Electricity and Magnetics I

Lecture No.(7)- Semester 1

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Capacitance and Dielectrics

- A capacitor is a device which stores electric charge.
- Capacitors vary in shape and size, but the basic configuration is two conductors carrying equal but opposite charges (Figure 5.1.1)

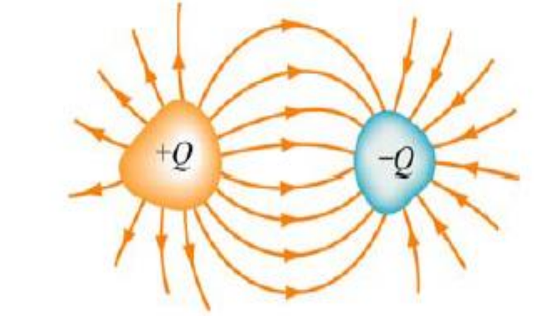


Figure 5.1.1 Basic configuration of a capacitor.

- Capacitors have many important applications in electronics. Some examples include storing electric potential energy, delaying voltage changes when coupled with resistors, filtering out unwanted frequency signals, forming resonant circuits and making frequency-dependent and independent voltage dividers when combined with resistors

- In the uncharged state, the charge on either one of the conductors in the capacitor is zero.
- During the charging process, a charge Q is moved from one conductor to the other one, giving one conductor a charge Q+, and the other one a charge .
- A potential difference ΔV is created, with the positively charged conductor at a higher potential than the negatively charged conductor. Note that whether charged or uncharged, the net charge on the capacitor as a whole is zero.

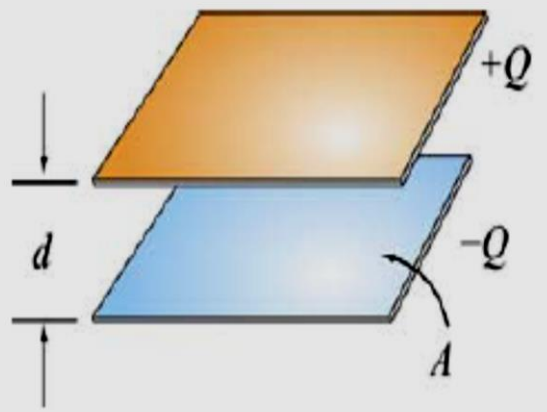


Figure 5.1.2 A parallel-plate capacitor

The simplest example of a capacitor consists of two conducting plates of areaA, which are parallel to each other, and separated by a distance d, as shown in Figure 5.1.2.

Experiments show that the amount of charge Q stored in a capacitor is linearly proportional to, the electric potential difference between the plates. Thus, we may write

$$Q = C\,|V\Delta| \qquad \text{--------} \ (1\text{-}1)$$

where C is a positive proportionality constant called capacitance. Physically, capacitance is a measure of the capacity of storing electric charge for a given potential difference ΔV. The SI unit of capacitance is the farad (F)

1 F= 1 farad= 1 coulomb/volt= 1 C/V

A typical capacitance is in the picofarad ($1\ \text{pF} = 10^{-12}\ \text{F}$) to millifarad range, ($1\ \text{mF} = 10^{-3}\ \text{F} = 1000\ \mu\text{F};\ 1\ \mu\text{F} = 10^{-6}\ \text{F}$).

Calculation of Capacitance

Example 5.1: Parallel-Plate Capacitor

Consider two metallic plates of equal area A separated by a distance d, as shown in Figure 5.2.1 below. The top plate carries a charge $+Q$ while the bottom plate carries a charge $-Q$. The charging of the plates can be accomplished by means of a battery which produces a potential difference. Find the capacitance of the system.

