محاضرات

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المرحلة الاولى

تفاضل وتكامل

II

الفصل الدراسي الثاني

٢٠١٦ - ٢٠١٧

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الفصل الثاني

المادة : تفاضل وتكامل II

Integration by partial fractions
"Quadratic factors"

$r(x) = p(x)/q(x)$

$q(x)$ has quadratic factors, such that $q(x)$ cann't be factored into linear terms $(x-x_1)(x-x_2)$, where x_1, x_2 are real numbers.

in general, the quadratic ax^2+bx+c is irreducible if and only if $b^2-4ac<0$.

Ex! $\int \frac{dx}{x^3+x}$

$$\frac{1}{x^3+x} = \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)x}{x(x^2+1)}$$

$$\frac{1}{x^3+x} = \frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)}$$

$$1 = (A+B)x^2+Cx+A$$

$A+B=0 \Rightarrow A=-B \Rightarrow B=-1$

$C=0$, $A=1$

$$\int \frac{dx}{x^3+x} = \int \frac{dx}{x} + \int \frac{-x}{x^2+1}dx = \ln|x| - \frac{1}{2}\ln|x^2+1| + c$$

$$= \ln\left|\frac{x}{\sqrt{x^2+1}}\right| + c$$

Ex! $\int \frac{x^2+2x+3}{(x^2+2x+2)(x-2)}dx$ $\quad \sqrt{b^2-4ac} = \sqrt{4-4\times1\times3} = \sqrt{4-8} < 0$

x^2+2x+2 irreducible, then we can write

$$\frac{x^2+2x+3}{(x-2)(x^2+2x+2)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+2}$$

$$\left.\frac{x^2+2x+3}{(x^2+2x+2)}\right|_{x=2} = A$$

$$\frac{4+4+3}{4+4+2} = \frac{11}{10}$$

$$x^2+2x+3 = A(x^2+2x+2)+(Bx+C)(x-2)$$
$$= Ax^2+2Ax+2A+Bx^2-2Bx+Cx-2C$$

$A+B=1$
$2A-2B+C=2$
$2A-2C=3$

$A=\frac{11}{10}$, $\quad B=1-A=1-\frac{11}{10}=\frac{-1}{10}$

$$2A-2C=3 \Rightarrow 2A-3=2C \Rightarrow \frac{2A-3}{2}=C \Rightarrow \frac{2\cdot\frac{11}{10}-3}{2}=\frac{\frac{11}{5}-3}{2}=-\frac{2}{5}$$

$$A+B=1 \Rightarrow A=1-B$$

$$2A-2B+C=2 \Rightarrow 2(1-B)-2B+C=2 \Rightarrow 2-2B-2B+C=2$$

$$2A \quad -2C=3 \qquad\qquad 2-4B+C=2 \Rightarrow 4B=C$$

$$A+B=1 \Rightarrow 4B=C \Rightarrow B=\frac{C}{4}$$

$$A+\frac{C}{4}=1 \Rightarrow 4A+C=4 \quad \text{—— (4)}$$

$$\mp 4A \pm 4C = \mp 8$$

$$+5C=-2 \Rightarrow C=\frac{-2}{5}=$$

$$2A=3+2C \Rightarrow A=\frac{3+2C}{2}=\frac{3+2\left(-\frac{2}{5}\right)}{2}$$

$$A=\frac{3-\frac{4}{5}}{2}=\frac{11}{10}$$

$$B=1-A=1-\frac{11}{10}=\frac{-1}{10}$$

$$\int\frac{x^2+2x+3}{(x^2+2x+2)(x-2)}\,dx=\int\frac{A}{x-2}\,dx+\int\frac{Bx+C}{x^2+2x+2}\,dx$$

$$=\frac{11}{10}\ln|x-2|-\int\frac{+\frac{1}{10}x+\frac{2}{5}}{x^2+2x+2}\,dx$$

$$=\frac{11}{10}\ln|x-2|-\left[\frac{1}{10}\int\frac{x\,dx}{x^2+2x+2}+\frac{2}{5}\int\frac{dx}{x^2+2x+2}\right]$$

$$\int \frac{\frac{1}{10}x+\frac{2}{5}}{x^2+2x+2}\,dx = \frac{1}{10}\int \frac{x\,dx}{x^2+2x+2} + \frac{2}{5}\int \frac{dx}{x^2+2x+2}$$

