محاضرات

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المرحلة الاولى

تفاضل وتكامل

II

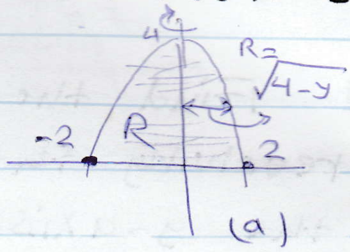
الفصل الدراسي الثاني

٢٠١٦ — ٢٠١٧

4) Let $\mathbb{R}$ be the region bounded by $y=4-x^2$ and $y=0$. Find the volume of the solids obtained by revolving $\mathbb{R}$ about each of the following

(a) Y-axis (b) the line $y=-3$, (c) the line $y=7$, (d) $x=3$.

(a) $V=\int_{y=0}^{y=4} \pi (g(y))^2\,dy = \int_0^4 \pi(4-y)\,dy = 8\pi$



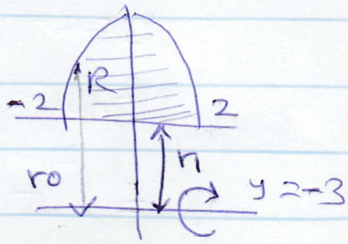
(b)

$r_0 = y-(-3) = y+3 = 4-x^2+3 = 7-x^2$

$r_1 = 0-(-3) = 3$

$V = \int_{-2}^{2} \pi(r_0^2 - r_1^2)\,dx$

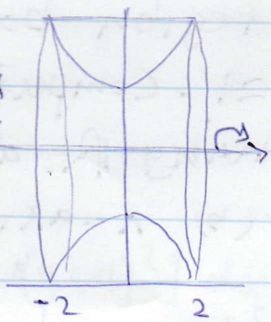
$= \int_{-2}^{2} \pi(7-x^2)^2\,dx - \pi\int_{-2}^{2}(3)^2\,dx = \frac{1472}{15}\pi$

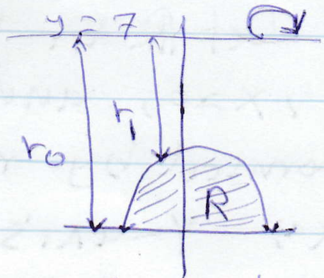


(c)

$V = \int_{x=-2}^{x=2} \pi(7)^2\,dx - \int_{-2}^{2}\pi(3+x^2)^2\,dx$

$= \frac{576}{5}\pi$





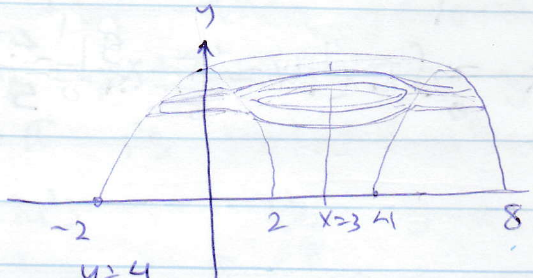
$r_0 = 7$ outer radius

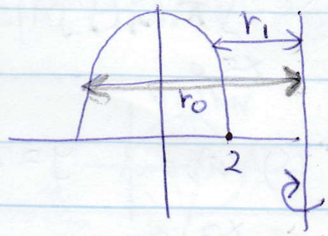
$r_1 = 7-4+x^2$

$= 3+x^2$ inner

(d) about $X=3$

$x = \pm\sqrt{4-y}$





$V = \int_{y=0}^{y=4} \pi(3+\sqrt{4-y})^2\,dy - \int_0^{y=4} \pi(3-\sqrt{4-y})^2\,dy$

$= 64\pi$

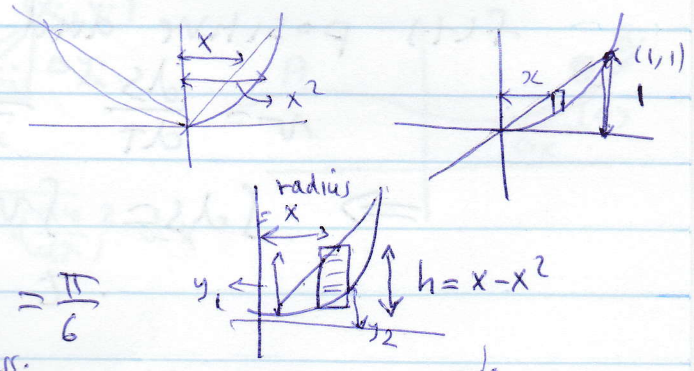
$r_1 = 3-\sqrt{4-y}$

$r_0 = 3-(-\sqrt{4-y}) = 3+\sqrt{4-y}$

H.W

Cylindrical Shell method

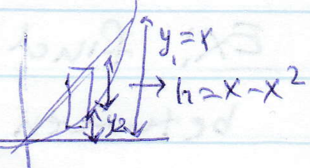
5) Revolve the region bounded by the graphs of $y=x$ and $y=x^2$ in the first quadrant about the y-axis.



