Electricity and Magnetics I

Lecture No.(8)- Semester 1

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EXAMPLE 1 **Capacitor calculations.** (*a*) Calculate the capacitance of a parallel-plate capacitor whose plates are 20 cm × 3.0 cm and are separated by a 1.0-mm air gap. (*b*) What is the charge on each plate if a 12-V battery is connected across the two plates? (*c*) What is the electric field between the plates? (*d*) Estimate the area of the plates needed to achieve a capacitance of 1 F, given the same air gap *d*.

SOLUTION (*a*) The area $A = (20 \times 10^{-2}\,\mathrm{m})(3.0 \times 10^{-2}\,\mathrm{m}) = 6.0 \times 10^{-3}\,\mathrm{m}^2$. The capacitance C is then

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}) \frac{6.0 \times 10^{-3}\,\mathrm{m}^2}{1.0 \times 10^{-3}\,\mathrm{m}} = 53\,\mathrm{pF}.$$

(*b*) The charge on each plate is

$$Q = CV = (53 \times 10^{-12}\,\mathrm{F})(12\,\mathrm{V}) = 6.4 \times 10^{-10}\,\mathrm{C}.$$

(*c*) For a uniform electric field, the magnitude of E is

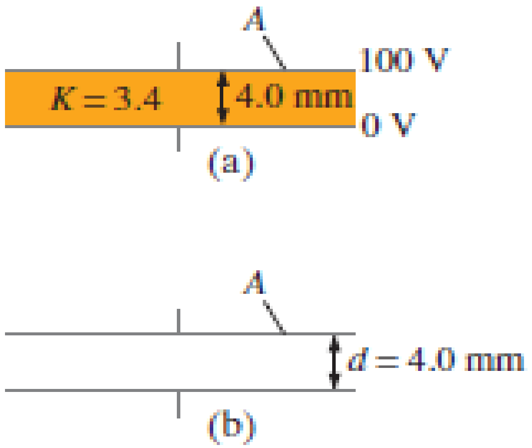
$$E = \frac{V}{d} = \frac{12\,\mathrm{V}}{1.0 \times 10^{-3}\,\mathrm{m}} = 1.2 \times 10^4\,\mathrm{V/m}.$$

(*d*) We solve for A in Eq. 2 and substitute $C = 1.0\,\mathrm{F}$ and $d = 1.0\,\mathrm{mm}$ to find that we need plates with an area

$$A = \frac{Cd}{\epsilon_0} \approx \frac{(1\,\mathrm{F})(1.0 \times 10^{-3}\,\mathrm{m})}{(9 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2})} \approx 10^8\,\mathrm{m}^2.$$

Example /

A parallel-plate capacitor, filled with a dielectric with $K = 3.4$, is connected to a 100-V battery (Fig. 16a). After the capacitor is fully charged, the battery is disconnected. The plates have area $A = 4.0\,\text{m}^2$, and are separated by $d = 4.0\,\text{mm}$. (*a*) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor. (*b*) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor (Fig. 16b). Find the new values of capacitance, electric field strength, voltage between the plates, and the energy stored in the capacitor.



SOLUTION (*a*) First we find the capacitance, with dielectric:

$$C = \frac{K\epsilon_0 A}{d} = \frac{3.4(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(4.0\,\text{m}^2)}{4.0 \times 10^{-3}\,\text{m}}$$

$$= 3.0 \times 10^{-8}\,\text{F}.$$

The charge Q on the plates is

$$Q = CV = (3.0 \times 10^{-8}\,\text{F})(100\,\text{V}) = 3.0 \times 10^{-6}\,\text{C}.$$

The electric field between the plates is

$$E = \frac{V}{d} = \frac{100\ \text{V}}{4.0 \times 10^{-3}\ \text{m}} = 25\ \text{kV/m}.$$

Finally, the total energy stored in the capacitor is

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(3.0 \times 10^{-8}\ \text{F})(100\ \text{V})^2 = 1.5 \times 10^{-4}\ \text{J}.$$

(b) The capacitance without dielectric decreases by a factor $K = 3.4$:

$$C_0 = \frac{C}{K} = \frac{(3.0 \times 10^{-8}\ \text{F})}{3.4} = 8.8 \times 10^{-9}\ \text{F}.$$

