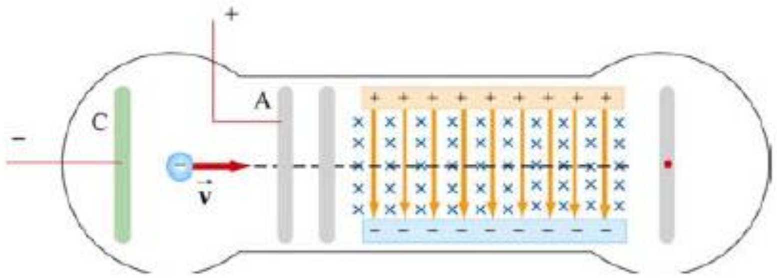


Figure 8.6.1 Thomson's apparatus

The electrons with charge $q = -e$ and mass m are emitted from the cathode C and then accelerated toward slit A. Let the potential difference between A and C be $V_A - V_C = \Delta V$. The change in potential energy is equal to the external work done in accelerating the electrons: $\Delta U = W_{ext} = q\Delta V = -e\Delta V$. By energy conservation, the kinetic energy gained is $\Delta K = -\Delta U = mv^2/2$. Thus, the speed of the electrons is given by

$$v = \sqrt{\frac{2e\Delta V}{m}} \tag{8.6.2}$$

If the electrons further pass through a region where there exists a downward uniform electric field, the electrons, being negatively charged, will be deflected upward. However, if in addition to the electric field, a magnetic field directed into the page is also applied, then the electrons will experience an additional downward magnetic force $-e\vec{\mathbf{v}}\times\vec{\mathbf{B}}$. When the two forces exactly cancel, the electrons will move in a straight path. From Eq. 8.6.1, we see that when the condition for the cancellation of the two forces is given by $eE = evB$, which implies

$$v = \frac{E}{B} \tag{8.6.3}$$

In other words, only those particles with speed $v = E / B$ will be able to move in a straight line. Combining the two equations, we obtain

$$\boxed{\frac{e}{m} = \frac{E^2}{2(\Delta V)B^2}} \tag{8.6.4}$$

By measuring E, ΔV and B, the charge-to-mass ratio can be readily determined. The most precise measurement to date is $e/m = 1.758820174(71)\times 10^{11}\ \text{C/kg}$.

8.6.2 Mass Spectrometer

Various methods can be used to measure the mass of an atom. One possibility is through the use of a mass spectrometer. The basic feature of a *Bainbridge* mass spectrometer is illustrated in Figure 8.6.2. A particle carrying a charge $+q$ is first sent through a velocity selector.

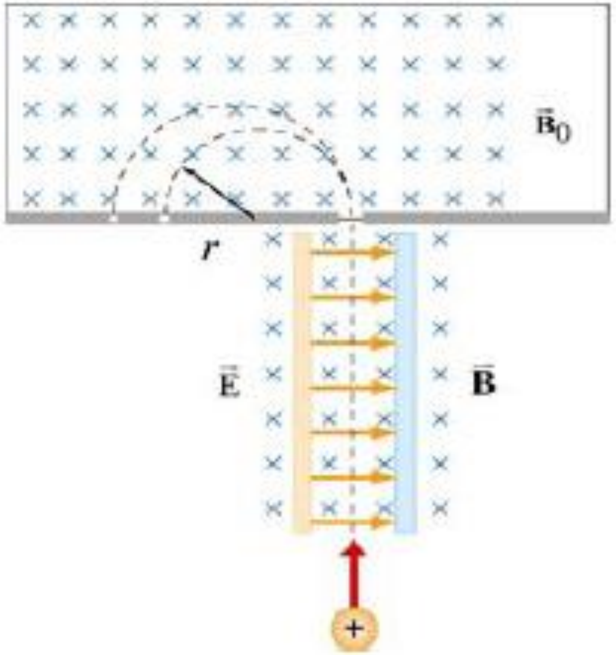


Figure 8.6.2 A Bainbridge mass spectrometer

The applied electric and magnetic fields satisfy the relation $E = vB$ so that the trajectory of the particle is a straight line. Upon entering a region where a second magnetic field $\vec{\mathbf{B}}_0$ pointing into the page has been applied, the particle will move in a circular path with radius r and eventually strike the photographic plate. Using Eq. 8.5.2, we have

$$r = \frac{mv}{qB_0} \tag{8.6.5}$$

Since $v = E/B$, the mass of the particle can be written as

$$m = \frac{qB_0 r}{v} = \frac{qB_0 B r}{E} \tag{8.6.6}$$

8.7 Summary

- The **magnetic force** acting on a charge q traveling at a velocity $\vec{\mathbf{v}}$ in a magnetic field $\vec{\mathbf{B}}$ is given by

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

- The magnetic force acting on a wire of length $\vec{\ell}$ carrying a steady current I in a magnetic field $\vec{\mathbf{B}}$ is

$$\vec{\mathbf{F}}_B = I\vec{\ell} \times \vec{\mathbf{B}}$$

- The magnetic force $d\vec{\mathbf{F}}_B$ generated by a small portion of current I of length $d\vec{\mathbf{s}}$ in a magnetic field $\vec{\mathbf{B}}$ is

$$d\vec{\mathbf{F}}_B = I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

- The **torque** $\vec{\boldsymbol{\tau}}$ acting on a close loop of wire of area A carrying a current I in a uniform magnetic field $\vec{\mathbf{B}}$ is

$$\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

where $\vec{\mathbf{A}}$ is a vector which has a magnitude of A and a direction perpendicular to the loop.