$$V=\int_{x=a}^{x=b} 2\pi x(\overset{f(x)}{y_1-y_2})\,dx$$

$$V=\int_0^1 2\pi \underbrace{x}_{\text{radius}}\underbrace{(x-x^2)}_{\text{hight}}\underbrace{dx}_{\text{thickness.}}=\frac{\pi}{6}$$

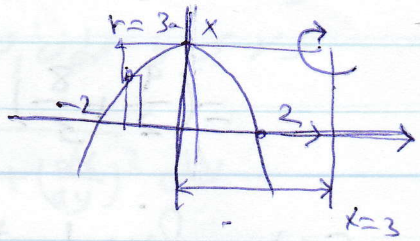
H.W



(6) Find the volume of the solid formed by revolving the region bounded by the curve $y=4-x^2$ and the x-axis about the line $x=3$.

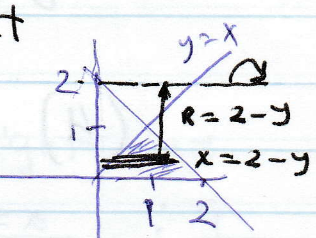
$$V=2\pi\int_{x=-2}^{2} x\,f(x)\,dx$$

$$=2\pi\int_{-2}^{2}(\overset{r}{3-x})(\overset{h}{4-x^2})\overset{w}{dx}=64\pi.$$



H.W

(7) let R be the region bounded by the graphs of $y=x$, $y=2-x$ and $y=0$. Compute the volume of the solid formed by revolving R about
(a) $y=2$, (b) $y=-1$, $c=x=3$.



(b) $V=\int_0^1 2\pi\,[\overset{r}{y-(-1)}][\overset{h}{(2-y)-y}]\,dy=\frac{8}{3}\pi$

(a) $V=\int_0^1 2\pi(2-y)[(2-y)-y]\,dy=\frac{10}{3}\pi$

(c) $V=\int_0^1 \pi[(3-y)^2-[3-(2-y)]^2]\,dy=4\pi.$

(3) How to find the distance

we calculate the distance traveled by a body moving with velocity $v = f(t)$, $a \le t \le b$, $f(t)$ positive and continuous.

$$v = \frac{ds}{dt} \Rightarrow ds = v\,dt$$

$$\Rightarrow \int ds = \int v\,dt \Rightarrow S = \int v\,dt$$

S هي المسافة المقطوعة في الزمن t.

<u>EX:</u> find the distance traveled by the body between $t=1$ and $t=3$, $v = t - \frac{8}{t^2}$

$$S = \int v\,dt = \int_{t=1}^{t=3} \left(t - \frac{8}{t^2}\right) dt = \frac{t^2}{2} + \frac{8}{t} \Big|_1^3$$

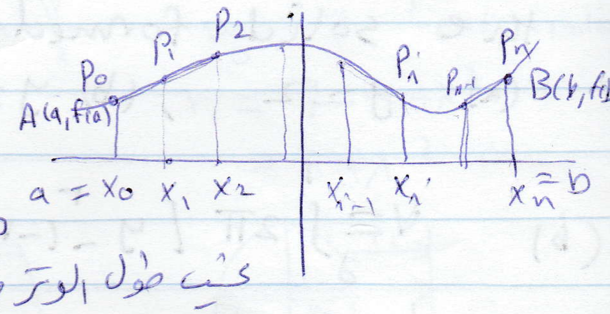
$$= \left(\frac{9}{2} + \frac{8}{3}\right) - \left(\frac{1}{2} + 8\right) = \frac{27+8}{6} - \frac{1+16}{2} = \frac{35}{6} - \frac{17}{2} = \frac{35-51}{6} = \frac{16}{6} = \frac{8}{3}$$

<u>EX:</u> find the total distance traveled by the body between $t=a$ and $t=b$

(i) $v = 6\sin 3t$, $0 \le t \le \frac{\pi}{2}$ (ii) $v = \sin t + \cos t$ $0 \le t \le \pi$

طول القوس لمنحني

(4) Arc length.



P : $a = x_0 < x_1 < x_2 < \cdots < x_i < \cdots < x_n = b$

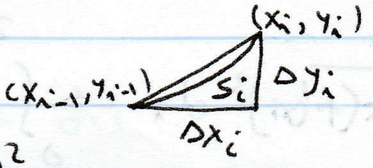
$y = f(x)$, نجد طول الوتر ولتكن $|P_{i-1}P_i|$

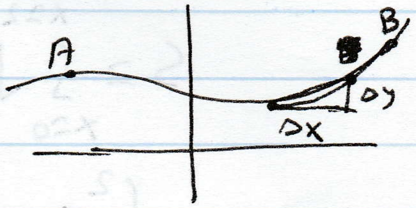
$$|P_{i-1}P_i| = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \quad \forall i = 0,1,2,\dots,n$$