$$= \frac{1}{10}\int \frac{x+1-1}{x^2+2x+2}\,dx + \frac{2}{5}\int \frac{dx}{x^2+2x+2}$$

$$= \frac{1}{10}\int \frac{x+1}{x^2+2x+2}\,dx - \frac{1}{10}\int \frac{dx}{x^2+2x+2} + \frac{2}{5}\int \frac{dx}{x^2+2x+2}$$

$$= \frac{1}{20}\int \frac{2x+2}{x^2+2x+2}\,dx - \frac{1}{10}\int \frac{dx}{x^2+2x+2} + \frac{2}{5}\int \frac{dx}{x^2+2x+2}$$

$$= \frac{1}{20}\int \frac{2x+2}{x^2+2x+2}\,dx + \frac{3}{10}\int \frac{dx}{x^2+2x+2}$$

$$= \frac{1}{20}\ln|x^2+2x+2| + \frac{3}{10}\int \frac{dx}{(x+1)^2+1}$$

$$= \frac{1}{20}\ln|x^2+2x+2| + \frac{3}{10}\tan^{-1}(x+1) + C$$

$$I = \frac{11}{10}\ln|x-2| - \frac{1}{20}\ln|x^2+2x+2| + \frac{3}{10}\tan^{-1}(x+1) + C$$

$$\int \frac{x^3-1}{x^3+1}\,dx = \int \left(1 - \frac{2}{x^3+1}\right) dx$$

$$\int \frac{2\,dx}{x^3+1} = \int \frac{2\,dx}{(x+1)(x^2-x+1)}$$

$$\frac{2}{x^3+1} = \frac{2}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} = A(x^2-x+1)+$$

$$2 = A(x^2-x+1) + (Bx+C)(x+1) = Ax^2 - Ax + A + Bx^2 + Bx + Cx + C$$

$$A = \left.\frac{2}{(x^2-x+1)}\right|_{x=-1} = \frac{2}{1+1+1} = \frac{2}{3}$$

$A + B = 0 \quad -(1) \Rightarrow \quad A = -B$

$-A + B + C = 0 \quad -(2)$

$A + C = 2 \quad -(3)$

$-A + B + C = 0$

$A + C = 2$

$B + 2C = 2$

$-A + 2C = 2$

$A + C = 2$

$3C = 4 \Rightarrow C = \frac{4}{3}$

$A = 2 - C = 2 - \frac{4}{3} = \frac{2}{3}, \quad B = \frac{-2}{3}$

$$\int \frac{A\,dx}{(x+1)} + \int \frac{Bx + C}{x^2 - x + 1}\,dx$$

$$\frac{2}{3}\int \frac{dx}{x+1} + \int \frac{-\frac{2}{3}x + \frac{4}{3}}{x^2 - x + 1}\,dx$$

$$-\frac{1}{3}\int \frac{2x}{x^2 - x + 1} + \frac{4}{3}\int \frac{dx}{x^2 - x + 1}$$

$$= -\frac{1}{3}\int \frac{2x - 1 + 1}{x^2 - x + 1}\,dx + \frac{4}{3}\int \frac{dx}{x^2 - x + 1}$$

$$= -\frac{1}{3}\int \frac{(2x-1)}{x^2 - x + 1}\,dx - \frac{1}{3}\int \frac{dx}{x^2 - x + 1} + \frac{4}{3}\int \frac{dx}{x^2 - x + 1}$$

$$= -\frac{1}{3}\ln|x^2 - x + 1| + \int \frac{dx}{x^2 - x + 1} = -\frac{1}{3}\ln|x^2 - x + 1| + \int \frac{dx}{(x - \frac{1}{2})^2 + \frac{3}{4}}$$

$$= -\frac{1}{3}\ln|x^2 - x + 1| + \frac{1}{\sqrt{3/4}} \tan \left(\frac{x - \frac{1}{2}}{\sqrt{3/4}}\right) + C$$

$$I = x - \frac{1}{3}\ln| \quad | + \underline{\quad\quad} + C$$

② H.W

$$\int \frac{x^3+3x^2+1}{(x^2+1)^2}\,dx$$

$$\frac{x^3+3x^2+1}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2} = \frac{(Ax+B)(x^2+1)+Cx+D}{(x^2+1)^2}$$