Because the battery has been disconnected, the charge Q can not change; when the dielectric is removed, $V = Q/C$ increases by a factor $K = 3.4$ to 340 V. The electric field is

$$E = \frac{V}{d} = \frac{340\ \text{V}}{4.0 \times 10^{-3}\ \text{m}} = 85\ \text{kV/m}.$$

The energy stored is

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(8.8 \times 10^{-9}\ \text{F})(340\ \text{V})^2$$
$$= 5.1 \times 10^{-4}\ \text{J}.$$

EXAMPLE 12 **Dielectric partially fills capacitor.** A parallel-plate capacitor has plates of area $A = 250\ \text{cm}^2$ and separation $d = 2.00\ \text{mm}$. The capacitor is charged to a potential difference $V_0 = 150\ \text{V}$. Then the battery is disconnected (the charge Q on the plates then won't change), and a dielectric sheet ($K = 3.50$) of the same area A but thickness $\ell = 1.00\ \text{mm}$ is placed between the plates as shown in Fig. 18. Determine (*a*) the initial capacitance of the air-filled capacitor, (*b*) the charge on each plate before the dielectric is inserted, (*c*) the charge induced on each face of the dielectric after it is inserted, (*d*) the electric field in the space between each plate and the dielectric, (*e*) the electric field in the dielectric, (*f*) the potential difference between the plates after the dielectric is added, and (*g*) the capacitance after the dielectric is in place.

SOLUTION (*a*) Before the dielectric is in place, the capacitance is

$$C_0 = \epsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(\frac{2.50 \times 10^{-2}\ \text{m}^2}{2.00 \times 10^{-3}\ \text{m}}\right) = 111\ \text{pF}.$$

(*b*) The charge on each plate is

$$Q = C_0 V_0 = \left(1.11 \times 10^{-10}\ \text{F}\right)(150\ \text{V}) = 1.66 \times 10^{-8}\ \text{C}.$$

(*c*) Equations 10 and 11 are valid even when the dielectric does not fill the gap, so (Eq. 11b)

$$Q_{\text{ind}} = Q\left(1 - \frac{1}{K}\right) = (1.66 \times 10^{-8}\,\text{C})\left(1 - \frac{1}{3.50}\right) = 1.19 \times 10^{-8}\,\text{C}.$$

(*d*) The electric field in the gaps between the plates and the dielectric (see Fig. 17c) is the same as in the absence of the dielectric since the charge on the plates has not been altered. The result of Example 13 in the "Electric Charge and Electric Field" Chapter can be used here, which gives $E_0 = \sigma/\epsilon_0$. [Or we can note that, in the absence of the dielectric, $E_0 = V_0/d = Q/C_0 d$ $\left(\text{since } V_0 = Q/C_0\right) = Q/\epsilon_0 A$ $\left(\text{since } C_0 = \epsilon_0 A/d\right)$ which is the same result.] Thus

$$E_0 = \frac{Q}{\epsilon_0 A} = \frac{1.66 \times 10^{-8}\,\text{C}}{(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.50 \times 10^{-2}\,\text{m}^2)} = 7.50 \times 10^4\,\text{V/m}.$$

(*e*) In the dielectric the electric field is (Eq. 10)

$$E_D = \frac{E_0}{K} = \frac{7.50 \times 10^4 \text{ V/m}}{3.50} = 2.14 \times 10^4 \text{ V/m}.$$

(*f*) To obtain the potential difference in the presence of the dielectric we use $V = -\int \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$, and integrate from the surface of one plate to the other along a straight line parallel to the field lines:

$$V = -\int \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = E_0(d - \ell) + E_D \ell,$$

which can be simplified to

$$\begin{aligned} V &= E_0\left(d - \ell + \frac{\ell}{K}\right) \\ &= (7.50 \times 10^4 \text{ V/m})\left(1.00 \times 10^{-3}\text{ m} + \frac{1.00 \times 10^{-3}\text{ m}}{3.50}\right) \\ &= 96.4 \text{ V}. \end{aligned}$$

(g) In the presence of the dielectric, the capacitance is

$$C = \frac{Q}{V} = \frac{1.66 \times 10^{-8}\text{ C}}{96.4 \text{ V}} = 172 \text{ pF}.$$

(II) Use Gauss's law to show that $\vec{E} = 0$ inside the inner conductor of a cylindrical capacitor (see Fig. 6 and Example 2 of "Capacitance, Dielectrics, Electric Energy Storage") as well as outside the outer cylinder.

