Kind of relations

Def:-

Assume R be a relation over A then:

① R is a revlexive relation.

$(x,x) \in R \quad \forall x \in A$

② R is a symmetric relation

If $(x,y) \in R$ then $(y,x) \in R \quad \forall x,y \in A$

③ R is a transtive relation

If $(x,y) \in R$ and $(y,z) \in R$ then $(x,z) \in R \quad \forall x,y,z \in A$

④ R is an anti-symmetric relation

If $(x,y) \in R$ and $(y,x) \in R$ then $x=y \quad \forall x,y \in A$

Remarks:-

1- R isn't reflexive relation if there exist $x \in A$ s.t $(x,x) \notin R$

2- R isn't symmetric relation if $(x,y) \in R$ s.t $(y,x) \notin R$

3- R isn't transtive relation if $(x,y) \in R$ and $(y,z) \in R$

s.t $(x,z) \notin R$

4- R isn't anti-symmetric relation if $(x,y) \in R$ and $(y,x) \in R$

s.t $x \neq y$

5- If R is a reflexive relation then the identity I_A be a sub set of R, i.e; $I_A \subseteq R$

Ex:-

Assume N be a set of natural number and

$R_1 = \{ (x,y) \in N \times N : x < y \}$

$R_2 = \{ (x,y) \in N \times N : x + y = 5 \}$

Is R_1 and R_2 be a reflexive, symmetric, transtive, and anti-

Sol:-

(a) with respect to R_1

1- R_1 isn't a reflexive relation

since $1 \in N$ and $1 \not< 1$ then $(1,1) \notin R$

2- R_1 isn't a symmetric relation

since $1 < 2$ but $2 \not< 1$

i.e.; $(1,2) \in R_1 \wedge (2,1) \notin R_1$

3- Assume $x, y, z \in N$ s.t $(x,y) \in R_1$ and $(y,z) \in R_1$ to proof that

$(x,z) \in R_1$

since $(x,y) \in R_1 \longrightarrow x < y$ --- (1)

since $(y,z) \in R_1 \longrightarrow y < z$

from (1) and (2), we get $x < z$

that is, $(x,z) \in R_1$

that is, R_1 be a transtive relation

4- Assume $x, y \in N$ s.t $(x,y) \in R_1$ and $(y,x) \in R_1$

T.P $x = y$

Since $(x,y) \in R_1$ then $x \leq y$ ---(1)

Since $(y,x) \in R_1$ then $y \leq x$ ---(2)

from (1) and (2) ; we get $x = y$

that is, R_1 is an anti-symmetric relation

(B) with respect to R_2

1- R_2 isn't reflexive relation

Since $1 \in N$ but $1+1 = 2 \neq 5$

that is, $(1,1) \notin R_2$

2- Assume $x, y \in N$ and $(x,y) \in R_2$

To proof that $(y,x) \in R_2$

since $(x,y) \in R_2$ then $x+y=5$

Since addition operation be a Commutiative   عملية جمع ابدالية

then $y+x=5$,

that is, $(y,x) \in R_2$

thus, R_2 be a symmetric relation

3- R_2 isn't a transtive relation

Since $(2,3) \in R_2$ and $(3,2) \in R_2$ then $(2,2) \notin R_2$

4- R_2 isn't anti-Symmetric relation

Since $(1,4) \in R_2$ and $(4,1) \in R_2$ but $1 \neq 4$

If S, R be two relation over A such that R and S be a reflexive, symmetric, transtive and anti-symmetric relations. Is $R \cup S$ and $R \cap S$ be a reflexive, symmetric, transtive and anti-symmetric relations and why?

proof:-

1- since S and R are reflexive relations

$\forall x \in A$, $(x,x) \in R \wedge (x,x) \in S$ then $(x,x) \in R \cap S$

that is, $R \cap S$ be a reflexive relation

2- Assume $(x,y) \in R \cap S$ then $(x,y) \in R \wedge (x,y) \in S$

Since R and S be two symmetric relation

$(y,x) \in R \wedge (y,x) \in S$ then $(y,x) \in R \cap S$

that is, $R \cap S$ be a symmetric relation

3- $(x,y) \in R \cap S$ and $(y,z) \in R \cap S$

$[(x,y) \in R \wedge (x,y) \in S] \wedge [(y,z) \in R \wedge (y,z) \in S)]$

$[(x,y) \in R \wedge (y,z) \in R] \wedge [(x,y) \in S \wedge (y,z) \in S]$

$(x,z) \in R \wedge (x,z) \in S$

$(x,z) \in R \cap S$

that is, $R \cap S$ be a transtive relation

4- Assume $(x,y) \in R \cap S$ and $(y,x) \in R \cap S$

$[(x,y) \in R \wedge (x,y) \in S] \wedge [(y,x) \in R \wedge (y,x) \in S]$

$[(x,y) \in R \wedge (y,x) \in R] \wedge [(x,y) \in S \wedge (y,x) \in S]$

Since R and S are an anti-symmetric relations

then $x=y$

that is, $R \cap S$ be an anti- symmetric relation.

"Equivalence relation" علاقة التكافؤ

Def:-

Assume R be a relation over A then R called by equivalence relation if it is satisfies the following conditions:-

1- reflexive 2- symmetric 3- transitive

Ex:-

let $\mathbb{R}$ be a set of real numbers and $R_1 = \{(x,y) \in \mathbb{R} \times \mathbb{R} : x=y\}$

Is R_1 be an equivelance relation? why?

Sol:-

1- let $x \in \mathbb{R}$ since $x=x$ then $(x,x) \in R_1$

that is, R_1 is a reflexive relation

2- let $x,y \in \mathbb{R}$ s.t $(x,y) \in R_1$

since $(x,y) \in R_1$ and $x=y$ then $y=x$ so. $(y,x) \in R_1$

that is, R_1 be a symmetric relation

3- let $x, y, z \in \mathbb{R}$ s.t $(x,y) \in R_1 \wedge (y,z) \in R_2$

since $(x,y) \in R_1$ then $x = y$ ---(1)

since $(y,z) \in R_1$ then $y = z$ ---(2)

From (1) and (2), we have $x = y = z$ then $x = z$

i.e.; $(x,z) \in R_1$

that is, R_1 be a transtive relation

Finally from 1, 2, and 3, we have R_1 be an equivalence relation over $\mathbb{R}$

Ex:-

Assume X be a nonempty set and $R = \{(A,B) \in P(X) \times P(X) ; A \cap B \neq \phi\}$

Is R be an equivalence relation? why?

Sol:-

Assume $X = \{1, 2, 3\}$ and $P(X) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, X\}$

1- $\exists A \in P(X) ; A \cap A \neq \phi$

let $\phi \in P(X)$ s.t $\phi \cap \phi = \phi$, then $(\phi, \phi) \notin R$

R isn't reflexive

2- $\exists A, B \in P(X)$ s.t $A \cap B \neq \phi \wedge B \cap A \neq \phi$ but $A \neq B$

2- $\exists A, B \in P(X)$ s.t $\{\{1,2\} \wedge \{1\} \neq \phi$

i.e.; $\{1\} \cap \{1,2\} \neq \phi \wedge$

then $\{1\} \neq \{1,2\}$ So R isn't symmetric

3-

$\{1\} \cap \{1,2\} \neq \phi \wedge \{1,2\} \cap \{2,3\} \neq \phi$

but $\{1\} \cap \{2,3\} = \phi$

R isn't transtive, then from (1), (2) and (3) R isn't equivalence relation.