$$x^3+3x^2+1 = Ax^3+Ax+Cx+D+Bx^2+B$$

$A = 1$

$B = 3$

$A + C = 0 \Rightarrow A = -C \Rightarrow C = -1$

$B + D = 1 \Rightarrow D = 1 - B = 1 - 3 = -2$

$$\int \frac{x^3+3x^2+1}{(x^2+1)^2}\,dx = \int \frac{x+3}{x^2+1}\,dx + \int \frac{-x-2}{(x^2+1)^2}\,dx$$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2+1} + 3\int \frac{dx}{x^2+1} - \int \frac{x+2}{(x^2+1)^2}\,dx$$

$$= \frac{1}{2}\ln|x^2+1| + 3\tan^{-1}x + \frac{1}{2(x^2+1)} - 2\int \frac{dx}{(x^2+1)^2}$$

$$= \frac{1}{2}\ln|x^2+1| + 3\tan^{-1}x - \frac{1}{2(x^2+1)} - \theta + \frac{1}{2}\sin 2\theta$$

$$\int \frac{dx}{(x^2+1)^2}, \quad \text{let } x = \tan\theta \Rightarrow dx = \sec^2\theta\,d\theta$$

$$\int \frac{\sec^2\theta\,d\theta}{(\tan^2\theta+1)^2} = \int \frac{\sec^2\theta\,d\theta}{\sec^4\theta} = \int \frac{1}{\sec^2\theta}\,d\theta = \int \cos^2\theta\,d\theta$$

$$= \int \frac{1+\cos 2\theta}{2}\,d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$$

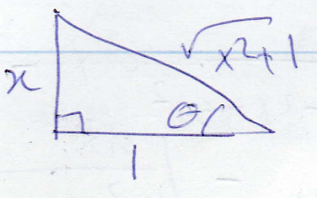
$$I = \frac{1}{2}\ln|x^2+1| + 3\tan^{-1}x - \frac{1}{2(x^2+1)} - \tan^{-1}x - \frac{x}{x^2+1} + C$$

from figure

$\tan\theta = x$

$\sin\theta = \frac{x}{\sqrt{x^2+1}}$

$\cos\theta = 1/\sqrt{x^2+1}$



* H.W

$$\int \frac{x^2-3x+18}{(x-1)(x^2-2x+5)^2}$$

$$\frac{x^2-3x+18}{(x-1)(x^2-2x+5)^2} = \frac{Ax+B}{(x^2-2x+5)} + \frac{Cx+D}{(x^2-2x+5)^2} + \frac{E}{x-1}$$

$$x^2-3x+18 = (Ax+B)(x^2-2x+5)(x-1) + (Cx+D)(x-1) + E(x^2-2x+5)^2$$

$$=(Ax+B)(x^3-3x^2+7x-5) + C(x+D)(x-1) + E(\quad)^2$$

$$= Ax^4-3Ax^3+7Ax^2-5Ax+Bx^3-3Bx^2+7Bx-5B+Cx^2-Cx+Dx-D$$
$$+Ex^4-4Ex^3+14Ex^2-20Ex+25E$$

$$A+E = 0 \implies A=-E$$
$$-3A+B-4E = 0 \implies -3A+B+4A = A+B=0$$
$$7A-3B+C-14E = 1$$
$$-5A+7B-C+D-20E = -3$$
$$-5B-D+25E = 18$$

$$A=-1, \quad B=1, \quad C=-3, \quad D=2, \quad E=1$$

$$I = \int \frac{-x+1}{x^2-2x+5}dx + \int \frac{-3x+2}{(x^2-2x+5)^2}dx + \int \frac{dx}{x-1}$$

$$\int \frac{-x+1}{x^2-2x+5}dx = -\frac{1}{2}\int \frac{2x-2}{x^2-2x+5}dx = -\frac{1}{2}\ln|x^2-2x+5|$$

$$\int \frac{-3x+2}{(x^2-2x+5)^2}dx = \int \frac{-2x+2-x}{(x^2-2x+5)^2} = \int \frac{-2x+2}{(x^2-2x+5)^2}dx - \int \frac{x\,dx}{(x^2-2x+5)}$$

$$= -\int \frac{2x-2}{(x^2-2x+5)^2}dx - \int \frac{x-1+1}{(x^2-2x+5)^2}dx$$

$$= \frac{1}{(x^2-2x+5)} + \frac{1}{2}\int \frac{2x-2}{(x^2-2x+5)^2}dx - \int \frac{dx}{(x^2-2x+5)^2}$$

$$= \frac{1}{x^2-2x+5} + \frac{1}{2(x^2-2x+5)} - \int \frac{dx}{(x^2-2x+5)^2}$$