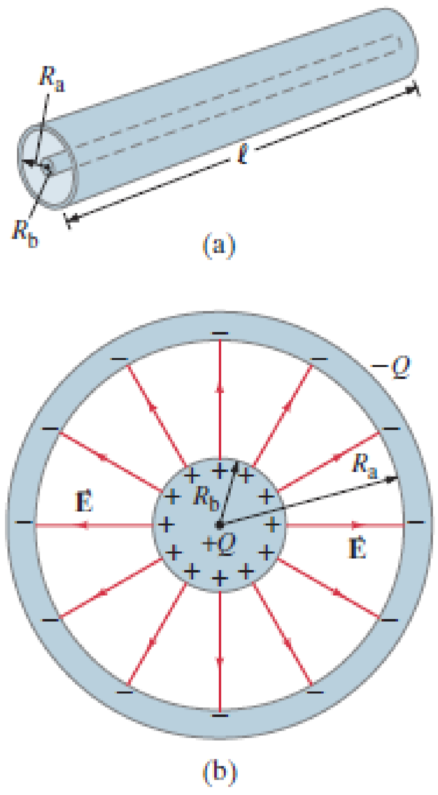


FIGURE 6
(a) Cylindrical capacitor consists of two coaxial cylindrical conductors.
(b) The electric field lines are shown in cross-sectional view.

Q1 Two point charges $q_A = -12.0\ \mu C$ and $q_B = 45.0\ \mu C$ and a third particle with unknown charge q_C are located on the x axis. The particle q_A is at the origin, and q_B is at $x = 15.0$ cm. The third particle is to be placed so that each particle is in equilibrium under the action of the electric forces exerted by the other two particles. (a) Is this situation possible? If so, is it possible in more than one way? Explain. Find (b) the required location and (c) the magnitude and the sign of the charge of the third particle.

Q2/ A negatively charged particle $-q$ is placed at the center of a uniformly charged ring, where the ring has a total positive charge Q as shown in Figure P23.82. The particle, confined to move along the x axis, is moved a small distance x along the axis (where $x << a$) and released. Show that the particle oscillates in simple harmonic motion with a frequency given by

$$f = \frac{1}{2\pi}\left(\frac{k_e q Q}{m a^3}\right)^{1/2}$$

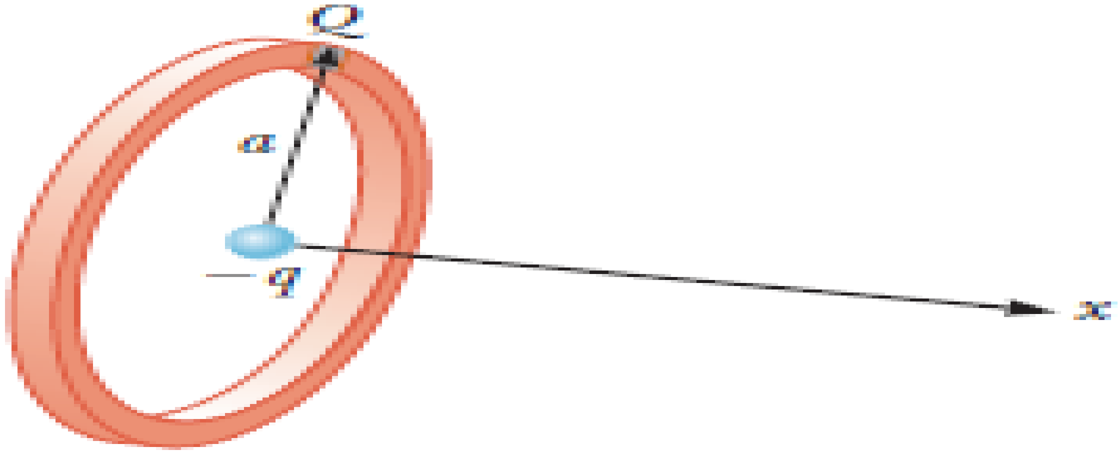


Figure P23.82

Q3/ Two particles, each with charge 52.0 nC, are located on the y axis at $y = 25.0$ cm and $y = -25.0$ cm. (a) Find the vector electric field at a point on the x axis as a function of x. (b) Find the field at $x = 36.0$ cm. (c) At what location is the field $1.00\hat{i}$ kN/C? You may need a computer to solve this equation. (d) At what location is the field $16.0\hat{i}$ kN/C?

