algebra of sets

firstly: The union

Def:-
If A and B be two sets then $A \cup B = \{x : x \in A \vee x \in B\}$

Ex:- If $A = \{x : 0 \leq x \leq 6\}$ and $B = \{x : 6 < x < 13\}$ then

$A \cup B = \{x : 0 \leq x < 13\}$

Theorem:-

let A and B be two sets then:

① $A \subseteq A \cup B \wedge B \subseteq A \cup B$

② $A \subseteq B \longleftrightarrow A \cup B = B$

proof:-
we'll proof $A \subseteq A \cup B$

let $x \in A \Rightarrow x \in A \vee x \in B \Rightarrow x \in A \cup B \Rightarrow A \subseteq A \cup B$

Now. To proof that $B \subseteq A \cup B$ using the same methat of last proof

② let $A \subseteq B$, and $x \in A \cup B$ then we have

$x \in A \cup B \Rightarrow x \in A \vee x \in B \Rightarrow x \in B \vee x \in B \longrightarrow x \in B$

$\therefore A \cup B \subseteq B \Rightarrow \because B \subseteq A \cup B \Rightarrow \therefore A \cup B = B$

($\Leftarrow$) let $A \cup B = B$ from (1), we have $A \subseteq A \cup B \Rightarrow A \subseteq B$

Theorem:-

let A, B, and C be a sets, then:

① $A \cup A = A$ ② $A \cup B = B \cup A$ ③ $A \cup (B \cup C) = (A \cup B) \cup C$.

Theorem:-

let A be a set then:

① $A \cup \phi = A$ ② $A \cup X = X$

s.t ϕ is an empty set and X is an universel set.

Secondly:-

Def:-

If A and B be two sets then $A \cap B = \{x : x \in A \land x \in B\}$

Ex:-

If $A = (-1, 6)$ and $B = (-\infty, -1)$ then $A \cap B = \phi$

Theorem:-

let A and B be two sets:

① $A \cap B \subseteq A$, $A \cap B \subseteq B$

② $A \subseteq B \leftrightarrow A \cap B = A$

proof:-

① To proof that $A \cap B \subseteq A$

let $x \in A \cap B \Rightarrow x \in A \land x \in B \Rightarrow x \in A \Rightarrow A \cap B \subseteq A$

By the same we proof that $A \cap B \subseteq B$

② $(\Rightarrow)$

let $x \in A$, $A \subseteq B$

$x \in A \land A \subseteq B \Rightarrow x \in B \Rightarrow x \in A \land x \in B \Rightarrow x \in A \cap B \Rightarrow A \subseteq A \cap B$

Since $A \cap B \subseteq A$ then $A \cap B = A$

$(\Leftarrow)$

let $A \cap B = A$ and since $A \cap B \subseteq B$ then $A \subseteq B$

Theorem:-

let A, B and C be sets then:-

① $A \cup B = A$ ② $A \cap (B \cap C) = (A \cap B) \cap C$

③ $A \cap B = B \cap A$

Def:-

let A and B be two sets then $A \Delta B = (A-B) \cup (B-A)$

Ex:-

prove that $A - B = A \cap B^c$

Now, to proof that $A - B \subseteq A \cap B^c$ and $A \cap B^c \subseteq A - B$

let $x \in A - B$

$x \in A \wedge x \notin B$

$x \in A \wedge x \in B^c$

$x \in A \cap B^c$

$A - B \subseteq A \cap B^c$

let $x \in A \cap B^c$

$x \in A \wedge x \in B^c$

$x \in A \wedge x \notin B$

$x \in A - B$

$A \cap B^c \subseteq A - B$

that is, $A - B = A \cap B^c$

power of sets:

If A be a nonempty set then the set which all element are the subsets of A and denoted by $P(A)$

i.e.; $P(A) = \{x : x \subseteq A\}$

Ex:

let $A = \{a, b, c\}$ then $P(A) = \{A, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \emptyset\}$