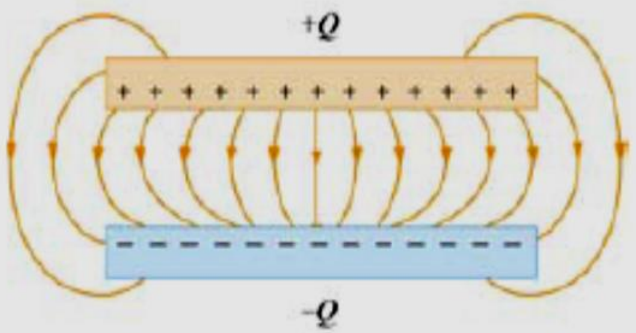


Figure 5.2.1 The electric field between the plates of a parallel-plate capacitor

To find the capacitance *C*

- **first need to know the electric field between the plates.**

In the limit where the plates are infinitely large, the system has planar symmetry and we can calculate the electric field everywhere using Gauss's law given in Eq. (4.2.5):

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{enc}}{\varepsilon_0}$$

- **By choosing a Gaussian "pillbox" with cap area *A′* to enclose the charge on the positive plate (see Figure 5.2.2), the electric field in the region between the plates is**

$$EA' = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma A'}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{\sigma}{\varepsilon_0}$$

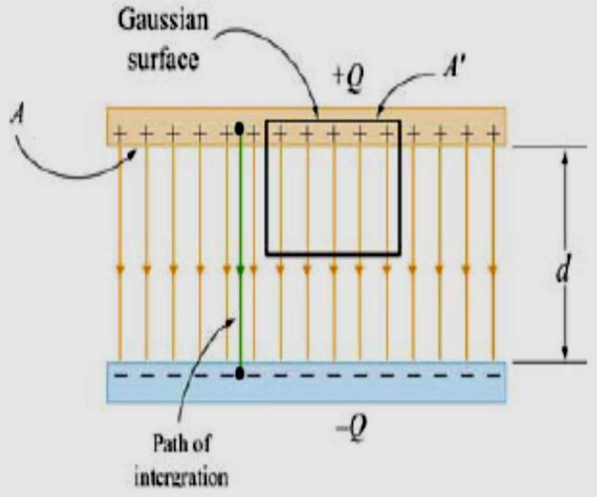


Figure 5.2.2 Gaussian surface for calculating the electric field between the plates.

- The potential difference between the plates is $$\Delta V = V_- - V_+ = -\int_+^- \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -Ed$$

- where we have taken the path of integration to be a straight line from the positive plate to the negative plate following the field lines

Since the electric field lines are always directed from higher potential to lower potential, -V< +V

- computing the capacitance C, the relevant quantity is the magnitude of the potential difference

$$|\Delta V| = Ed$$

- From the definition of capacitance, we have $$\boxed{C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_0 A}{d} \qquad \text{(parallel plate)}}$$

- **The capacitance C increases linearly with the area A since for a given potential difference VΔ, a bigger plate can hold more charge. On the other hand, C is inversely proportional to d, the distance of separation because the smaller the value of d, the smaller the potential difference |VΔ| for a fixed Q.**

Example 5.2: Cylindrical Capacitor

Consider next a solid cylindrical conductor of radius a surrounded by a coaxial cylindrical shell of inner radius b, as shown in Figure 5.2.4. The length of both cylinders is L and we take this length to be much larger than b− a, the separation of the cylinders, so that edge effects can be neglected. The capacitor is charged so that the inner cylinder has charge +Q while the outer shell has a charge –Q. What is the capacitance

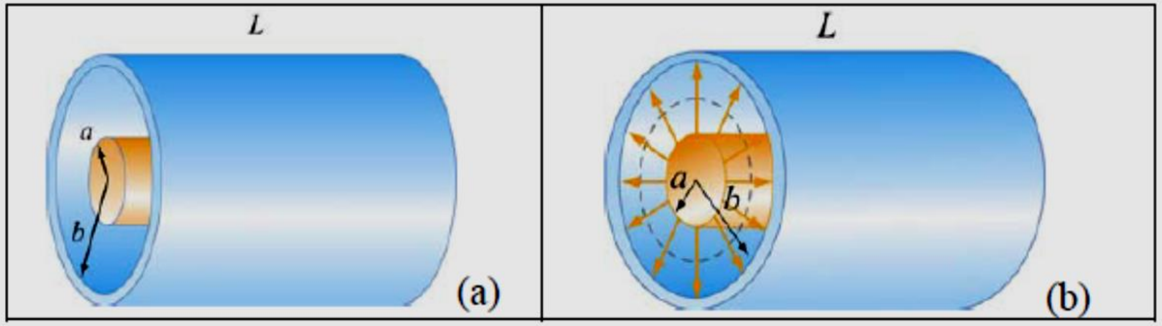


Figure 5.2.4 (a) A cylindrical capacitor. (b) End view of the capacitor. The electric field is non-vanishing only in the region $a < r < b$.

