محاضرات

الأستاذ المساعد

موسى مكي كريدي

المرحلة الأولى

تفاضل وتكامل

II

الفصل الدراسي الثاني

٢٠١٦ - ٢٠١٧

manuals

$$\int \frac{du}{\sqrt{u^2+1}} = \sinh^{-1} u + C$$

$$\int \frac{du}{\sqrt{u^2-1}} = \cosh^{-1} u + C; \quad u > 1$$

$$\int \frac{du}{1-u^2} = \begin{cases} \tanh^{-1} u + C; & |u| < 1 \\ \coth^{-1} u + C; & |u| > 1 \end{cases} = \frac{1}{2} \ln \left| \frac{1+u}{1-u} \right| + C$$

$$\int \frac{du}{u\sqrt{1-u^2}} = -\operatorname{sech}^{-1}|u| + C = -\cosh^{-1}\left(\frac{1}{|u|}\right) + C$$

$$\int \frac{du}{u\sqrt{1+u^2}} = -\operatorname{csch}^{-1}|u| + C = -\sinh^{-1}\left(\frac{1}{|u|}\right) + C$$

Ex: $$\int \frac{dx}{9-x^2} = \int \frac{dx}{9\left[1-\left(\frac{x}{3}\right)^2\right]} = \frac{3}{9}\int \frac{\frac{1}{3}\,dx}{\left[1-\left(\frac{x}{3}\right)^2\right]}$$

$$= \frac{1}{3}\tanh^{-1}\left(\frac{x}{3}\right) + C$$

$$\int \frac{dx}{\sqrt{1+4x^2}} = \int \frac{dx}{\sqrt{1+(2x)^2}} \quad \#\#\# = \frac{1}{2}\int \frac{2\,dx}{\sqrt{1+(2x)^2}}$$

$$= \frac{1}{2}\sinh^{-1}(2x) + C$$

$$\int \frac{dx}{x\sqrt{4+x^2}} = \int \frac{dx}{2x\sqrt{1+\left(\frac{x}{2}\right)^2}} = \frac{2}{2}\int \frac{\frac{1}{2}\,dx}{x\sqrt{1+\left(\frac{x}{2}\right)^2}}$$

$$= -\frac{1}{2}\operatorname{csch}^{-1}\left(\frac{x}{2}\right) + C$$

H.W! Calculate

$$\int \frac{e^x\,dx}{\sqrt{1+e^{2x}}} \; ; \int \frac{\sin x\,dx}{1-\cos^2 x} \; , \int \frac{\sec^2\theta\,d\theta}{\sqrt{\tan^2\theta - 1}} \; , \int \frac{dx}{\sinh 3x + \cosh 3x}$$

تمارين عامة

1) $\int \frac{dx}{\sqrt{25-16x^2}} = \int \frac{dx}{\sqrt{5^2-(4x)^2}} = \frac{1}{5}\int \frac{dx}{\sqrt{1-(\frac{4}{5}x)^2}}$

$= \frac{1}{5}\int \frac{\frac{4}{5}dx}{\sqrt{1-(\frac{4}{5}x)^2}} = \frac{1}{4}\sin^{-1}(\frac{4x}{5}) + C$

2) $* \int \frac{dx}{x\sqrt{4x^2-9}} = \int \frac{2dx}{2x\sqrt{(2x)^2-3^2}} = \frac{1}{3}\int \frac{\frac{2}{3}dx}{\frac{2}{3}x\sqrt{(\frac{2}{3}x)^2-1}}$

$= \frac{1}{3}\sec^{-1}(\frac{2}{3}x) + C$

$* \int \frac{dx}{x\sqrt{x^4-1}} = \frac{1}{2}\int \frac{2x\,dx}{x^2\sqrt{(x^2)^2-1}} = \frac{1}{2}\sec^{-1}(x^2) + C$

$= \frac{1}{2}\cos^{-1}(\frac{1}{x^2}) + C$

3) $\int \frac{dx}{\sqrt{4-(x+2)^2}} = \int \frac{dx}{2\sqrt{1-(\frac{x+2}{2})^2}}$

$= 2\int \frac{\frac{1}{2}dx}{2\sqrt{1-(\frac{x+2}{2})^2}} = \sin^{-1}(\frac{x+2}{2}) + C$

4) $* \int \frac{dy}{y^2+10y+30} = \int \frac{dy}{y^2+10y+25-25+30} = \int \frac{dy}{(y+5)^2+5}$

$= \frac{\sqrt{5}}{5}\int \frac{\frac{1}{\sqrt{5}}dy}{[1+(\frac{y+5}{\sqrt{5}})^2} = \frac{1}{\sqrt{5}}\tan^{-1}(\frac{y+5}{5}) + C$

$* \int \frac{dx}{\sqrt{20+8x-x^2}} = \int \frac{dx}{\sqrt{20-(x^2-8x)}} = \int \frac{dx}{\sqrt{20-(x^2-8x+16-16)}}$