Theorem:-

If A and B be two sets then

① $A \subseteq B \leftrightarrow P(A) \subseteq P(B)$

② $P(A \cap B) = P(A) \cap P(B)$

proof:- ① $(\Rightarrow)$

let $A \subseteq B$, Now to proof $P(A) \subseteq P(B)$

let $x \in A \Rightarrow x \in B \Rightarrow x \in p(B)$

Then $p(A) \subseteq p(B)$

$(\Leftarrow)$

let $p(A) \subseteq p(B)$ Now, to proof that $A \subseteq B$

let $x \in A$

$\{x\} \subseteq A \Rightarrow \{x\} \in p(x)$

but we have $p(A) \subseteq p(B)$

$\{x\} \in p(B)$

$\{x\} \subseteq B$

$\therefore A \subseteq B$

(أمير يوسف رفيعة)

Def:-

let $\{A_i\}_{i \in I}$ be a family of sets and be a finite if I is a finite set

and if I is an infinte then $\{A_i\}_{i \in I}$ be an infinite.

Ex:-

let $X = \{a, b, c\}$ then find the family index set

Sol:-

$\{\phi, X, \{a\}, \{b, c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

Def:-

The intersection and the union of index family of sets

If $\{A_i\}_{i \in I}$ be an index family of sets then $\bigcap_{i \in I} A_i = \{x : x \in A_i, \forall i \in I\}$

$\bigcup_{i \in I} A_i = \{x : \exists i \in I, x \in A_i\}$

Theorem:-

If $A_i \subseteq B \ \forall i \in I$ then $\bigcup_{i \in I} A_i \subseteq B$

proof:-

let $A_i \subseteq B \ \forall i \in I$

Now, To proof $\bigcup_{i\in I} A_i \subset B$

let $x \in \bigcup_{i\in I} A_i \rightarrow \exists\, i \in I \ni x \in A_i \Rightarrow x \in B$ (since $A_i \subset B$)

$\therefore \bigcup_{i\in I} A_i \subset B$

Theorem:

If $\{A_i\}_{i\in I}$ be an index family of sets then $(\bigcup_{i\in I} A_i)^c = \bigcap_{i\in I} A_i^c$

proof:-

let $x \in (\bigcup_{i\in I} A_i)^c \Leftrightarrow x \notin \bigcup_{i\in I} A_i \Leftrightarrow \forall i \in I : x \notin A_i \Leftrightarrow \forall i \in I : x \in A_i^c$

$\Leftrightarrow x \in \bigcap_{i\in I} A_i^c$

Def:- (cartiasin product)

If A and B be two sets then $A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$

Remark:-

① If $A = \phi$ then $A \times B = \phi$ and $B \times A = \phi$

② $A \times B = B \times A$

③ If the number of element of A is equal to n and the number of element of B is equal to n then the number of element of $A \times B$ is equal nm

④ The Cartiasin product of more than set

$\prod_{i\in I}^{n} A_i = A_1 \times A_2 \times A_3 \times \cdots \times A_n$

Theorem:

If A, B and C be a sets then:

① $A \times (B - C) = (A \times B) - (A \times C)$

② $A \times (B \cap C) = (A \times B) \cap (A \times C)$

④ $A \times (B \cup C) = (A \times B) \cup (A \times C)$

⑤ $A \times (B \times C) = (A \times B) \times C$

proofs:-

① let $(x,y) \in A \times (B - C)$ $\Leftrightarrow x \in A \wedge y \in B - C$

$\Leftrightarrow x \in A \wedge (y \in B \wedge y \notin C)$

$\Leftrightarrow (x \in A \wedge y \in B) \wedge (x \in A \wedge y \notin C)$

$\Leftrightarrow (x,y) \in A \times B \wedge (x,y) \notin A \times C$

$\Leftrightarrow (x,y) \in (A \times B) - (A \times C)$

Binary relation

Def:-

If A and B be two set then the relation R over set A to B as a subset $A \times B$

Ex:-

let $A = \{1,2\}$ and $B = \{2,4,6\}$. what is the following relations

① $R = \{(x,y) \in A \times B : x \leq y\}$ ② $S = \{(x,y) \in A \times B : y = 2x\}$

Sol:-

① $R = \{(1,2), (1,4), (1,6), (2,2), (2,4), (2,6)\}$

② $S = \{(1,2), (2,4)\}$