محاضرات

الاستاذ المساعد

موسى مكي كريدي

المرحلة الاولى

تفاضل وتكامل

II

الفصل الدراسي الثاني

٢٠١٦ - ٢٠١٧

Application of the definite integral

Volumes of Revolution لحجوم الدورانية 1-1

(2) Volumes:

Volume of a box: $V = \text{length} \times \text{width} \times \text{height}$

V of a sphere: $V = \frac{4}{3}\pi r^3$

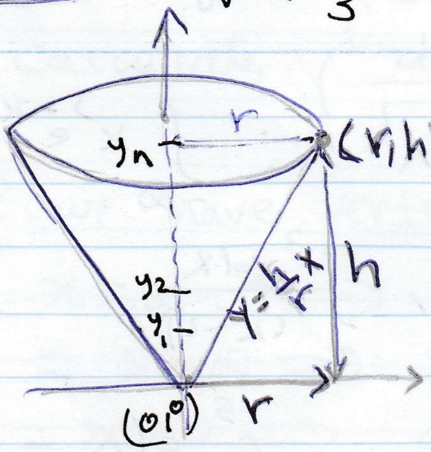
Volume of a right circular cylindre

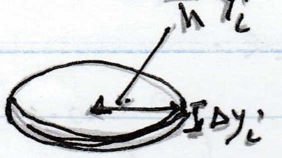
$V = \underbrace{\pi r^2}_{\text{cross-sectional area}} h$ ← height

in general, the volume of any cylinder is found by

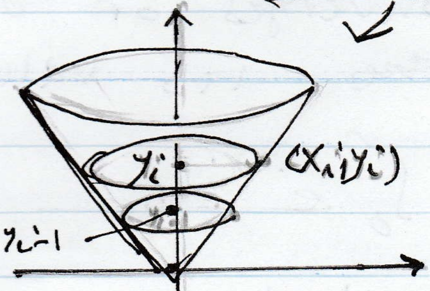
$$V = (\text{cross-sectional area}) \times \text{height}$$
$$= \pi r^2 h$$

Cone: $V = \frac{1}{3}\pi r^2 h$





$\boxed{\text{Disk Method}}$



right circular cone.

$$0 = y_0 < y_1 < y_2 < \cdots < y_n = h$$

$$V_i = \text{volume of } i\text{th disk} \approx \pi\left(\frac{r}{h}y_i\right)^2 \Delta y_i$$
$$= \frac{\pi r^2}{h^2} y_i^2 \Delta y_i$$

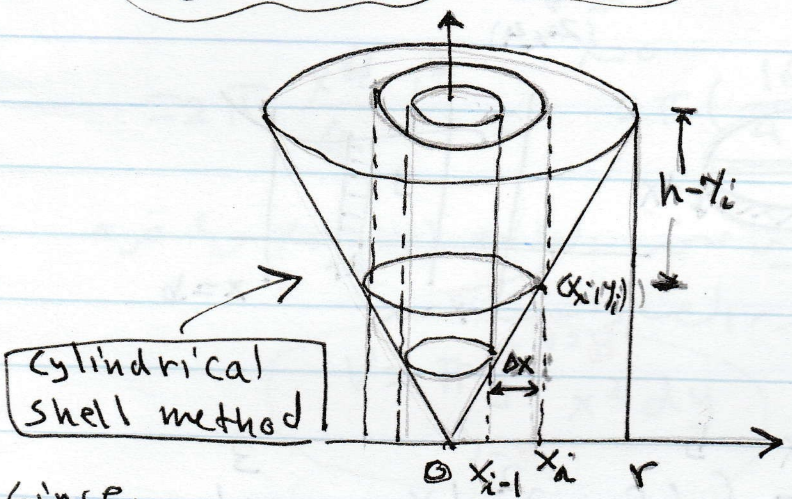
Then the approximate volume of the cone is

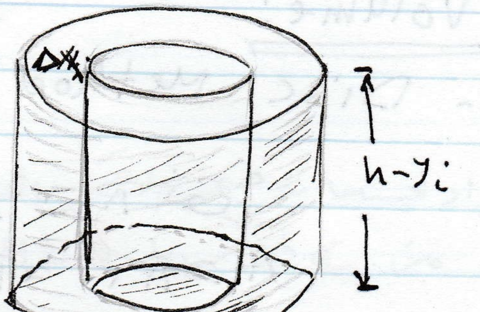
$$V = V_1 + V_2 + \cdots + V_n \approx \frac{\pi r^2}{h^2}\left(y_1^2 \Delta y_1 + y_2^2 \Delta y_2 + \cdots + y_n^2 \Delta y_n\right)$$

$$V = \frac{\pi r^2}{h^2} \lim_{\|P\| \to 0} \left(y_1^2 \Delta y_1 + y_2^2 \Delta y_2 + \cdots + y_n^2 \Delta y_n\right)$$

$$V=\frac{\pi r^2}{h^2}\int_0^h y^2 dy=\frac{\pi r^2}{h^2}\frac{y^3}{3}\Big|_0^h=\frac{\pi r^2}{3h^2}h^3=\frac{1}{3}\pi r^2 h$$

we calculate the same volume in different way, from partition P of the x-axis between 0 and $x=r$ suchthat $0=x_0<x_1<x_2<\cdots<x_n=r$, $\Delta x_i=x_i-x_{i-1}$
this dividing the cone into what are called Cylindrical shells.





