محاضرات

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المرحلة الاولى

تفاضل وتكامل

II

الفصل الدراسي الثاني

٢٠١٦ – ٢٠١٧

المرحلة الاولى

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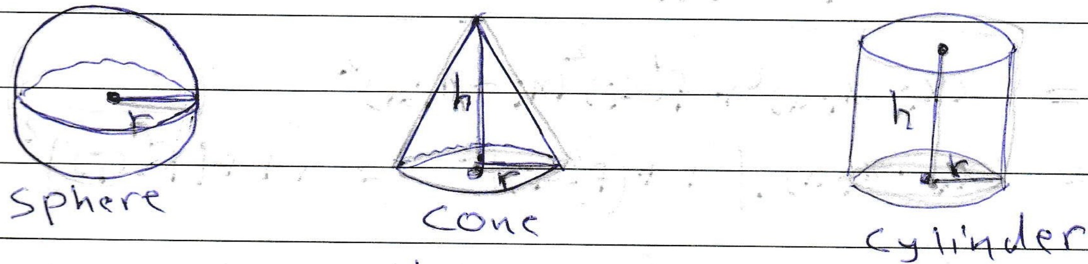
Surface Area

تطبيقات التكامل II

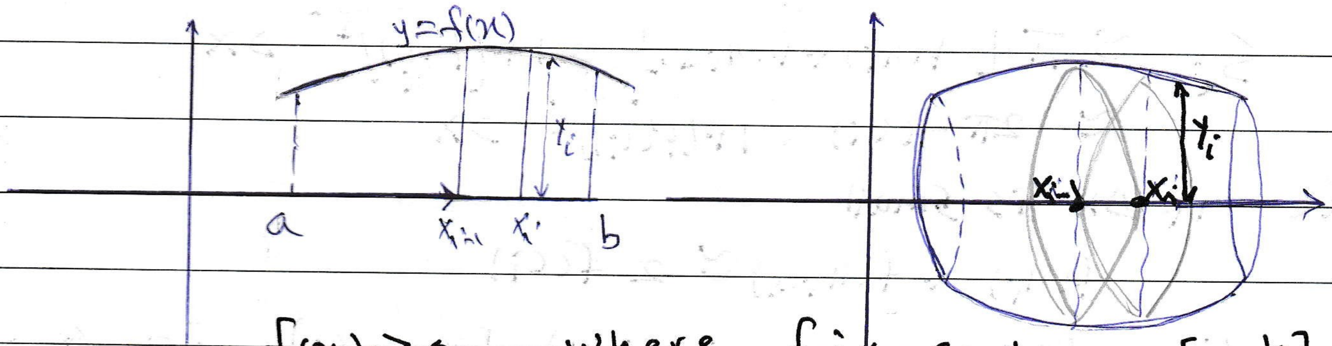
Sphere: $V = \frac{4}{3}\pi r^3$, Surface Area $= 4\pi r^2$

cone: $V = \frac{1}{3}\pi r^2 h$, $SA = \pi r\sqrt{r^2+h^2}$

cylinder: $V = \pi r^2 h$ $\quad SA = 2\pi r h$



How to calculate the surface Area of solid of revolution obtained by rotating a curve around a given line.



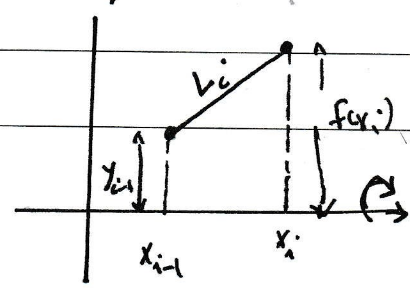
$f(x) \geq 0$, where f is cont. on $[a,b]$ and differentiable on (a,b).

partition $[a,b]$ into n pieces of equal size

$$a = x_0 < x_1 < x_2 < \cdots < x_{i-1} < x_i < \cdots < x_n = b$$

where $\Delta x = x_i - x_{i-1} = \dfrac{b-a}{n}$, for each $i = 1, 2, \ldots, n$

On each subinterval $[x_{i-1}, x_i]$, we can approximate the curve by the straight line segment joining the point (x_{i-1}, y_{i-1}) , and (x_i, y_i) as in figure below revolving L_i about x-axis which is generated the (frustum of a cone)



$$L_i = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$

from mean value theorem , $c_i \in (x_{i-1}, x_i)$

$$f'(c_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \Rightarrow f(x_i) - f(x_{i-1}) = f'(c_i)(x_i - x_{i-1})$$

$$L_i = \sqrt{(x_i - x_{i-1})^2 + f'(c_i)^2 (x_i - x_{i-1})^2}$$

$$= (x_i - x_{i-1}) \sqrt{1 + (f'(c_i))^2} = \sqrt{1 + [f'(c_i)]^2}\, \Delta x$$

The surface area S_i of that portion of the surface on $[x_{i-1}, x_i]$ is approximetly the surface area of the frustum of the cone.