فاذا كان العدد n كبيرجدا اي ان $\|P\|$ صغيرجدا فان $\sum_{i=1}^{n} |P_{i-1}P_i|$ هو تقريب جيد لطول المنحنى AB وبصورة اكثر دقة فاننا نعرف طول المنحنى AB بانه

$$\lim_{\|P\| \to 0} \sum_{i=1}^{n} |P_{i-1}P_i|$$ بشرط وجود الغاية

Suppose that $y = f(x)$ is cont. and possesses a cont.-derivative at each point of the curve from $A(a, f(a))$ to $B(b, f(b))$.





$$\Delta s_i \cong \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

$\Delta s \to 0$ $$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\boxed{L_A^B = S = \int_A^B \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx}$$

1) $$S = \int_{x=a}^{x=b} \sqrt{1 + [f'(x)]^2}\, dx$$

في بعض الأحيان المعادلات ، من الصعب ايجاد $\frac{dx}{dy}$ بدلا من $\frac{dy}{dx}$
ولايجاد طول المنحنى نستعمل العلاقة التالية

$$S = \int_A^B \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \int_{y=c}^{y=d} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

وفي حالات أخرى فإن المنحنى يعطى على شكل معادلتين أي أن
x, y تعطى بدلالة متغير ثالث وليكن مثلا t فإننا نحصل على

$$S = \int_A^B \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

<u>EX</u>: calculate the length of the straight line $y = x$ from $x = 0$ to $x = 1$.

1) $(0, 0)$, $(1, 1)$, $L = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

or
$$L = \int_{x=0}^{x=1} \sqrt{1 + (f'(x))^2}\, dx = \int_0^1 \sqrt{1+1}\, dx = \sqrt{2}\int_0^1 dx = \sqrt{2}\, x\Big|_0^1 = \sqrt{2}.$$

2) Calculate the length of arc of the curve $y=x^2$ between $x=0$ and $x=2$.

$$S = \int_{x=0}^{x=2} \sqrt{1+(f'(x))^2}\, dx = \int_0^2 \sqrt{1+4x^2}\, dx$$

$$= \int_0^2 2\sqrt{\tfrac{1}{4}+x^2}\, dx = \int_0^2 2\sqrt{x^2+1/4}\, dx$$

let $x = \frac{1}{2}\tan\theta$, then, $dx = \frac{1}{2}\sec^2\theta d\theta$ $\quad 2x = \tan\theta$

$$S = \int_0^{\tan^{-1}4} 2\sqrt{\tfrac{1}{4}\tan^2\theta + 1/4}\, d\theta = \int_0^{\tan^{-1}4} \tfrac{1}{2}\sqrt{\tan^2\theta+1}\, d\theta = \int_0^{\tan^{-1}4} \tfrac{1}{2}\sec^3\theta\, d\theta$$

$$= \frac{1}{4}\left(\ln|\sec\theta+\tan\theta| + \sec\theta\tan\theta\right|_0^{\tan^{-1}4}$$

$\theta = \tan^{-1}4 \Rightarrow \tan\theta = 4$ and $\sec\theta = \sqrt{17}$ $\quad (1+\tan^2\theta) = \sec^2\theta$

$$S = \frac{1}{4}\left[\ln|\sqrt{17}+4| + 4\sqrt{17}\right]$$

3) Calculate the length of arc of $y = x^{3/2}$ from $(0,0)$ to $(4,8)$.

4) " " " " " " $9x^2 = 4y^3$ from $(0,0)$ to $(2\sqrt{3}, 3)$

$$9x^2 = 4y^3 \Rightarrow x^2 = \frac{4}{9}y^3 \Rightarrow x = \frac{2}{3}y^{3/2}$$

$$\frac{dx}{dy} = \frac{3}{2}\cdot\frac{2}{3}y^{1/2} = y^{1/2}$$

$$S = \int_{y=0}^{y=3} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy = \int_0^3 \sqrt{1+(y^{1/2})^2}\, dy$$

$$= \int_0^3 \sqrt{(1+y)}\, dy = --$$

alculate the arc length of the curve given by

$x = \frac{t^2}{2}, \quad y = \frac{1}{3}(2t+1)^{3/2}$, from $t=0$ to $t=4$

$$S = \int_{t=0}^{t=4} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^4 \sqrt{t^2 + \left[(2t+1)^{1/2}\right]^2}\, dt = \int_0^4 \sqrt{t^2 + (2t+1)}\, dt$$

$$= \int_0^4 \sqrt{t^2 + 2t + 1}\, dt = \int_0^4 \sqrt{(t+1)^2}\, dt$$

$$= \int_0^4 (t+1)\, dt = \text{——}$$

H.W:

(1) Calculate the length of the loop of the $y = \sin x$, between $x=0$ and $x=\pi/2$.

$$S = \int_0^{\pi/2} \sqrt{1+\cos^2 x}\, dx = \text{——}$$

(2) $y = \ln x$, $x=1$ to $x=\sqrt{3}$

(3) $(y+1)^2 = 4x^3$, $x=0$ to $x=1$

مطبعة المتنبي - ديوانية - هـ ٦٥٢٠٤٦