- To calculate the capacitance, we first compute the electric field everywhere. Due to the cylindrical symmetry of the system

- we choose our Gaussian surface to be a coaxial cylinder with length $\ell < L$ and radius r where a<r<b.

- Using Gauss's law, we have $$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = E(2\pi r \ell) = \frac{\lambda \ell}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

where λ= Q/L is the charge per unit length. Notice that the electric field is non-vanishing only in the region a<r<b. For a<r , the enclosed charge is q=0 since any net charge in a conductor must reside on its surface

Similarly, for $r > b$, the enclosed charge is $q_{enc} = \lambda\ell - \lambda\ell = 0$ since the Gaussian surface encloses equal but opposite charges from both conductors. The potential difference is given by

$$\Delta V = V_b - V_a = -\int_a^b E_r\, dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

where we have chosen the integration path to be along the direction of the electric field lines. As expected, the outer conductor with negative charge has a lower potential. This gives

$$\boxed{C = \frac{Q}{|\Delta V|} = \frac{\lambda L}{\lambda \ln(b/a)/2\pi\varepsilon_0} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}}$$

we see that the capacitance C depends only on the geometrical factors, L, a and b.

Example 5.3: Spherical Capacitor

As a third example, let's consider a spherical capacitor which consists of two concentric spherical shells of radii *a* and *b*, as shown in Figure 5.2.5. The inner shell has a charge +Q uniformly distributed over its surface, and the outer shell an equal but opposite charge –Q. What is the capacitance of this configuration?

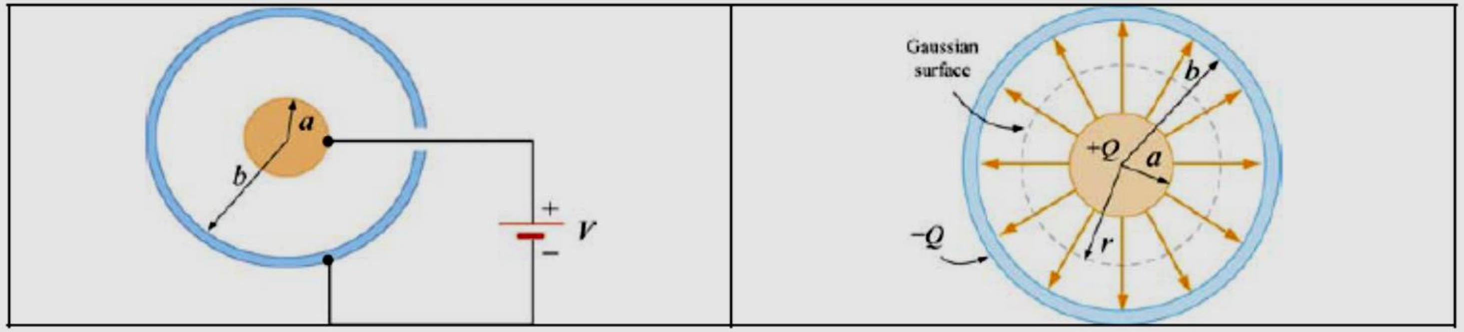


Figure 5.2.5 (a) spherical capacitor with two concentric spherical shells of radii *a* and *b*. (b) Gaussian surface for calculating the electric field.

