محاضرات

الاستاذ المساعد

موسى مكي كريدي

المرحلة الاولى

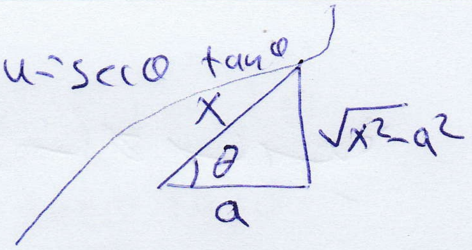
تفاضل وتكامل

II

الفصل الدراسي الثاني

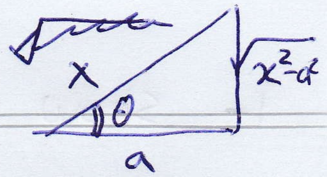
٢٠١٦ ـ ٢٠١٧

$\int \sec^3\theta \, d\theta$, $u = \sec\theta \Rightarrow du = \sec\theta \tan\theta$



$dv = \sec^2 x \, dx$

$\Rightarrow v = \int \sec^2 x \, dx = \tan x$



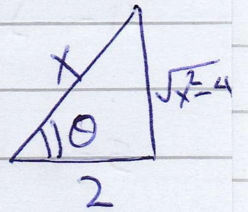
__Ex!__ $\int \sqrt{x^2-a^2} \, dx$,

$\int \sqrt{x^2-4} \, dX$ $\quad x = a\sec\theta \Rightarrow dx = a\sec\theta\tan\theta \, d\theta$

$\frac{x}{2} = \sec\theta \quad x = 2\sec\theta \Rightarrow dx = 2\sec\theta\tan\theta \, d\theta$

$\int \sqrt{x^2-4} \, dx = \int \sqrt{4\sec^2\theta - 4} \; 2\sec\theta \tan\theta \, d\theta$

$= \int 2\sqrt{\sec^2\theta - 1} \; 2\sec\theta \tan\theta \, d\theta$



$= \int 2 \tan\theta \; 2\sec\theta \tan\theta \, d\theta$

$= \int 4 \tan^2\theta \sec\theta \, d\theta$

$= \int 4 (\sec^2\theta - 1) \sec\theta \, d\theta$

$= \int 4\sec^3\theta - \int 4\sec\theta \, d\theta$

$\int \sec^3\theta = \int \sec^2\theta \sec\theta \, d\theta = \int \sec\theta (1 + \tan^2\theta) \, d\theta$

$= \int \sec\theta \, d\theta + \int \sec\theta \tan^2\theta \, d\theta$

$\int \sec\theta \tan^2\theta \, d\theta = \int \sec\theta \tan\theta \tan\theta \, d\theta$

let $u = \tan\theta \Rightarrow du = \sec^2\theta \, d\theta$

$dv = \sec\theta \tan\theta \, d\theta \Rightarrow v = \sec\theta$

$\int \sec\theta \tan^2\theta \, d\theta = \tan\theta \sec\theta - \int \sec\theta \sec^2\theta \, d\theta$

$\int \sec^3\theta \, d\theta = \ln|\sec\theta + \tan\theta| + \tan\theta \sec\theta - \int \sec^3\theta \, d\theta + c$

$2\int \sec^3\theta \, d\theta = \frac{1}{2}\left[\ln|\sec\theta + \tan\theta| + \tan\theta \sec\theta\right] + c$

التمارين الاساسية

1- Show that $\int \sqrt{3-2x-x^2}\,dx = \frac{x+1}{2}\sqrt{3-2x-x^2} + 2\sin^{-1}\frac{x+1}{2} + c$

2- $\int \sqrt{4x^2-4x+5}\,dx = \frac{2x-1}{4}\sqrt{4x^2-4x+5} + \ln(2x-1+\sqrt{4x^2-4x+5}) + c$

3- $\int b \sec ax \tan ax\,dx = \frac{b}{a}\sec ax + c$

4- $\int \cot^4 3x \csc^2 3x\,dx = -\frac{1}{15}\cot^5 3x + c$

5- $\int \frac{x\,dx}{\sqrt{27+6x-x^2}} = -\sqrt{27+6x-x^2} + 3\sin^{-1}\left(\frac{x-3}{6}\right) + c$