- The **magnetic dipole moment** of a closed loop of wire of area A carrying a current I is given by

$$\vec{\boldsymbol{\mu}} = I\vec{\mathbf{A}}$$

- The torque exerted on a magnetic dipole $\vec{\boldsymbol{\mu}}$ placed in an external magnetic field $\vec{\mathbf{B}}$ is

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$$

- The potential energy of a magnetic dipole placed in a magnetic field is

$$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$$

- If a particle of charge q and mass m enters a magnetic field of magnitude B with a velocity $\vec{\mathbf{v}}$ perpendicular to the magnetic field lines, the radius of the circular path that the particle follows is given by

$$r = \frac{mv}{|q|B}$$

and the angular speed of the particle is

$$\omega = \frac{|q|B}{m}$$

8.8 Problem-Solving Tips

In this Chapter, we have shown that in the presence of both magnetic field $\vec{\mathbf{B}}$ and the electric field $\vec{\mathbf{E}}$, the total force acting on a moving particle with charge q is $\vec{\mathbf{F}} = \vec{\mathbf{F}}_e + \vec{\mathbf{F}}_B = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$, where $\vec{\mathbf{v}}$ is the velocity of the particle. The direction of $\vec{\mathbf{F}}_B$ involves the cross product of $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$, based on the right-hand rule. In Cartesian coordinates, the unit vectors are $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ which satisfy the following properties:

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}, \quad \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}, \quad \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}, \quad \hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}}, \quad \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

For $\vec{\mathbf{v}} = v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}} + v_z\,\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = B_x\,\hat{\mathbf{i}} + B_y\,\hat{\mathbf{j}} + B_z\,\hat{\mathbf{k}}$, the cross product may be obtained as

$$\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = (v_y B_z - v_z B_y)\hat{\mathbf{i}} + (v_z B_x - v_x B_z)\hat{\mathbf{j}} + (v_x B_y - v_y B_x)\hat{\mathbf{k}}$$

If only the magnetic field is present, and $\vec{\mathbf{v}}$ is perpendicular to $\vec{\mathbf{B}}$, then the trajectory is a circle with a radius $r = mv / |q| B$, and an angular speed $\omega = |q| B / m$.

When dealing with a more complicated case, it is useful to work with individual force components. For example,

$$F_x = ma_x = qE_x + q(v_y B_z - v_z B_y)$$

8.9.1 Rolling Rod

A rod with a mass m and a radius R is mounted on two parallel rails of length a separated by a distance ℓ, as shown in the Figure 8.9.1. The rod carries a current I and rolls without slipping along the rails which are placed in a uniform magnetic field $\vec{\mathbf{B}}$ directed into the page. If the rod is initially at rest, what is its speed as it leaves the rails?

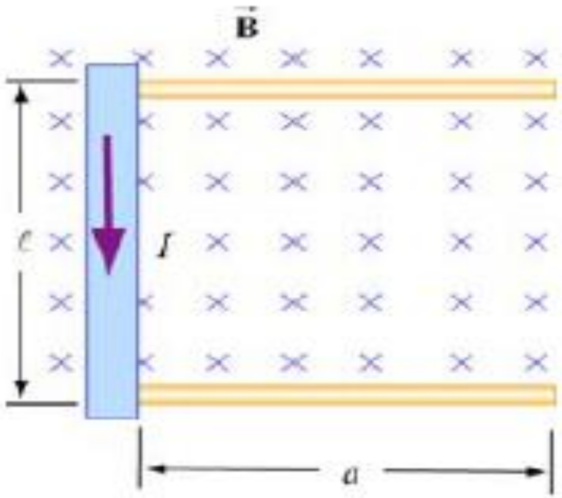
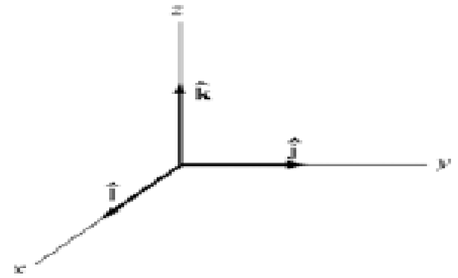


Figure 8.9.1 Rolling rod in uniform magnetic field

Solution:

Using the coordinate system shown on the right, the magnetic force acting on the rod is given by

$$\vec{\mathbf{F}}_B = I\vec{\ell}\times\vec{\mathbf{B}} = I(\ell\,\hat{\mathbf{i}})\times(-B\,\hat{\mathbf{k}}) = I\ell B\,\hat{\mathbf{j}} \tag{8.9.1}$$