Solution:

The electric field is non-vanishing only in the region *a<r<b*. Using Gauss's law, we

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E_r A = E_r \left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

Or

$$E_r = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$

Therefore, the potential difference between the two conducting shells is:

$$\Delta V = V_b - V_a = -\int_a^b E_r dr = -\frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

which yields

$$\boxed{C = \frac{Q}{|\Delta V|} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)}$$

Again, the capacitance C depends only on the physical dimensions, a and b.

$$\lim_{b\to\infty} C = \lim_{b\to\infty} 4\pi\varepsilon_0 \left(\frac{ab}{b-a}\right) = \lim_{b\to\infty} 4\pi\varepsilon_0 \frac{a}{\left(1-\frac{a}{b}\right)} = 4\pi\varepsilon_0 a$$

Thus, for a single isolated spherical conductor of radius R, the capacitance is

$$C = 4\pi\varepsilon_0 R$$

The above expression can also be obtained by noting that a conducting sphere of radius R with a charge Q uniformly distributed over its surface has $V = Q/4\pi\varepsilon_0 R$, using infinity as the reference point having zero potential, $V(\infty) = 0$. This gives

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{Q/4\pi\varepsilon_0 R} = 4\pi\varepsilon_0 R$$

As expected, the capacitance of an isolated charged sphere only depends on its geometry, namely, the radius R.

System	Capacitance
Isolated charged sphere of radius R	$C = 4\pi\varepsilon_0 R$
Parallel-plate capacitor of plate area A and plate separation d	$C = \varepsilon_0 \frac{A}{d}$
Cylindrical capacitor of length L, inner radius a and outer radius b	$C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$
Spherical capacitor with inner radius a and outer radius b	$C = 4\pi\varepsilon_0 \frac{ab}{(b-a)}$

Capacitors in Electric Circuits

A capacitor can be charged by connecting the plates to the terminals of a battery, which are maintained at a potential difference ΔV called the terminal voltage.

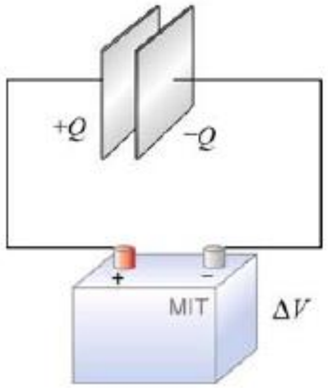


Figure 5.3.1 Charging a capacitor.

