محاضرات

الاستاذ المساعد

موسى مكي كريدي

المرحلة الاولى

تفاضل وتكامل

II

الفصل الدراسي الثاني

٢٠١٦ - ٢٠١٧

H.W: calculate

1- $\int \sqrt{x^2-a^2}\,dx$, 2- $\int_0^1 \frac{x^3}{(3+x^2)^{5/2}}\,dx$, 3) $\int_0^{1/3} \sqrt{4-9x^2}\,dx$

4/ Show that by integration

$\sinh^{-1} x = \ln[x+\sqrt{1+x^2}]$

$\int \frac{dx}{\sqrt{x^2+1}} = \sinh^{-1} x$

$\int \frac{dx}{\sqrt{x^2+1}}$, $x = \tan\theta$, then $\sqrt{x^2+1} = \sqrt{\tan^2\theta+1} = \sec\theta$

$dx = \sec^2\theta\, d\theta$

$\int \frac{dx}{\sqrt{x^2+1}} = \int \frac{\sec^2\theta\, d\theta}{\sec\theta} = \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + c$

$\int \frac{dx}{\sqrt{x^2+1}} = \ln|\sqrt{x^2+1} + x| + c_1$

H.W: 1) prove by integration that $\cosh^{-1} x = \ln[x+\sqrt{x^2-1}]$

2) prove " " " $\tanh^{-1} x = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$

3) (i) Graph the curve $y = \sqrt{1-x^2}$ for $0 \le x \le 1$

(ii) Find the area bounded by the curve and x and y-axes.

4): Integration by Partial Fractions (linear Factor):

$$r(x) = \frac{p(x)}{q(x)}, \quad q(x) \neq 0$$

$$\int r(x)\,dx = \int \frac{p(x)}{q(x)}\,dx$$

Ex: calculate $\int \frac{dx}{x^2+x}$

$$\frac{1}{x^2+x} = \frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$, to find constants A & B

$$\frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1)+Bx}{x(x+1)} = \frac{(A+B)x+A}{x(x+1)}$$

$$\frac{(A+B)x+A}{x(x+1)} = \frac{1}{x(x+1)} = \frac{0 \cdot x+1}{x(x+1)}$$

$$(A+B)x + A = 1 \Rightarrow A+B=0 \Rightarrow A=-B$$

$$A=1 \Rightarrow B=-1$$

$$\int \frac{dx}{x^2+x} = \int \frac{A}{x}\,dx + \int \frac{B}{x+1} = \int \frac{dx}{x} - \int \frac{dx}{x+1}$$

$$= \ln|x| - \ln|x+1| + c = \ln\left|\frac{x}{x+1}\right| + c$$

Ex(2): $$\int \frac{x^5+4x^4-14x^3-31x^2+57x-72}{x^2+2x-15}\,dx$$

$$= \int \left[x^3+2x^2-3x+5 + \frac{2x+3}{x^2+2x-15}\right] dx$$

$$\frac{2x+3}{x^2+2x-15} = \frac{2x+3}{(x+5)(x-3)} = \frac{A}{x+5} + \frac{B}{x-3}$$

$$\frac{2x+3}{(x+5)(x-3)} = \frac{A(x-3)+B(x+5)}{(x+5)(x-3)}$$

$$A(x-3) + B(x+5) = 2x+3$$

$$(A+B)x + (5B-3A) = 2x+3$$

$$A+B=2 \qquad 3A+3B=6$$
$$5B-3A=3 \qquad -3A+5B=3$$
$$8B=9 \Rightarrow B=\frac{9}{8}, \quad A=2-\frac{9}{8}=\frac{16-9}{8}=\frac{7}{8}$$

$$\int \frac{2x+3}{x^2+2x-15}\,dx = \int \frac{7/8\,dx}{x+5} + \int \frac{9/8}{x-3}\,dx$$
$$= 7/8 \ln|x+5| + 9/8 \ln|x-3|$$
$$= \frac{1}{8}\left[\ln|(x+5)^7 (x-3)^9|\right] + C$$

$$I = \frac{x^4}{4} + \frac{2x^3}{3} - \frac{3x^2}{2} + 5x + \frac{1}{8}\ln|(x+5)^7(x-3)^9| + C.$$

Note: (i) If degree $p(x) \geq$ degree $q(x)$, divide to obtain

$r(x) = \frac{p(x)}{q(x)} = s(x) + \frac{t(x)}{q(x)}$, where ~~deg~~ degree $t(x) <$ deg.$(q(x))$, $s(x)$ is poly.

