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المرحلة الأولى

تفاضل وتكامل

II

الفصل الدراسي الثاني

٢٠١٦ - ٢٠١٧

Integration of the inverse of Trigonometric function.

1) $\int \frac{dx}{\sqrt{1-x^2}} = \begin{cases} \sin^{-1} x + C \\ -\cos^{-1} x + C \end{cases}$

2) $\int \frac{dx}{1+x^2} = \begin{cases} \tan^{-1} x + C \\ -\cot^{-1} x + C \end{cases}$

3) $\int \frac{dx}{x\sqrt{x^2-1}} = \begin{cases} \sec^{-1} x + C \\ -\csc^{-1} x + C \end{cases}$

Ex: Calculate $\int \frac{dx}{\sqrt{9-x^2}}$; $\int \frac{dx}{a^2+x^2}$

1) $\int \frac{dx}{\sqrt{9-x^2}} = \int \frac{dx}{3\sqrt{1-(\frac{x}{3})^2}} = \frac{1}{3}\int \frac{dx}{\sqrt{1-(\frac{x}{3})^2}} = \int \frac{1/3\, dx}{\sqrt{1-(\frac{x}{3})^2}}$

$= \sin^{-1}\frac{x}{3} + C$

2) $\int \frac{dx}{a^2+x^2} = \frac{1}{a^2}\int \frac{dx}{1+(\frac{x}{a})^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

3) $\int \frac{dx}{9x^2+4} = \int \frac{dx}{4+9x^2} = \int \frac{dx}{4(\frac{9}{4}x^2+1)}$

$= \int \frac{dx}{4\left(1+(\frac{3}{2}x)^2\right)} = \int \frac{\frac{2}{3}\cdot\frac{3}{2}\, dx}{4\left[1+(\frac{3}{2}x)^2\right]}$

$= \frac{2}{3}\frac{1}{4}\int \frac{\frac{3}{2}\, dx}{\left[1+(\frac{3}{2}x)^2\right]} = \frac{1}{6}\tan^{-1}(\frac{3}{2}x) + C$

4) $\int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x\, dx}{e^{2x}+1} = \int \frac{e^x\, dx}{1+(e^x)^2} = \tan^{-1}(e^x) + C$

5) $\int \frac{\cos^{-1} x\, dx}{\sqrt{1-x^2}} = -\frac{(\cos^{-1} x)^2}{2} + C$

6) $\int \frac{\sqrt{\tan^{-1} x}\, dx}{1+x^2} = \int (\tan^{-1} x)^{1/2} \frac{dx}{1+x^2} = \frac{2(\tan^{-1} x)^{3/2}}{3} + C$

③ $\int x e^{x^2} dx = \frac{1}{2}\int 2x e^{x^2} dx = \frac{1}{2} e^{x^2} + C$

$\int e^u du = e^u + C$, Let $u = x^2 \Rightarrow du = 2x\,dx$

$\int x e^{x^2} dx = \int \frac{1}{2} e^u du$ $\qquad \frac{du}{2} = x\,dx$

$= \frac{1}{2}\int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$

④ $\int \frac{dx}{e^x + 1} = \int \frac{e^{-x} dx}{e^{-x}(e^x + 1)} = \int \frac{e^{-x} dx}{1 + e^{-x}} = -\ln|1 + e^{-x}| + C$

Integration of hyperbolic Function and their inverses.

$\int \sinh u\, du = \cosh u + C$; $\int \cosh u\, du = \sinh u + C$

$\int \operatorname{sech}^2 u\, du = \tanh u + C$

$\int \operatorname{csch}^2 u\, du = -\coth u + C$;

$\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + C$;

$\int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u + C$

Ex: ① $\int \tanh u\, du = \int \frac{\sinh u}{\cosh u} du = \ln|\cosh u| + C$

② $\int \cosh(3x-1) dx = \frac{1}{3}\int \cos(3x-1)\, 3 dx = \frac{1}{3}\sinh(3x-1) + C$

(3) $\int \frac{\sinh x}{\cosh^3 x}\,dx = \int \frac{\sinh x}{\cosh x}\cdot\frac{1}{\cosh^2 x}\,dx = \int \tanh x\,\operatorname{sech}^2 x\,dx$

$= \frac{\tanh^2 x}{2} + C.$

(4) $\int \tanh^2 x\,dx$; $\quad 1-\tanh^2 x = \operatorname{sech}^2 x$

$= \int (1+\operatorname{sech}^2 x)\,dx = \int dx + \int \operatorname{sech}^2 x\,dx$

$= x + \tanh x + C$

$\cosh^2 x - \sinh^2 x = 1$; $\quad 1-\tanh^2 x = \operatorname{sech}^2 x$

$\sinh 2x = 2\sinh x\cosh x$ $\quad \coth^2 x - 1 = \operatorname{csch}^2 x$

$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 2\sinh^2 x + 1$

$e^x = \cosh x + \sinh x$; $\quad e^{-x} = \cosh x - \sinh x$

$\cosh(-x) = \cosh x$; $\quad \sinh(-x) = -\sinh x$

$\sinh(x+y) = \sinh x\cosh y + \cosh x\sinh y$

$\cosh(x+y) = \cosh x\cosh y + \sinh x\sinh y$

(5) $\int \cosh^2 x\,dx = \int \left(\frac{e^x+e^{-x}}{2}\right)^2 dx = \frac{1}{4}\int (e^x+e^{-x})^2\,dx$

$= \frac{1}{4}\int (e^{2x} + 2 + e^{-2x})\,dx = \frac{1}{4}\int e^{2x}\,dx + \frac{1}{2}\int dx + \frac{1}{4}\int e^{-2x}\,dx$

$= \frac{1}{8}e^{2x} + \frac{1}{2}x - \frac{1}{8}e^{-2x} + C$

(6) $\int \frac{\sinh\sqrt{x}}{\sqrt{x}}\,dx = \int \sinh\sqrt{x}\; x^{-1/2}\,dx = 2\int \sinh\sqrt{x}\,\frac{1}{2}x^{-1/2}\,dx$

$= 2\cosh\sqrt{x} + C$

(7) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx = \int \frac{\sinh x}{\cosh x}\,dx = \ln|\cosh x| + C.$