Parallel Connection

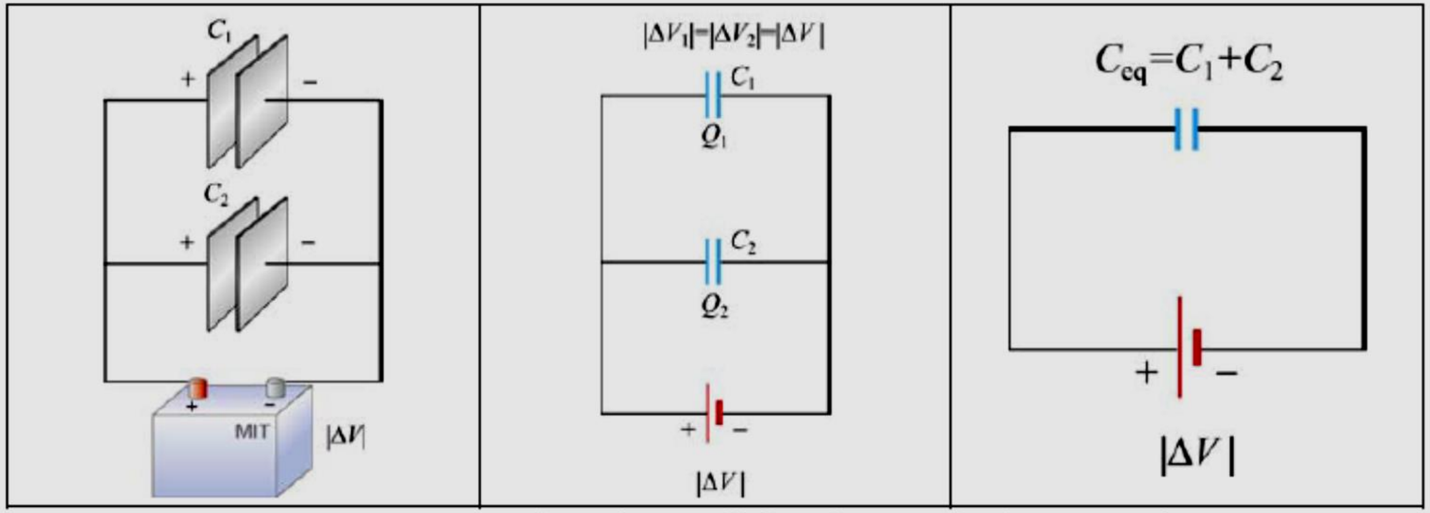


Figure 5.3.2 Capacitors in parallel and an equivalent capacitor.

The left plates of both capacitors C1 and C2 are connected to the positive terminal of the battery and have the same electric potential as the positive terminal. Similarly, both right plates are negatively charged and have the same potential as the negative terminal. Thus, the potential difference |ΔV| is the same across each capacitor. This gives

$$C_1 = \frac{Q_1}{|\Delta V|}, \qquad C_2 = \frac{Q_2}{|\Delta V|}$$

These two capacitors can be replaced by a single equivalent capacitor Ceq with a total charge Q supplied by the battery. However, since Q is shared by the two capacitors, we must have

$$Q = Q_1 + Q_2 = C_1 |\Delta V| + C_2 |\Delta V| = (C_1 + C_2)|\Delta V|$$

The equivalent capacitance is then seen to be given by

$$C_{eq} = \frac{Q}{|\Delta V|} = C_1 + C_2$$

Thus, capacitors that are connected in parallel add. The generalization to any number of capacitors is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N = \sum_{i=1}^{N} C_i \qquad \text{(parallel)}$$

5.3.2 Series Connection

Suppose two initially uncharged capacitors C_1 and C_2 are connected in series, as shown in Figure 5.3.3. A potential difference $|\Delta V|$ is then applied across both capacitors. The left plate of capacitor 1 is connected to the positive terminal of the battery and becomes positively charged with a charge $+Q$, while the right plate of capacitor 2 is connected to the negative terminal and becomes negatively charged with charge $-Q$ as electrons flow in. What about the inner plates? They were initially uncharged; now the outside plates each attract an equal and opposite charge. So the right plate of capacitor 1 will acquire a charge $-Q$ and the left plate of capacitor $+Q$.

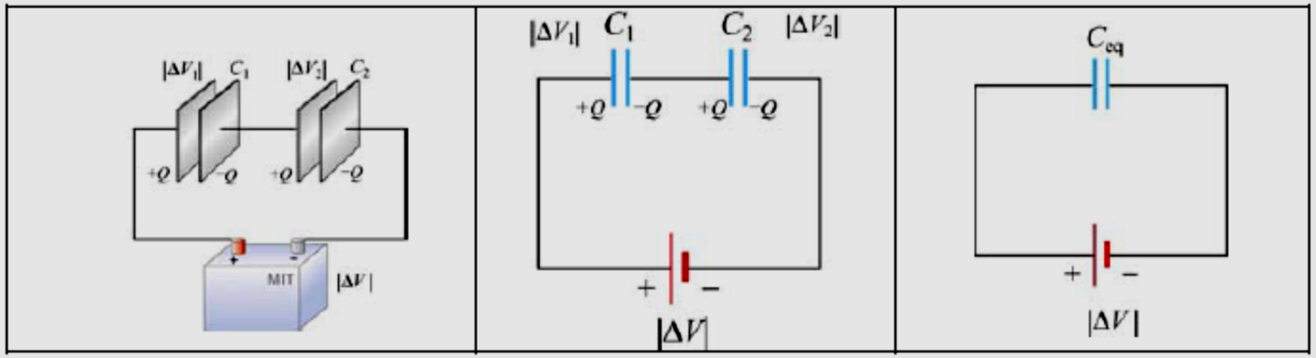


Figure 5.3.3 Capacitors in series and an equivalent capacitor

The potential differences across capacitors C_1 and C_2 are

$$|\Delta V_1| = \frac{Q}{C_1}, \quad |\Delta V_2| = \frac{Q}{C_2} \tag{5.3.5}$$

respectively. From Figure 5.3.3, we see that the total potential difference is simply the sum of the two individual potential differences:

$$|\Delta V| = |\Delta V_1| + |\Delta V_2| \tag{5.3.6}$$

In fact, the total potential difference across any number of capacitors in series connection is equal to the sum of potential differences across the individual capacitors. These two capacitors can be replaced by a single equivalent capacitor $C_{\mathrm{eq}} = Q/|\Delta V|$. Using the fact that the potentials add in series,

$$\frac{Q}{C_{\mathrm{eq}}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

and so the equivalent capacitance for two capacitors in series becomes

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N} = \sum_{i=1}^{N} \frac{1}{C_i} \qquad (\text{series})}$$

Example 5.4: Equivalent Capacitance

Find the equivalent capacitance for the combination of capacitors shown in Figure 5.3.4(a)

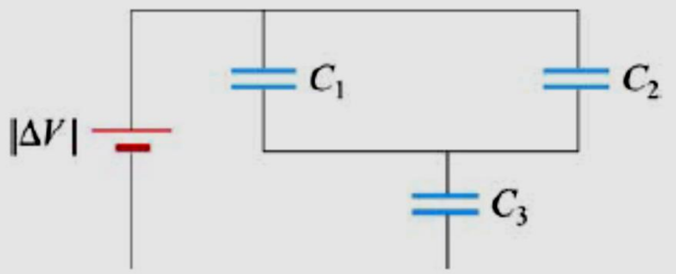


Figure 5.3.4 (a) Capacitors connected in series and in parallel

Solution:

Since C_1 and C_2 are connected in parallel, their equivalent capacitance C_{12} is given by

$$C_{12} = C_1 + C_2$$

Figure 5.3.4 (b) and (c) Equivalent circuits.

Now capacitor C_{12} is in series with C_3, as seen from Figure 5.3.4(b). So, the equivalent capacitance C_{123} is given by

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

or

$$\boxed{C_{123} = \frac{C_{12}C_3}{C_{12}+C_3} = \frac{(C_1+C_2)C_3}{C_1+C_2+C_3}}$$

5.4 Storing Energy in a Capacitor

capacitors can be used to stored electrical energy. The amount of energy stored is equal to the work done to charge it. During the charging process, the battery does work to remove charges from one plate and deposit them onto the other.

We start out at the bottom plate, fill our magic bucket with a charge +dq , carry the bucket up the stairs and dump the contents of the bucket on the top plate, charging it up positive to charge +dq. However, in doing so, the bottom plate is now charged to -dq

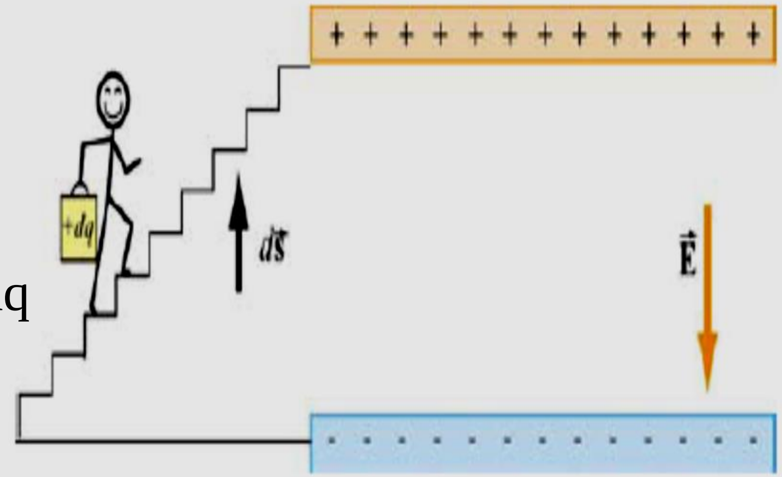


Figure 5.4.1 Work is done by an external agent in bringing $+dq$ from the negative plate and depositing the charge on the positive plate.

