Strict order relation

Assume R be a relation over A, R is called strict relation if it is satisfies the following conditions:

1- R is an irreflexive
2- R is an anti-Symmetric relation
3- R is a transitive relation

Ex:-

Assume $\mathbb{R}$ be a set of real numbers then

$R=\{(x,y)\in \mathbb{R}\times\mathbb{R} \,/\, x<y\}$. Is R is a strict relation over $\mathbb{R}$.

Sol:-

1) Assume $x\in\mathbb{R}$, since $x \nless x$ then $(x,x)\notin R$

then R is an irreflexive relation

2) Assume $x,y\in\mathbb{R}$ s.t $(x,y)\in R$ and $(y,x)\in R$

since $(x,y)\in R \longrightarrow x<y$ --- (1)

since $(y,x)\in R \longrightarrow y<x$ --- (2)

from (1) and (2), we have $x=y$

that is, R is an anti-symmetric relation

3) let $x, y, z \in \mathbb{R}$ s.t $(x,y) \in R$ and $(y,z) \in R$

Since $(x,y) \in R \longrightarrow x < y$ --- (1)

since $(y,z) \in R \longrightarrow y < z$ --- (2)

from (1) and (2), we have $x < z$

that is, $(x, z) \in R$

R is a transitive relation

from (1), (2) and (3), we have that R is a strict order relation over $\mathbb{R}$

Def:-

Assume R is a partially order relation over A we said that (A, R) is partial order set.

Remark:-

Some time we denoted to partially order relation over A by symbole $\precsim$ and every pair in the relation and written by $x \precsim y$ and reading by y follow x

Def:-

Assume $(A, \precsim)$ be a partial order set and let $x, y \in A$ we said that x, y are comparable if

If $x \precsim y$ or $y \precsim x$ except that we said that x, y not Comparable

Def:-

Assume $(A, \precsim)$ be a partial order set, we said that $(A, \precsim)$ be a totally order set if every two elements in A are Comparable

Ex:-

proof that $\mathbb{Z}$ be a set of integer numbers be a totally order set.

proof:- $\precsim = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x \leq y\}$

$\precsim$ is a partial order set

let $x, y \in \mathbb{Z}$

From properties of integer numbers

① $x < y$

② $y < x$

③ $x = y$

* If $x < y$

Since $x \leq y$ then $(x, y) \in \precsim$

So that $x \precsim y$

* If $y < x$

Since $y \leq x$ then $(y,x) \in \precsim$

So that $y \precsim x$

* If $x = y$

Since $x \leq y$ then $(x,y) \in \precsim$

So that $x \precsim y$

that is, x and y are comparable

$\precsim$ is a totally order relation

Def:-

Assume (A,R) be a partial order set then:

1- the elemen $b \in A$ called by the smallest element belong to R iff $(b,x) \in R$ $\forall x \in A$

2- the element $b \in A$ called by the greatest element belong to R iff $(x,b) \in R$ $\forall x \in A$

Ex:-

Find the smallest and greatest element for the following relation that defined on A s.t $A = \{3, 6, 9, 12, 15\}$

$R = \{(x,y) \in A \times A ; x \leq y\}$

Sol:-

Firstly R is a partial order relation

Now, To find the smallest element in R

Take $3 \in A$ and since $3 \leq x \quad \forall x \in A$

then $(3, x) \in R$

That is, 3 be the smallest element in R

Now, To find the greatest element in R

Take $15 \in A$ and $15 \geq x \quad , \forall x \in A$

Then $(x, 15) \in R$

that is, 15 be the greatest elemen in R

Theorem:-

Assume (A, R) be a partial order set then:-

① the smallest element is a unique

② The greatest element is a unique

proof:-

① let a_1, a_2 be a two elements in A that the smallest element w.r.t R

Since a_1 is a smallest element in relation R

then $(a_1, x) \in R \quad \forall x \in A$

Since $a_2 \in A$ then $(a_1, a_2) \in R$

Since a_2 is Smallest element in R

then $(a_2, x) \in R \quad \forall x \in A$

Since $a_1 \in A$

then $(a_2, a_1) \in R$

Then, we have (a_2, a_1) and $(a_1, a_2) \in R$

Since R is a partial order set then R is an anti-

Symmetric relation

So that $a_1 = a_2$

that is, smallest element is a unique

The mappings or functions

Def:-

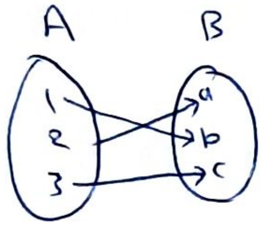
Assume $F: A \longrightarrow B$ s.t for all $x \in A$ there exist a unique element $y \in B$ s.t $(x, y) \in F$

Ex:-

Assume $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$ and $F: A \longrightarrow B$

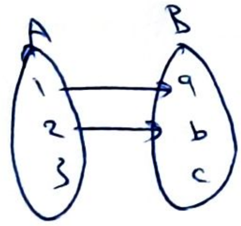
defined by;

(1)



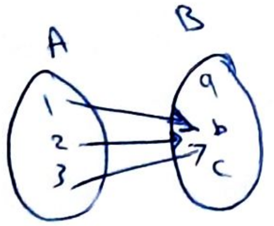
, $F: A \longrightarrow B$ is a mapping

(2)



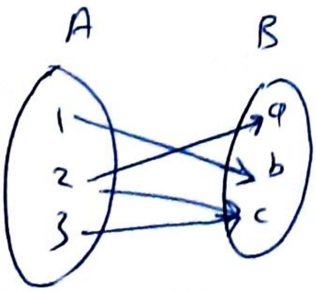
, $F: A \longrightarrow B$ isn't a mapping

(3)



, $F: A \longrightarrow B$ is a mapping

(4)



, $F: A \longrightarrow B$ isn't a mapping

Def:-

Assume $F: A \longrightarrow B$ is a mapping, the A is called be a domain and B is a codomain.

The Kind of mappings (functions)

1- surjective mapping

The function $F: A \longrightarrow B$ is a surjective or (onto)

$\forall y \in B \ \exists x \in A$ s.t $F(x) = y$

Ex:-

If $F: \mathbb{R} \longrightarrow \mathbb{R}$ defined by $F(x) = 5x+1$

sol:-

It is clear that F is a surjective mapping

2- Injective mapping

The function $F: A \longrightarrow B$ is an injective or (one to one)

if $F(x_1) = F(x_2) \longrightarrow x_1 = x_2$

also, $x_1 \neq x_2 \longrightarrow F(x_1) \neq F(x_2)$

Ex:-

If $F: \mathbb{R} \longrightarrow \mathbb{R}$ defined by $F(x) = 5x+1$

Sol:-

The mapping is one to one (injective)

Since $1 \neq 2$ and $F(1) \neq F(2)$

3- Bijective mapping

The function $F: A \longrightarrow B$ called bijective mapping

If F is a surjective and injective