(ii) Integrate $s(x)$

(iii) It is always possible to factor $q(x)$ into linear and quadratic factors. there are four cases.

(1) distinct linear factors
$q(x) = (x-x_1)(x-x_2)\cdots(x-x_n)$, where no two of the x_j's are equal.

(2) Repeated linear factors; Some of the roots x_j are the same.

(3) distinct Quadratic factors: Here $q(x)$ can be factored into linear terms (like $(x-x_j)$) and quadratic terms of the form x^2+ax+b

(4) Repeated Quadratic factors

Case(1): $q(x)=(x-x_1)(x-x_2)\cdots(x-x_n)$ Then we can write

$$\frac{f(x)}{q(x)}=\frac{A_1}{x-x_1}+\frac{A_2}{(x-x_2)}+\cdots+\frac{A_n}{(x-x_n)}$$

Ex: Calculate $\int \frac{2x^3-3x^2-5x+11}{x^3-2x^2-x+2}\,dx$

$$\int \frac{(2x^3-3x^2-5x+11)}{(x^3-2x^2-x+2)}\,dx=\int\left[2+\frac{x^2-3x+7}{x^3-2x^2-x+2}\right]dx$$

$$\frac{x^2-3x+7}{x^3-2x^2-x+2}=\frac{x^2-3x+7}{(x-1)(x+1)(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x+1)}+\frac{C}{(x-2)}$$

$$=\frac{A(x+1)(x-2)+B(x-1)(x-2)+C(x-1)(x+1)}{(x-1)(x+1)(x-2)}$$

$$=\frac{A(x^2-x-2)+B(x^2-3x+2)+C(x^2-1)}{(x-1)(x+1)(x-2)}$$

$$\frac{x^2-3x+7}{x^3-2x^2-x+2}=\frac{x^2(A+B+C)+(-A-3B)x+(-2A+2B-C)}{(x-1)(x+1)(x-2)}$$

$$\left.\begin{aligned}A+B+C&=1\\-A-3B&=-3\\-2A+2B-C&=7\end{aligned}\right\}\Rightarrow\quad \begin{aligned}\cancel{A}+B+C&=1\\\cancel{-A}-3B&=-3\\\hline -2B+C&=-2\end{aligned}$$

$$\begin{aligned}-2A+2B-C&=7\\-2B+C&=-2\\\hline -2A&=5\end{aligned}$$

$-2A=5 \Rightarrow A=-\frac{5}{2}$

$-A-3B=-3 \Rightarrow -3B=-3+A=-3-\frac{5}{2}=\frac{-6-5}{2}=\frac{-11}{2}$

$-3B=\frac{-11}{2} \Rightarrow B=\frac{11}{6}$

~~A+B~~ $C=1-A-B=1+\frac{5}{2}-\frac{11}{6}=\frac{6+15-11}{6}=\frac{10}{6}=\frac{5}{3}$

<u>or</u>: $(x^3-2x^2-x+2)=(x+1)(x-1)(x-2)$

$$A_1=\left.\frac{x^2-3x+7}{(x-1)(x-2)}\right|_{x=-1}=\frac{11}{6}\quad , A_2=\left.\frac{x^2-3x+7}{(x+1)(x-2)}\right|_{x=1}=\frac{-5}{2}$$

$$A_3=\left.\frac{x^2-3x+7}{(x+1)(x-1)}\right|_{x=2}=\frac{5}{3}$$

$$\int\frac{x^2-3x+7}{x^3-2x^2-x+2}\,dx=\int\frac{A_1}{x+1}\,dx+\int\frac{A_2}{(x-1)}\,dx+\int\frac{A_3}{(x-2)}\,dx$$