The total work done by the magnetic force on the rod as it moves through the region is

$$W = \int \vec{\mathbf{F}}_B \cdot d\,\vec{\mathbf{s}} = F_B a = (I\ell B)a \tag{8.9.2}$$

By the work-energy theorem, W must be equal to the change in kinetic energy:

$$\Delta K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \tag{8.9.3}$$

where both translation and rolling are involved. Since the moment of inertia of the rod is given by $I = mR^2/2$, and the condition of rolling with slipping implies $\omega = v/R$, we have

$$I\ell Ba = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2 \tag{8.9.4}$$

Thus, the speed of the rod as it leaves the rails is

$$v = \sqrt{\frac{4I\ell Ba}{3m}} \tag{8.9.5}$$

8.9.2 Suspended Conducting Rod

A conducting rod having a mass density λ kg/m is suspended by two flexible wires in a uniform magnetic field $\vec{\mathbf{B}}$ which points out of the page, as shown in Figure 8.9.2.

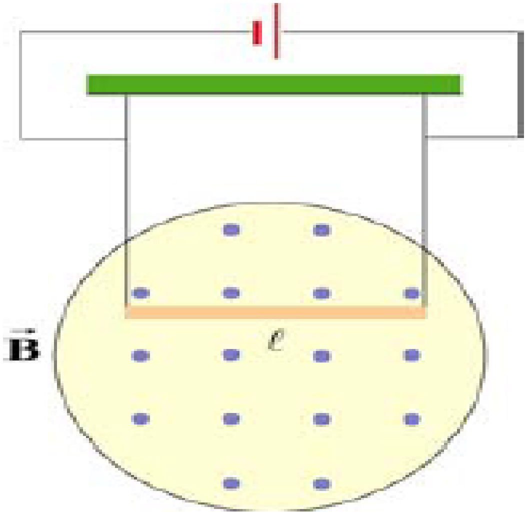
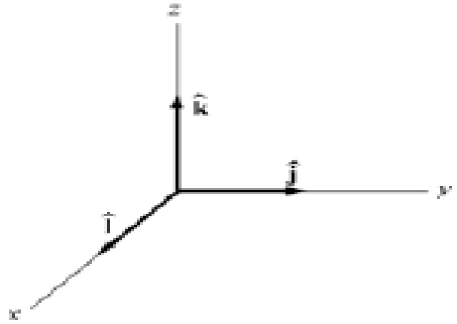


Figure 8.9.2 Suspended conducting rod in uniform magnetic field

If the tension on the wires is zero, what are the magnitude and the direction of the current in the rod?

Solution:

In order that the tension in the wires be zero, the magnetic force $\vec{\mathbf{F}}_B = I\vec{\ell}\times\vec{\mathbf{B}}$ acting on the conductor must exactly cancel the downward gravitational force $\vec{\mathbf{F}}_g = -mg\hat{\mathbf{k}}$.	

For $\vec{\mathbf{F}}_B$ to point in the $+z$-direction, we must have $\vec{\ell} = -\ell\,\hat{\mathbf{j}}$, i.e., the current flows to the left, so that

$$\vec{\mathbf{F}}_B = I\vec{\ell}\times\vec{\mathbf{B}} = I(-\ell\,\hat{\mathbf{j}})\times(B\,\hat{\mathbf{i}}) = -I\ell B(\hat{\mathbf{j}}\times\hat{\mathbf{i}}) = +I\ell B\,\hat{\mathbf{k}} \qquad (8.9.6)$$

The magnitude of the current can be obtain from

$$I\ell B = mg \qquad (8.9.7)$$

or

$$I = \frac{mg}{B\ell} = \frac{\lambda g}{B} \qquad (8.9.8)$$

8.9.3 Charged Particles in Magnetic Field

Particle A with charge q and mass m_A and particle B with charge $2q$ and mass m_B, are accelerated from rest by a potential difference ΔV, and subsequently deflected by a uniform magnetic field into semicircular paths. The radii of the trajectories by particle A and B are R and $2R$, respectively. The direction of the magnetic field is perpendicular to the velocity of the particle. What is their mass ratio?

Solution:

The kinetic energy gained by the charges is equal to

$$\frac{1}{2}mv^2 = q\Delta V \qquad (8.9.9)$$

which yields

$$v = \sqrt{\frac{2q\Delta V}{m}} \qquad (8.9.10)$$

The charges move in semicircles, since the magnetic force points radially inward and provides the source of the centripetal force:

$$\frac{mv^2}{r} = qvB \qquad (8.9.11)$$

The radius of the circle can be readily obtained as:

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}} \tag{8.9.12}$$

which shows that r is proportional to $(m/q)^{1/2}$. The mass ratio can then be obtained from

$$\frac{r_A}{r_B} = \frac{(m_A/q_A)^{1/2}}{(m_B/q_B)^{1/2}} \quad \Rightarrow \quad \frac{R}{2R} = \frac{(m_A/q)^{1/2}}{(m_B/2q)^{1/2}} \tag{8.9.13}$$

which gives

$$\frac{m_A}{m_B} = \frac{1}{8} \tag{8.9.14}$$