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(1) $\int \frac{(3x+1)}{x-3}\,dx = \int \frac{3x+9-9+1}{(x-3)}\,dx = \int \frac{3(x-3)+10}{(x-3)}\,dx$

$= \int 3\,dx + \int \frac{10}{x-3}\,dx = 3x + 10\ln|x-3| + c$

or

$\int \frac{3x+1}{x-3}\,dx = \int \left[3 + \frac{10}{x-3}\right]dx = 3x + 10\ln|x-3| + c$

(2) $\int \sqrt{1+\sin 2x}\,dx$; $\quad 1+\sin 2x = \sin^2 x + \cos^2 x + 2\sin x\cos x$

$= (\sin x + \cos x)^2$

$= \int \sqrt{(\sin x+\cos x)^2}\,dx$

$= \int (\sin x + \cos x)\,dx = -\cos x + \sin x + c$

(3) $\int \frac{dx}{\sqrt{1+\sin 2x}} = \int \frac{dx}{\sin x + \cos x}$

(4) $\int \frac{\sin x\,dx}{1+\sin x} = \sec x - \tan x + x + c$

(5) $\int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx = \int \tan x \sec x\,dx = \sec x + c$

$$\int \sec^3\theta \, d\theta = \int \sec^2\theta \sec\theta \, d\theta$$

$$u = \sec\theta \Rightarrow du = \sec\theta \tan\theta$$

$$dv = \sec^2\theta \, d\theta \Rightarrow v = \tan\theta$$

$$I = \sec\theta \tan\theta - \int \tan^2\theta \sec\theta \, d\theta$$

$$= \sec\theta \tan\theta - \int (\sec^2\theta - 1) \sec\theta \, d\theta$$

$$= \sec\theta \tan\theta - \int \sec^3\theta \, d\theta - \int \sec\theta \, d\theta$$

$$2I = \sec\theta \tan\theta - \ln|\sec\theta + \tan\theta| + c$$

$$I = \frac{1}{2} \left[\sec\theta \tan\theta - \underline{\ln} |\sec\theta + \tan\theta| + c \right.$$

Given fn	standard form
$\frac{\sin x}{\cos^2 x}$	$\frac{1}{\cos x}\frac{\sin x}{\cos x} = \sec x \tan x$
$\frac{\cos x}{\sin^2 x}$	$\frac{1}{\sin x}\frac{\cos x}{\sin x} = \csc x \cot x$
$\frac{1}{1+\sin x}$	$\frac{1}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x} = \frac{1-\sin x}{\cos^2 x} = \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} = \sec^2 x - \sec x \tan x$
$\frac{1}{1-\sin x}$	$\frac{1}{1-\sin x}\frac{1+\sin x}{1+\sin x} = \frac{1+\sin x}{1-\sin^2 x} = \frac{1+\sin x}{\cos^2 x} = \sec^2 x + \sec x \tan x$
$\frac{1}{1+\cos x}$	$\frac{1}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x} = \frac{1-\cos x}{1-\cos^2 x} = \frac{1-\cos x}{\sin^2 x} = \frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x} = \csc^2 x - \cot x \csc x$
$\frac{1}{1-\cos x}$	$\csc^2 x + \csc x \cot x$
$\frac{\sin x}{1+\sin x}$	$\sec x \tan x - \sec^2 x + 1$
$\frac{\cos x}{1+\cos x}$	$\csc x \cot x - \csc^2 x + 1$

$$\sin^3 x = \frac{3\sin x - \sin 3x}{4} \Rightarrow 4\sin^3 x = 3\sin x - \sin 3x$$

$$\sin 3x = 3\sin x - 4\sin^3 x$$

$$\cos^3 x = \frac{\cos 3x + 3\cos x}{4} \; ; \quad \cos 3x = 4\cos^3 x - 3\cos x$$

$$\sin 2x = 2\sin x \cos x = \frac{2\tan x}{1+\tan^2 x}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$