$$S_i \approx \pi [f(x_i) + f(x_{i-1})] \sqrt{1 + (f'(c_i))^2}\, \Delta x$$

$$\approx 2\pi f(c_i) \sqrt{1 + [f'(c_i)]^2}\, \Delta x$$

since if Δx is small

$$f(x_i) + f(x_{i-1}) \approx 2 f(c_i)$$

Repeating for each subinterval $[x_{i-1}, x_i]$, $i = 1, 2, \dots, n$

, total surface area S approximatly is

$$S \approx \sum_{i=1}^{n} 2\pi f(c_i) \sqrt{1 + (f'(c_i))^2}\, \Delta x$$

as $n \to \infty$

$$S = \lim_{n \to \infty} \sum_{i=1}^{n} 2\pi f(c_i) \sqrt{1 + [f'(c_i)]^2}\, \Delta x$$

cylinder

$$S = \int_a^b 2\pi \underbrace{f(x)}_{\text{radius}} \underbrace{\sqrt{1 + [f'(x)]^2}}_{h\,=\,\text{height}}\, dx.$$

Ex: Find the surface area of the surface generated by revolving $y = x^4$ for $0 \le x \le 1$ about x-axis.

$$S = \int_{x=0}^{x=1} 2\pi f(x) \sqrt{1 + (f'(x))^2}\, dx$$

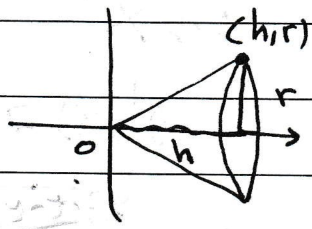
$$= \int_0^1 2\pi x^4 \sqrt{1 + (4x^3)^2}\, dx = \int_0^1 2\pi x^4 \sqrt{1 + 16x^6}\, dx$$

$$\simeq 3.4365$$

المساحة الجانبية

Ex: Calculate the lateral surface area of right circular cone with radius r and height h.

The cone is generated by revolving the area under the line $y = \frac{r}{h} x$ about the x-axis. Then



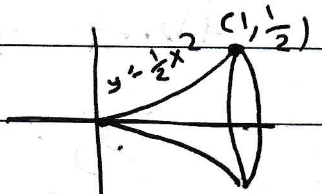
$$S = \int_0^h 2\pi \left(\frac{r}{h}\right) x \sqrt{1 + \left(\frac{r}{h}\right)^2}\, dx$$

$$= 2\pi \frac{r}{h} \sqrt{1 + \left(\frac{r}{h}\right)^2} \int_0^h x\, dx = 2\pi \frac{r}{h} \sqrt{1 + \left(\frac{r}{h}\right)^2}\, \frac{x^2}{2}\Big|_0^h$$

$$= 2\pi \frac{r}{h} \sqrt{1 + \frac{r^2}{h^2}} \cdot \frac{h^2}{2} = 2\pi \frac{r}{h} \sqrt{\frac{h^2 + r^2}{h^2}}\, \frac{h^2}{2}$$

$$= \pi r \sqrt{h^2 + r^2}$$

Ex: The area under the curve $y = \frac{1}{2} x^2$ between $x = 0$ and $x = 1$ is rotated about x-axis. Find the lateral surface area of the solid generated.

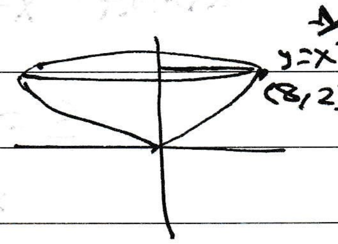


$$f'(x) = x$$

$$S = \int_0^1 2\pi \left(\tfrac{1}{2} x^2\right) \sqrt{1 + x^2}\, dx$$

let $x = \tan\theta$. $\quad S = \int_0^{\pi/4} \pi \tan^2\theta \sec^3\theta\, d\theta = \_\_\_$

Ex: Calculate the lateral surface area of the solid obtained by revolving the curve $y = x^{1/3}$ about y-axis between $x=0$ and $x=8$



Sol^n: When the area between curve $x=g(y)$ and y-axis is rotated about y-axis between $y=c$ & $y=d$, the lateral surface area is given by

$$S = \int_{y=c}^{y=d} 2\pi\, g(y)\sqrt{1+[g'(y)]^2}\, dy$$

$$y = x^{1/3} \Rightarrow y^3 = x \quad , \quad x = g(y) = y^3$$

$$g'(y) = 3y^2$$

$$S = \int_{y=0}^{y=2} 2\pi\, y^3 \sqrt{1+9y^4}\, dy = 2\pi \int_0^2 y^3 (1+9y^4)^{1/2}\, dy$$

$$= \frac{3\cdot 2}{2\cdot 36}\pi\,(1+9y^4)^{3/2}\Big|_1^2 = \frac{\pi}{12}(1+9y^4)^{3/2}\Big|_1^2 = \underline{\qquad}$$

$$= \underline{\qquad}$$

Note: If $x = f(t)$, $y = g(t)$, is rotated about x-axis, then

$$S_x = 2\pi \int_{t=t_0}^{t=t_1} y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

about y-axis

$$S_y = 2\pi \int_{t_0}^{t_1} x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$