Since,
Circumference of a circle of radius r is $2\pi r$, the approximate volume of each shell is its circumference $(2\pi x_i)$ times its height times its thickness, so that

$$V_i\approx 2\pi x_i(h-y_i)\Delta x_i=2\pi x_i\left(h-\frac{h}{r}x_i\right)\Delta x_i$$

$$V=\lim_{|P|\to 0}(V_1+V_2+\cdots+V_n)$$

$$=\lim_{|P|\to 0}\left[2\pi x_1\left(h-\frac{h}{r}x_1\right)\Delta x_1+2\pi x_2\left(h-\frac{h}{r}x_2\right)\Delta x_2+\cdots\right]$$

$$=\int_0^r 2\pi x\left(h-\frac{h}{r}x\right)dx=2\pi h\int_0^r\left(x-\frac{x^2}{r}\right)dx$$

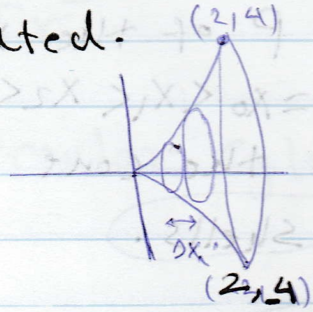
$$=2\pi h\left(\frac{x^2}{2}-\frac{x^3}{3r}\right)\Big|_0^r=2\pi h\left(\frac{r^2}{2}-\frac{r^3}{3r}\right)$$

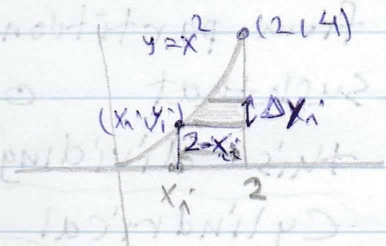
$$=2\pi h\left(\frac{r^2}{2}-\frac{r^2}{3}\right)=2\pi h\frac{r^2}{6}$$

$$=\frac{1}{3}\pi h r^2$$

①

<u>EX</u>: The region bounded by the curve $y = x^2$, the x-axis and the line $x = 2$ is revolved تدوير about the x-axis. Calculate the volume of the solid which is generated.

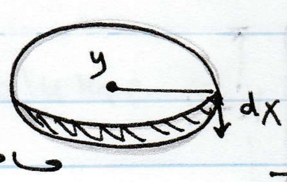


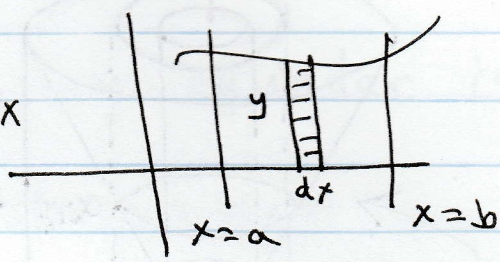


~~Disk method dx~~

Volume!

1- Disc Method





مساحة = π نق²

$A = \pi r^2 = \pi y^2$

الحجم = المساحة × السمك

$V = A \cdot dx = \pi y^2 \, dx$

$V = \pi \int_a^b y^2 \, dx = \pi \int_a^b [f(x)]^2 \, dx$ unit^3

(i) Disc Method:
If f is cont, nonegative on $[a,b]$, then the volume of the solid generated by revolving the area of f about x-axis, from $x=a$ to $x=b$ is given by

$$V = \int_a^b \pi [f(x)]^2 \, dx.$$ Disk Method.

(ii) If the area is revolving about y-axis, then the volume is given

$$V = \int_{x=a}^{x=b} 2\pi x f(x) \, dx$$

<u>Shell Method</u>

about x-axis!

EX (1)

$$V=\int_{x=0}^{x=2} \pi f(x)^2 dx=\int_0^2 \pi x^4 dx = \pi \frac{x^5}{5}\Big|_0^2 = \pi\left(\frac{32}{5}\right)$$

$$=\frac{32\pi}{5}$$

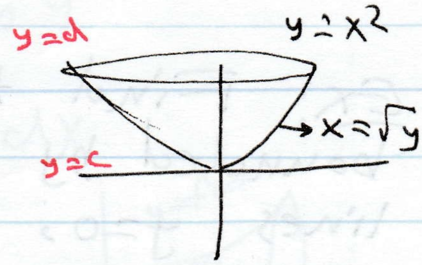
about y-axis

$$V=\int_{x=0}^{x=2} 2\pi\, x\, x^{\cancel{4}\,3}\, dx = 2\pi \int_0^2 x^3 dx$$

$$=2\pi \frac{x^{\cancel{4}}}{\cancel{4}}\Big|_0^2 = 2\pi\left(\frac{16}{4}\right) = \underline{\underline{8\pi}}$$