Q4/

A particle with charge q is located inside a cubical gaussian surface. No other charges are nearby. (i) If the particle is at the center of the cube, what is the flux through each one of the faces of the cube? (a) 0 (b) $q/2\epsilon_0$ (c) $q/6\epsilon_0$ (d) $q/8\epsilon_0$ (e) depends on the size of the cube (ii) If the particle can be moved to any point within the cube, what maximum value can the flux through one face approach? Choose from the same possibilities as in part (i).

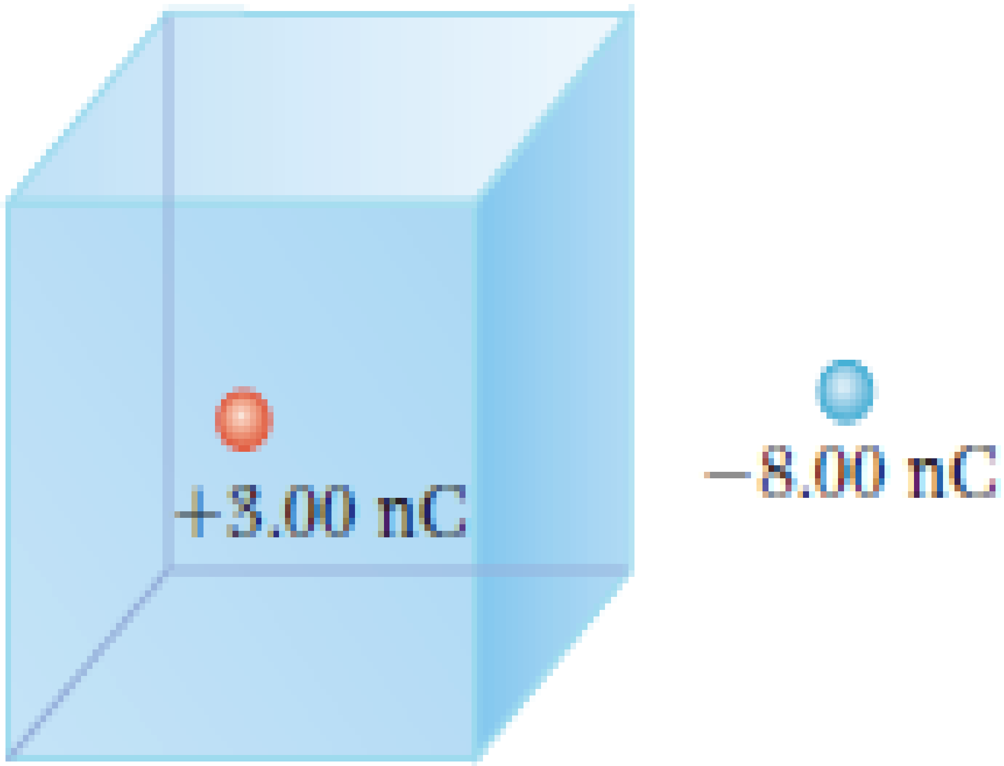
Q5/

Charges of 3.00 nC, −2.00 nC, −7.00 nC, and 1.00 nC are contained inside a rectangular box with length 1.00 m, width 2.00 m, and height 2.50 m. Outside the box are charges of 1.00 nC and 4.00 nC. What is the electric flux through the surface of the box? (a) 0 (b) $-5.64 \times 10^2\ N \cdot m^2/C$ (c) $-1.47 \times 10^3\ N \cdot m^2/C$ (d) $1.47 \times 10^3\ N \cdot m^2/C$ (e) $5.64 \times 10^2\ N \cdot m^2/C$

Q6/ A uniform electric field of 1.00 N/C is set up by a uniform distribution of charge in the *xy* plane. What is the electric field inside a metal ball placed 0.500 m above the *xy* plane? (a) 1.00 N/C (b) −1.00 N/C (c) 0 (d) 0.250 N/C (e) varies depending on the position inside the ball

Q7/

(a) Find the net electric flux through the cube shown in Figure P24.15. (b) Can you use Gauss's law to find the electric field on the surface of this cube? Explain.



Q8/

Assume the magnitude of the electric field on each face of the cube of edge $L = 1.00$ m in Figure P24.32 is uniform and the directions of the fields on each face are as indicated. Find (a) the net electric flux through the cube and (b) the net charge inside the cube. (c) Could the net charge be a single point charge?

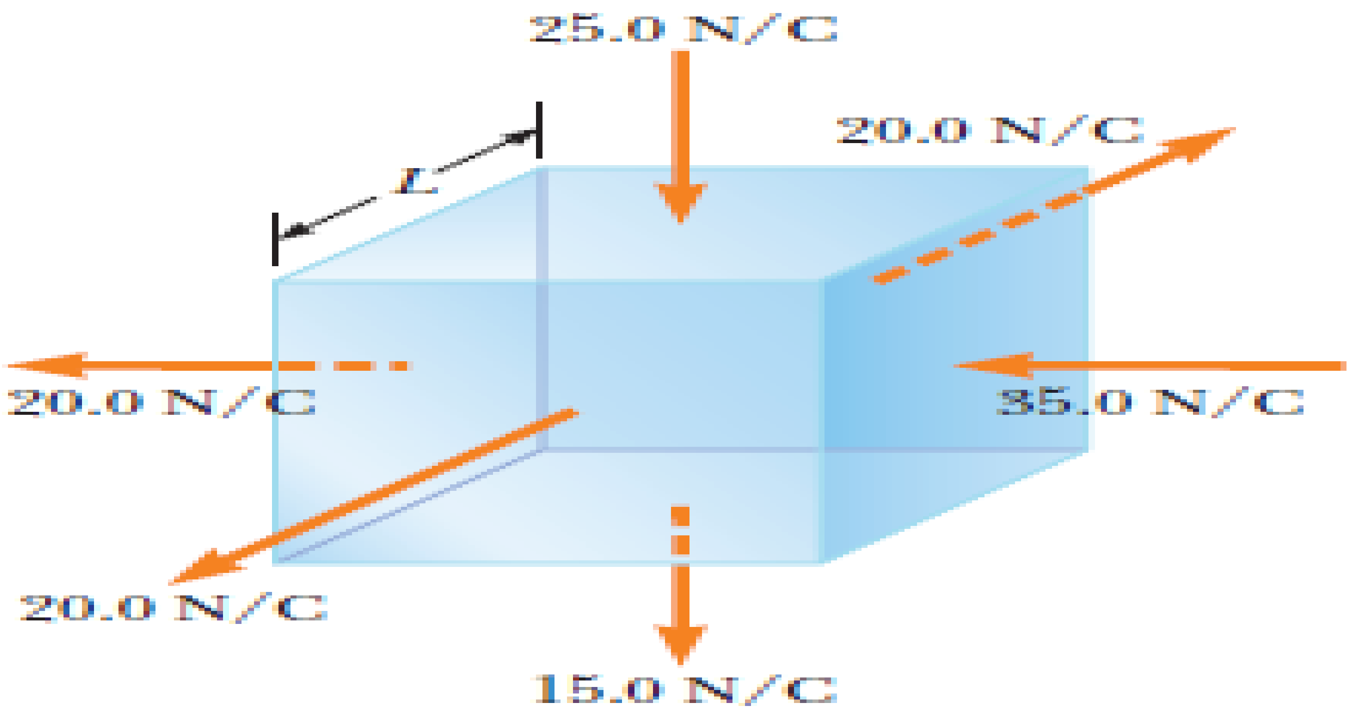


Figure P24.32

Q9/

A cylindrical shell of radius 7.00 cm and length 2.40 m has its charge uniformly distributed on its curved surface. The magnitude of the electric field at a point 19.0 cm radially outward from its axis (measured from the midpoint of the shell) is 36.0 kN/C. Find (a) the net charge on the shell and (b) the electric field at a point 4.00 cm from the axis, measured radially outward from the midpoint of the shell.

Q10/

A solid sphere of radius 40.0 cm has a total positive charge of 26.0 μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere.

Q11/

Two infinite, nonconducting sheets of charge are parallel to each other as shown in Figure P24.56. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density $-\sigma$. Calculate the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets. (d) **What If?** Find the electric fields in all three regions if both sheets have *positive* uniform surface charge densities of value σ.

