Mathematical proof

Firstly:-

proof of sentence $P \to Q$ (ie.; If P then Q) then there exists two method

A- Conditional proof:-

To the statement $P \to Q$ follow the following:-

i- suppose that P is true

ii- By using P, theorems and hypothesis we get Q

Ex:-

If a is an even number then a^2 be an even number.

proof:-

The statement of kind $P \to Q$

P : a is an even number , Q : a^2 is an even number

$a = 2K, \ K \in Z$

$a^2 = (2K)^2$

$a^2 = 4K^2$

$a^2 = 2(2K^2)$

$a^2 = 2t \quad , t = 2K^2 , t \in Z$

a^2 is even number

-B-

Contra postive method

To proof the statement of kind $P \to Q$ using contra postive method using logical equivalence $P \to Q \equiv \sim Q \to \sim P$

Ex:-

If a^2 is an even number then a be an even number.

Sol:-

P: a^2 is even number, Q: a is even number

$\sim P$: a^2 is odd number, $\sim Q$: a is odd number

$a = 2k+1$, $k \in \mathbb{Z}$

$a^2 = (2k+1)^2$, $k \in \mathbb{Z}$

$a^2 = 4k^2 + 4k + 1$, $k \in \mathbb{Z}$

$a^2 = 2(2k^2 + 2k) + 1$, $k \in \mathbb{Z}$

$a^2 = 2t + 1$, $t = 2k^2 + 2k$, $k, t \in \mathbb{Z}$

a^2 is an odd number

Secondly:-

To proof statement of kind $P \leftrightarrow Q$

(i.e.; proof sentence of kind P if and only if Q)

To proof sentence of $P \leftrightarrow Q$ using logical equivelence

$$P \longleftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$$

Ex:-

proof that a is odd number iff a+1 is even number

Sol:-

P: a is odd number , Q: a+1 is even number

The statement of kind $P \leftrightarrow Q$

$P \leftrightarrow Q \equiv (P \rightarrow Q \wedge Q \rightarrow P)$

Firstly, using to proof $P \rightarrow Q$ conditional proof

$a = 2k+1 \quad , k \in Z$

$a+1 = (2k+1)+1 \quad , k \in Z$

$a+1 = 2k+2 \quad , k \in Z$

$a+1 = 2(k+1), \; k \in Z$

$a+1 = 2t \quad , t = k+1, \quad t, k \in Z$

a+1 is even number

secondly: using contra postive

P: a is odd , Q: a+1 is even number

~P: a is even number , ~Q: a+1 is odd number

$a = 2k, \; k \in Z$

$a+1 = 2k+1 \quad , k \in Z$

a+1 is odd number

So, $\sim P \rightarrow \sim Q$ is true

Since $\sim P \rightarrow \sim Q \equiv Q \rightarrow P$

So that; $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

Thirdly:

To proof sentence of kind $p \vee Q \longrightarrow R$

proof that $p \longrightarrow R$ and $Q \longrightarrow R$ true

Ex:-

let $a, b \in \mathbb{Z}$, If a or b is an even number then a.b is an even number

Sol:-

p: a is even number a.b is even number, R: a.b is even number

the statement of kind $p \vee Q \longrightarrow R$

To proof that $p \to R$ and $Q \longrightarrow R$

To proof $p \longrightarrow R$ using conditional proof

$a = 2k \quad , k \in \mathbb{Z}$

$a.b = 2k.b = 2.t \quad , \quad t = k.b \quad , k, t \in \mathbb{Z}$

a.b is even number

To proof $Q \longrightarrow R$ is true

$b = 2k \quad , k \in \mathbb{Z}$

$b.a = 2k.a = 2t \quad , t = k.a \, , \; t, k \in \mathbb{Z}$

b.a is even number

$p \vee Q \longrightarrow R$

Fourthly:-

The proof by contradiction method

To proof the statement is true then using some logical Basic concepts

Ex:-

If x is an even number and $x \neq 0$ then $x^2 \neq 0$

proof:-

P: x is real number and $x \neq 0$

Q: $x^2 \neq 0$

To proof $P \to Q$ using contradiction method

suppose that P and $\sim Q$ is true (i.e.; x is real number and $x^2 = 0$)

since $x^2 = 0$ then $x \cdot x = 0$

either $x = 0$ or $x = 0$ but $x \neq 0$ C!

(since if x is real number and $x \neq 0$ then $x^2 \neq 0$ is true)

Fifthly:-

To proof statement of kind $\forall x \in A$, $p(x)$

suppose that x is an element of the set A and $p(x)$ is true

Ex:-

let Z^+ be set postive integer number and $p(x) = 2x + 1 \geq 1$

prove that $\forall x \in Z^+$ then $p(x)$ is true

sol:-

$Z^+ = \{0, 1, 2, \ldots\}$

let $x \in Z^+$

$x \geq 0$ $(\times 2)$

$2x \geq 0$ $(+1)$

$2x + 1 \geq 1$

$p(x)$ is true so that $p(x) = 2x + 1 \geq 1$ is true

Sixthly:-

proof statement of kind $\exists x \in A$, $p(x)$

Ex:-

let N be the set of real number prove that $\exists x \in N$ s.t

$2x + 1 \leq 3$

proof:-

$\exists 1 \in N \ni (2)(1) + 1 = 3 \leq 3$