او لحساب حجم الشكل الدوراني المحدد بين منحنى الدالة $x=g(y)$ والمستمرة من $y=a$ الى $y=b$ حول محور الصادات

$$V=\pi\int_{y=a}^{y=b} x^2 dy$$



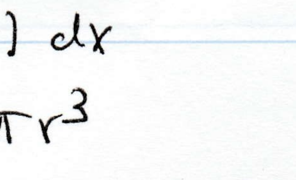
$y=x^2, \quad x=0, \quad x=2$

$y=x^2 \Longrightarrow x=\sqrt{y}$,

~~g(y)~~ $x=0,\ y=0, \quad x=2,\ y=4$

$$V=\pi\int_{y=0}^{y=4} y\, dy = \pi \frac{y^2}{2}\Big|_0^4 = \pi[8-0] = 8\pi$$

EX! calculate the volume of a sphere of radius r.



$x^2+y^2=r^2 \Longrightarrow y^2=r^2-x^2$

$y=\sqrt{r^2-x^2}$

$$V=\pi\int_{-r}^{r}\left(\sqrt{r^2-x^2}\right)^2 dx = \pi\,(r^2-x^2)\,dx$$

$$=\pi\left(r^2x-\frac{x^3}{3}\right)\Big|_{-r}^{r} = \frac{4}{3}\pi r^3$$

cylindrical shell Method:

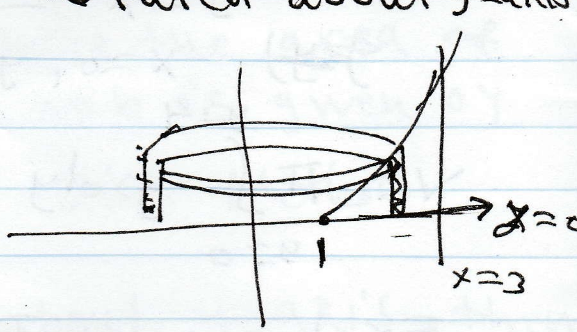
$$V = \int_a^b 2\pi x \, f(x) dx$$

$$= \int_{-r}^{r} 2\pi x \sqrt{r^2 - x^2} \, dx = \int_{-r}^{r} 2\pi x (r^2 - x^2)^{1/2} dx$$

$$= -\frac{2}{3}\pi (r^2 - x^2)^{3/2} \Big|_{-r}^{r} = -\frac{2}{3}\pi [r^2 -$$

Ex: Find the volume generated by the area bounded by the curve $y = (x-1)^2$ and the lines $y = 0$, $x = 3$ which is rotated about y-axis.

$$V = 2\pi \int_{x=1}^{x=3} x \, f(x) \, dx$$



$$= 2\pi \int x (x-1)^2 dx$$

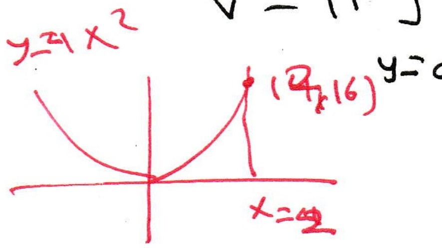
$$= 2\pi \int x (x^2 - 2x + 1) \, dx$$

$$= 2\pi \int_1^3 (x^3 - 2x^2 + x) dx = 2\pi \left(\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} \right) \Big|$$

$$=$$

EX: find the volume of the solid generated from rotated the area bounded by $y=4x^2$ and a line $y=16$ about y-axis.

$$V=\pi\int^{b}[x]^2\,dy=\pi\int_{y=0}^{y=16}\frac{y}{4}\,dy=\frac{\pi}{8}y^2\Big|_0^{16}$$

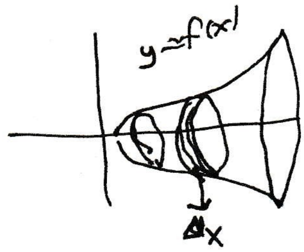


$$=\frac{\pi}{8}(16)^2=32\pi.$$

The Volume of a Solid of Revolution

1) The volume of a solid generated by revolving a curvilinear trapezoid (a curve $y=f(x)$), about the x-axis is determined by

$$V=\lim_{\Delta x\to 0}\sum \pi y^2\Delta x=\int_{x_1}^{x_2}\pi y^2\,dx$$



2) The volume of a solid generated by revolving about y-axis is determine by

$$V=\lim_{\Delta y\to 0}\sum \pi x^2\Delta y$$

$$=\int_{y_1}^{y_2}\pi x^2\,dy$$