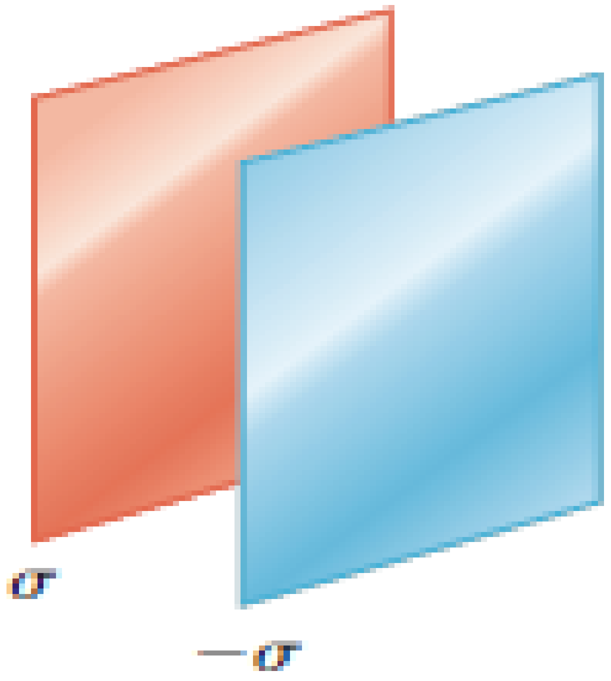


Figure P24.56

Q12 /

A closed surface with dimensions $a = b = 0.400$ m and $c = 0.600$ m is located as shown in Figure P24.63. The left edge of the closed surface is located at position $x = a$. The electric field throughout the region is nonuniform and is given by $\vec{E} = (3.00 + 2.00x^2)\hat{i}$ N/C, where x is in meters. (a) Calculate the net electric flux

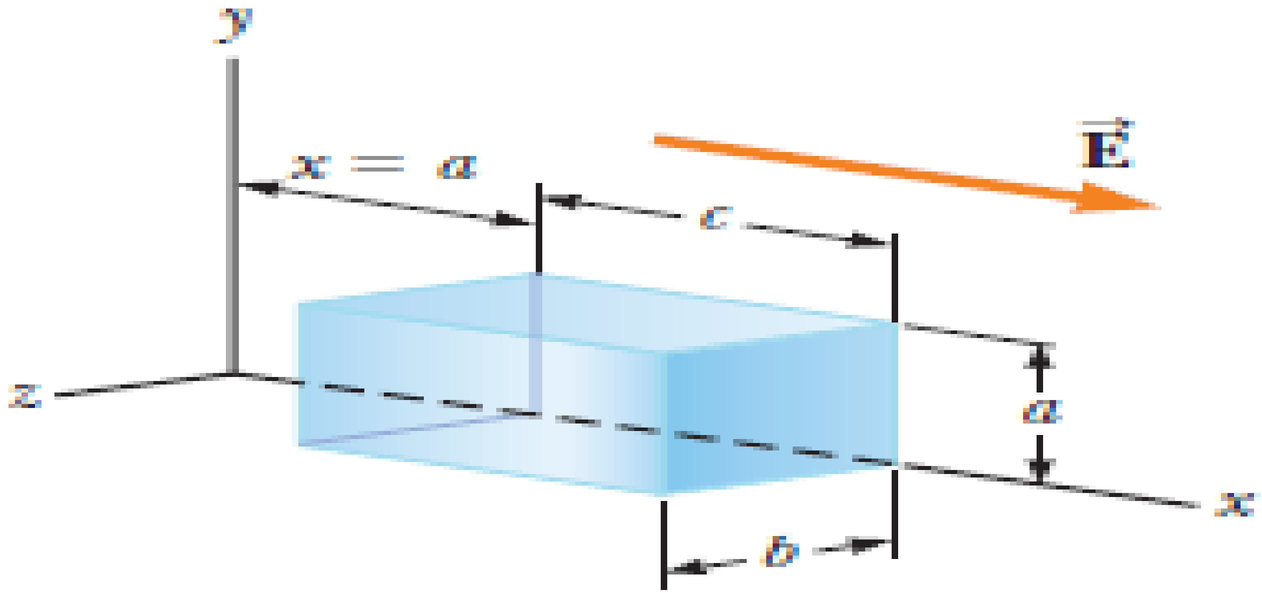


Figure P24.63