$= \int \frac{dx}{\sqrt{20-[(x-4)^2-16]}} = \int \frac{dx}{\sqrt{36-(x-4)^2}}$

$= \frac{1}{6}\int \frac{dx}{\sqrt{1-(\frac{x-4}{6})^2}} = \sin^{-1}(\frac{x-4}{6}) + C$

$$* \int \frac{dx}{2x^2+2x+5} = \int \frac{2\,dx}{4x^2+4x+10} = \int \frac{2\,dx}{(2x+1)^2+9}$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{2x+1}{3}\right) + C$$

$$\int \frac{x+1}{x^2-4x+8} = \frac{1}{2}\int \frac{(2x+2)}{x^2-4x+8}\,dx = \frac{1}{2}\int \frac{(2x-4)+6}{x^2-4x+8}\,dx$$

$$= \frac{1}{2}\int \frac{(2x-4)}{x^2-4x+8}\,dx + 3\int \frac{dx}{x^2-4x+8}$$

$$= \frac{1}{2}\int \frac{(2x-4)}{x^2-4x+8}\,dx + 3\int \frac{dx}{(x-2)^2+4}$$

$$= \frac{1}{2}\ln|x^2-4x+8| + \frac{3}{2}\tan^{-1}\frac{x-2}{2} + C$$

H.W

$$* \int \frac{dx}{\sqrt{28-12x-x^2}} = \sin^{-1}\frac{x+6}{8} + C$$

$\sqrt{28-(x^2+12x+36-36)}$
$= \sqrt{28-[(x-6)^2-36]}$
$= \sqrt{64-(x-6)^2}$

$$* \int \frac{2x+3}{9x^2-12x+8}\,dx = \frac{1}{9}\int \frac{18x+27}{9x^2-12x+8}\,dx$$

$$= \frac{1}{9}\int \frac{(18x-12)+39}{9x^2-12x+8}\,dx = \frac{1}{9}\int \frac{18x-12}{9x^2-12x+8}\,dx + \frac{1}{9}\int \frac{39}{9x^2-12x+8}$$

$$= \frac{1}{9}\int \frac{18x-12}{9x^2-12x+8}\,dx + \frac{13}{3}\int \frac{dx}{9x^2-12x+8}$$

$$= \frac{1}{9}\int \frac{18x-12}{9x^2-12x+8}\,dx + \frac{13}{3}\int \frac{dx}{(3x-2)^2+4}$$

$$= \frac{1}{9}\ln(9x^2-12x+8) + \frac{13}{18}\tan^{-1}\left(\frac{3x-2}{2}\right) + C$$

Answer

$$* \int \frac{2-x}{4x^2+4x-3}\,dx = -\frac{1}{8}\int \frac{(8x-16)}{4x^2+4x-3}\,dx = -\frac{1}{8}\int \frac{8x+4-20}{4x^2+4x-3}\,dx$$

$$= -\frac{1}{8}\int \frac{8x+4}{4x^2+4x-3}\,dx + \frac{20}{8}\int \frac{dx}{4x^2+4x-3} \qquad \rightarrow \int \frac{du}{u^2-a^2} = \frac{1}{2a}$$

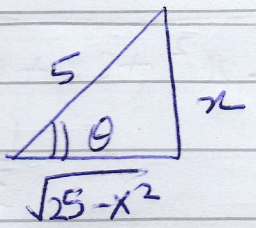
$$= -\frac{1}{8}\ln|4x^2+4x-3| + \frac{5}{2\times 2}\int \frac{2\,dx}{(2x+1)^2-4} \rightarrow \frac{5}{4}\frac{1}{(4)}\ln\left|\frac{2x+1-2}{2x+1+2}\right| + C$$

$$\ln\left|\frac{u-a}{u+a}\right|$$

$\sqrt{a^2-x^2}$, $x = a\sin\theta$

$\sqrt{a^2+x^2}$, $x = a\tan\theta$

$\sqrt{x^2-a^2}$, $x = a\sec\theta$



Ex! $\int \sqrt{25-x^2}\,dx =$, $x = a\sin\theta$

$x = 5\sin\theta \Rightarrow dx = 5\cos\theta\,d\theta$

$\frac{x}{5} = \sin\theta \Rightarrow \sin^{-1}\frac{x}{5} = \theta$

$$\int \sqrt{25-x^2}\,dx = \int \sqrt{25-25\sin^2\theta}\;5\cos\theta\,d\theta$$

$$= \int 5\sqrt{1-\sin^2\theta}\;5\cos\theta\,d\theta$$

$$= \int 25\cos^2\theta\,d\theta = 25\int \frac{1+\cos 2\theta}{2}\,d\theta$$

$$= \frac{25}{2}\theta + \frac{25}{4}\sin 2\theta + C$$

$$= \frac{25}{2}\sin^{-1}\frac{x}{5} + \frac{25}{4}\,2\sin\theta\cos\theta$$

$$= \frac{25}{2}\sin^{-1}\frac{x}{5} + \frac{25}{2}\,\frac{x}{5}\cdot\frac{\sqrt{25-x^2}}{5}$$

$$= \frac{25}{2}\sin^{-1}\frac{x}{5} + \frac{x}{2}\sqrt{25-x^2} + C$$

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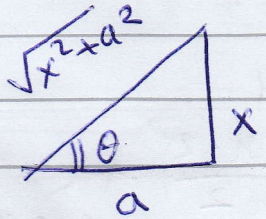
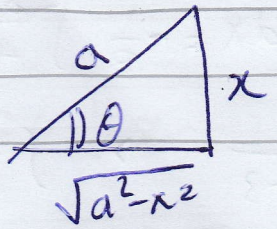
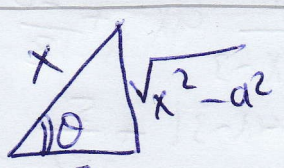
$x = a\tan\theta$





$x = a\sin\theta$



$x = a\sec\theta$.