Mathematical logic

① Natural number which denoted by N s.t $N = \{0, 1, 2, 3, \cdots\}$

② Integer numbers which denoted by Z s.t $Z = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$

③ even integer numbers which denoted by Z_e s.t $Z_e = \{x \in Z : x = 2t, t \in Z$

④ odd integer numbers which denoted by Z_o s.t

$Z_o = \{x \in Z : x = 2t + 1, t \in Z\}$

$Z_o = \{\cdots, -3, -1, 1, 3, \cdots\}$

⑤ Rational numbers which denoted by Q s.t

$Q = \{d : d = \frac{a}{b}, b \neq 0, a, b \in Z\}$

⑥ Real numbers with denoted by R

Defintions and examples

The set is

The set is an indefine Mathematical Concept and denoted by $A, B, C, \cdots$ and denoted for the elements of set by $a, b, c, \cdots$.

If $\underline{a}$ is an element in the set $\underline{A}$ that is mean $a \in A$

If $\underline{a}$ isn't element in the set A that is mean $a \notin A$

Ex:

① $A = \{2, 3, 4\}$

② the set of real number

The subset

let A is a set we say that B is a subset of A if ~~that~~ every elements of B exists in A and denoted by $B \subseteq A$

i.e (If $x \in B$ then $x \in A$)

the proper subset B of A denoted by $B \subset A$

If there exist $x \in A$ but $x \notin B$

Ex:

If $A = \{0,1,2,3\}$ and $B = \{3,1,2\}$, Is $B \subset A$?

Sol:

let $A = \{0,1,2,3\}$ and $B = \{3,2,1\}$

we note that for all $x \in B$ then $x \in A$, i.e; $B \subseteq A$

since $0 \in A$ and $0 \notin B$ then $B \subset A$

Definition:

let A, B be a set

① the union of A with B denoted by $A \cup B$ and defined by

$A \cup B = \{x : x \in A \vee x \in B\}$

② the intersection of A with B which denoted by $A \cap B$ and defined by

$A \cap B = \{x : x \in A \wedge x \in B\}$

Ex:

let $A=\{4,5,6,...\}$ and $B=\{2,3,5,7\}$

Find ① $A\cup B$ ② $A\cap B$

Sol:-

let $A=\{4,5,6,...\}$ and $B=\{2,3,5,7\}$

$A\cup B=\{2,3,4,5,6,7,...\}$

$A\cap B=\{5,7\}$

Definition:

The Statement is a declarative sentence which conveys factual information. If the information is correct then we say the statement is true, and if the information is incorrect, then we say the statement is false

Ex:

① the number 11 is smallest than the number 12

is statement since declarative sentence and true

② goto the school, isn't statement

Defintion:

the variable is a symbol ~~den~~ denoted to all element in a set

for examble $(2x-3=0)$ the variable is x

Defintion:

the open sentence:

let A is a set we say that $p(x)$ is an open set of x defind on A if $p(a)$ is true statement or false statement for all $a\in A$.

Ex:

let $A = \{1,2,3\}$ and $p(x) = x-1 \geq 0$, Is $p(x)$ open sentence?

Sol:-

$p(x) = x-1 \geq 0$

$p(1) = 1-1 = 0 \geq 0$ (T)

$p(2) = 2-1 = 1 \geq 0$ (T)

$p(3) = 3-1 = 2 \geq 0$ (T)

i.e.; $p(a)$ is true statement $\forall a \in A$

So that $p(x)$ is an open sentence

Defintion:

the set of solution

let A is a set and let $p(x)$ be an open sentence over A, we say that $p(a)$ is a solution of open sentence if $p(a)$ is true statement for all $a \in A$ and the set of all solutions of open sentence is the set of solution and denoted by T_p

i.e.;

$T_p = \{a \in A : p(a)\}$ is True statement.

Ex:

let $A = \{-1, 0, 1\}$ and let $p(x) = x^3 \geq 1$ is open sentence over A

Find T_p?

$p(-1) = (-1)^3 = -1 \geq 1$ (false statement)

$p(0) = (0)^3 = 0 \geq 1$ (false statement)

$p(1) = (1)^3 = 1 \geq 1$ (True statement)

$T_p = \{a \in A : a \geq 1\} \Rightarrow T_p = \{1 \in A : 1 \geq 1\}$

Ex:

let $\mathbb{R}$ be a set of real numbers and let $p(x) = |x-1| < 3$ find T_p?

Defintion

logical statement

If p is a statement then the negation of a logical statement is a new logical statement which says the opposite of the orginal statement

Ex:

① the negation of $x = -1$ is $x \neq -1$.

② the negation of $x > 3$ is $x \leq 3$.

③ the negation of $a \in A$ is $a \notin A$.

Compound statement

A Compound statement is a sentence that consist of two or more statements separated by logical connectors as $(\wedge , \vee , \rightarrow , \leftrightarrow)$

① (Conjunction) ($\wedge$, and)

let p, q are statements the compound statement (p and q) denoted by $(p \wedge q)$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex: Write the truth table of the following statement

① $\sim p \wedge \sim q$ ② $p \wedge \sim q$ (H.W)

② disjuction ($\vee$, or)

let p, q are statements the compound statement (p or q) denoted by ($p \vee q$). A truth table for $p \vee q$ is shown below.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Ex: write the truth taple of the following statements:

① $\sim p \vee q$ ② $\sim p \vee (p \wedge \sim q)$ (H.W)

③ Conditional statement denoted by $p \rightarrow q$ is an if-then statement in which p is a hypothesis and q is a conclusion the logical connector in a conditional statement is dentoed by the symbole $\rightarrow$. the conditional is defined to be true unless a true hypothesis leads to a false conclusion. A truth table for $p \rightarrow q$ is shown below.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(4) Bi Conditional Statement

A biconditional statement is defined to be true whenever both parts have the same truth value. the biconditional operator The biconditional operator is denoted by a double-headed arrows $\leftrightarrow$. the biconditional $p \leftrightarrow q$ represents ("p if and only if q") where p is a hypothesis and q is a conclusion.

The following is truth table for biconditional $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Ex: write the truth table of the following statements

(1) $(p \leftrightarrow q) \wedge (\sim p \wedge \sim q)$

(2) $[(p \vee q) \leftrightarrow (p \rightarrow q)] \wedge \sim q$

(3) $(\sim p \rightarrow \sim q) \vee (p \leftrightarrow \sim q)$

(4) $p \vee q \leftrightarrow p \wedge q$ (H.W)