then the total amount of work done in this process is

$$W = \int_0^Q dq\,|\Delta V| = \int_0^Q dq\,\frac{q}{C} = \frac{1}{2}\frac{Q^2}{C}$$

This is equal to the electrical potential energy U_E of the system:

$$\boxed{U_E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Q\,|\Delta V| = \frac{1}{2}C\,|\Delta V|^2}$$

5.5 Dielectrics

In many capacitors there is an insulating material such as paper or plastic between the plates. Such material, called a dielectric, can be used to maintain a physical separation of the plates.

Experimentally it was found that capacitance C increases when the space between the conductors is filled with dielectrics

When a dielectric material is inserted to completely fill the space between the plates, the capacitance increases to $C = \kappa_e C_0$

Ke where is called the ***dielectric constant***

Material	κ_e	Dielectric strength (10^6V/m)
Air	1.00059	3
Paper	3.7	16
Glass	4–6	9
Water	80	–

Note that every dielectric material has a characteristic dielectric strength which is the maximum value of electric field before breakdown occurs and charges begin to flow.

5.5.2 Dielectrics without Battery

As shown in Figure 5.5.5, a battery with a potential difference $|\Delta V_0|$ across its terminals is first connected to a capacitor C_0, which holds a charge $Q_0 = C_0|\Delta V_0|$. We then disconnect the battery, leaving $Q_0 = \text{const}$.

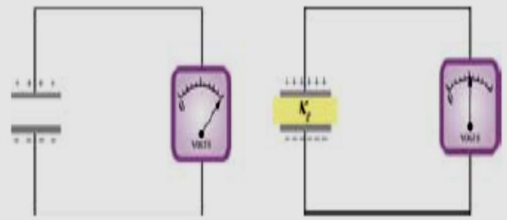


Figure 5.5.5 Inserting a dielectric material between the capacitor plates while keeping charge Q_0 constant

If we then insert a dielectric between the plates, while keeping the charge constant, experimentally it is found that the potential difference decreases by a factor of κ_e:

$$|\Delta V| = \frac{|\Delta V_0|}{\kappa_e}$$

This implies that the capacitance is changed to

$$\boxed{C = \frac{Q}{|\Delta V|} = \frac{Q_0}{|\Delta V_0|/\kappa_e} = \kappa_e \frac{Q_0}{|\Delta V_0|} = \kappa_e C_0}$$

Thus, we see that the capacitance has increased by a factor of κ_e .The electric field within the dielectric is now

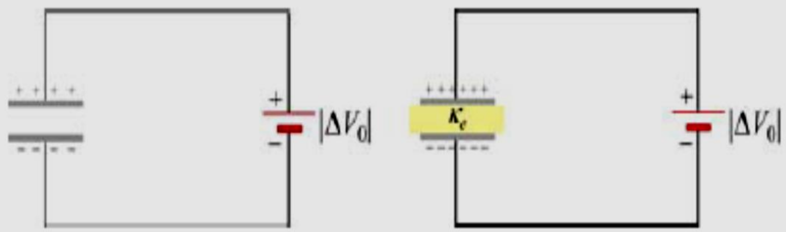
$$E = \frac{|\Delta V|}{d} = \frac{|\Delta V_0| / \kappa_e}{d} = \frac{1}{\kappa_e}\left(\frac{|\Delta V_0|}{d}\right) = \frac{E_0}{\kappa_e}$$

5.5.3 Dielectrics with Battery

Consider a second case where a battery supplying a potential difference $|\Delta V_0|$ remains connected as the dielectric is inserted. Experimentally, it is found (first by Faraday) that the charge on the plates is increased by a factor κ_e :

$$Q = \kappa_e Q_0$$

where Q_0 is the charge on the plates in the absence of any dielectric.



The capacitance becomes

$$\boxed{C = \frac{Q}{|\Delta V_0|} = \frac{\kappa_e Q_0}{|\Delta V_0|} = \kappa_e C_0}$$