$$=\int\frac{11/6}{x+1}\,dx+\int\frac{-5/2\,dx}{x-1}+\int\frac{5/3}{(x-2)}\,dx$$

$$=\frac{11}{6}\ln|x+1|-\frac{5}{2}\ln|x-1|+\frac{5}{3}\ln|x-2|+C$$

$$I=2x+\frac{11}{6}\ln|x+1|-\frac{5}{2}\ln|x-1|+\frac{5}{3}\ln|x-2|+C.$$

<u>Ex</u>: $\int\frac{dx}{x(x-1)^2}$

$x(x^2-2x+1)=x^3-2x^2+x$

$x=0,\ x=1,1$

$$\frac{1}{x(x-1)^2}=\frac{A_1}{x}+\frac{A_2}{x-1}+\frac{A_3}{(x-1)^2}$$

$1=A_1(x-1)^2+A_2(x-1)x+A_3x$

$1=A_1x^2-2A_1x+A_1+A_2x^2-A_2x+A_3x$

$$(A_1+A_2)x^2+(-2A_1-A_2+A_3)x+A_1=1$$

$(A_1+A_2)=0$

$-2A_1-A_2+A_3=0$

$A_1=1$

$$\begin{aligned}A_1+A_2&=0\\-2A_1-A_2+A_3&=0\\ \cancel{A_3}\ A_1&=1\end{aligned}\quad\Longrightarrow\quad A_1=1,\ A_2=-1,\ A_3=1$$

$$\int\frac{dx}{x(x-1)^2}=\int\frac{dx}{x}-\int\frac{dx}{x-1}+\int\frac{dx}{(x-1)^2}$$

$$=\ln|x|-\ln|x-1|-\frac{1}{(x-1)}+C$$

$$=\ln\left|\frac{x}{x-1}\right|-\frac{1}{x-1}+C.$$

H-W:

1) calculate the given Integral

$\int \frac{dx}{x(x-1)}$; $\int \frac{x+1}{x^2+x} dx$; $\int_{-1}^{0} \frac{dx}{(x-1)(x-2)(x-3)}$, $\int \frac{x^5}{x^3-x} dx$

$\int \frac{x}{(x-4)^3} dx$, $\int \frac{dx}{x^2(x+2)^2}$, $\int \frac{x^3+x^2-2}{x^4} dx$

$\int \frac{x^4-x^3+x^2-7x+2}{x^3+x^2-14x-24} dx$; $\int \frac{x^3}{x^4-x^2} dx$

2) Consider $\int \frac{t(x)}{q(x)} dx = \int \frac{x^2+2}{(x-1)^3} dx$

and write $\frac{x^2+2}{(x-1)^3} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$

(a) show that $(x-1)^3 \cdot \frac{x^2+2}{(x-1)^3} \Big|_{x=1} = C$

(b) 〃 〃 $\frac{d}{dx}\left[(x-1)^3 \frac{x^2+2}{(x-1)^3}\right]\Big|_{x=1} = B$

(c) 〃 〃 $\frac{d^2}{dx^2}\left[(x-1)^3 \cdot \frac{x^2+2}{(x-1)^3}\right]\Big|_{x=1} = 2A$

(d) calculate the given Integral

$\frac{a_1x+b_1}{(a_2x+b_2)(a_3x+b_3)} = \frac{A}{a_2x+b_2} + \frac{B}{a_3x+b_3}$,

$\int \frac{dx}{x^2+x-2} \Rightarrow A=\frac{1}{3}, B=-\frac{1}{3}$

$=\frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C$

partial fraction: distinct linear factors

$\frac{p(x)}{(a_1x+b_1)(a_2x+b_2)\cdots(a_nx+b_n)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \cdots + \frac{A_n}{a_nx+b_n}$

$\int \frac{3x^2-7x-2}{x^3-x} = \int\left[\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}\right]dx = \int\left(\frac{2}{x} - \frac{3}{x-1} + \frac{4}{x+1}\right)dx = \cdots$

$\int \frac{2x^3-4x^2-15x+5}{x^2-2x-8} = \int\left[2x + \frac{x+5}{x^2-2x-8}\right]dx = x^2 + \frac{3}{2}\ln|x-4| - \frac{1}{2}\ln|x+2| + C$

partial fraction: repeated linear factors

$\frac{p(x)}{(ax+b)^n} = \frac{A_1}{(ax+b)} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_n}{(ax+b)^n}$

Ex: $\int \frac{5x^2+20x+6}{x^3+2x^2+x} dx = \int \frac{5x^2+20x+6}{x(x+1)^2} dx = \int\left[\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\right]dx$

$A=6,\ B=-1,\ C=9$

Q: find the partial fractions decomposition only: