1- Minterms and Maxterms

Minterm

A product term containing all n variables of the function in either true or complemented form is called the minterm. Each minterm is obtained by an AND operation of the variables in their true form or complemented form. For a two-variable function, four different combinations are possible, such as, A′B′, A′B, AB′, and AB. These product terms are called the fundamental products or standard products or minterms. The minterm is formed using the values of the variables which make the value of the minterm equal to 1, hence m_0 = A′B′. The other three minterms are obtained in the same way.

A	B	Minterms	Maxterms
0	0	$m_0 = \bar{A}\bar{B}$	$M_0 = A + B$
0	1	$m_1 = \bar{A}B$	$M_1 = A + \bar{B}$
1	0	$m_2 = A\bar{B}$	$M_2 = \bar{A} + B$
1	1	$m_3 = AB$	$M_3 = \bar{A} + \bar{B}$

Note that, if the number of variables is n, then the possible number of minterms is 2^n.

Maxterm

A sum term containing all n variables of the function in either true or complemented form is called the maxterm. Each maxterm is obtained by an OR operation of the variables in their true form or complemented form. Four different combinations are possible for a two-variable function, such as, A′ + B′, A′ + B, A + B′, and A + B. These sum terms are called the standard sums or maxterms. The maxterms are formed using the values of the variables which make the value of the maxterm equal to 0. Note that, if the number of variables is n, then the possible number of maxterms is 2^n.

Now, for $A = 0$ and $B = 0$ we have that

$$m_0 = \bar{A}\bar{B} = 1, \text{ and } \bar{m}_0 = \overline{\bar{A}\bar{B}} = 0,$$

giving

$$\bar{m}_0 = M_0 = A + B,$$

i.e. the maxterm is the logical complement of its corresponding minterm.

One important property of minterms is that the logical OR of all 2n minterms is equal to logical 1, i.e.

$$\sum_{i=0}^{2^n-1} m_i = 1$$

The dual of this equation is

$$\prod_{i=0}^{2^n-1} M_i = 0$$

where ∏ signifies the Boolean product (AND), so that the logical product of all the maxterms is equal to logical zero.

2- Canonical and Standard forms:

A. Canonical form

- A truth table represents inputs and outputs. If there is 'n' number of input variables, then there are 2^n number of outputs or combinations or ones and zeros.
- Canonical forms are used to obtain the function from the given truth table.
- There are two methods in canonical form to represent an output variable. They are:
  - Canonical SoP form.
  - Canonical PoS form.

1- Canonical Sum of Product (SoP)

Canonical SoP stands for Canonical Sum of Products. This form considers the minterms. It is called sum of minterms form. First, it is necessary to recognize the min terms that have 1 as the output variable. After identifying the minterms, the logical OR is used to find the Boolean expression equivalent to the output variable.

In other words, A Boolean function F may be expressed algebraically as a sum of minterms from a given truth table by:
Step1: forming a minterm for each combination of the variables which produce 1 in the function.
Step2: OR all of the minterms in step1.

Example: From the given truth table express F and as a sum of minterms (SoP).

Given				Solution	
Inputs			Output	Step1	Step2
x	y	z	F	minterms	Sum of minterms
0	0	0	0		F= x'y'z+ x'yz+ xyz
0	0	1	**1**	x'y'z	$F=m_1+m_3+m_7$
0	1	0	0		F(x,y,z)=Σ(1,3,7)
0	1	1	**1**	x'yz	
1	0	0	0		
1	0	1	0		
1	1	0	0		
1	1	1	**1**	xyz	

Also, from the table F' can be expressed as a sum of minterms as follows:
Step1: forming a minterm for each combination of the variables which produce 0 in the function.
Step2: OR all of the minterms in step1.

Example: From the given truth table express F' as a sum of minterms

Given				Solution	
Inputs			Output	Step1	Step2
x	y	z	F	minterms	Sum of minterms
0	0	0	**0**	x'y'z'	F'= x'y'z'+ x'yz'+ xy'z'+ xy'z+ xyz'
0	0	1	1		$F'=m_0+m_2+m_4+m_5+m_6$
0	1	0	**0**	x'yz'	$F'(x,y,z)=\Sigma(0,2,4,5,6)$
0	1	1	1		
1	0	0	**0**	xy'z'	
1	0	1	**0**	xy'z	
1	1	0	**0**	xyz'	
1	1	1	1		

2- Canonical Product of Sum (PoS)

Canonical PoS stands for Canonical Product of Sums. This form concerns the maxterms. It is called the Product of Maxterms form. First, it is necessary to recognize the maxterms that have 0 as the output variable. After identifying the maxterms, the logical AND is used to find the Boolean expression equivalent to the output variable.

In other words, A Boolean function may be expressed algebraically as a product of maxterms from a given truth table by:

Step1: forming a maxterms for each combination of the variables which produce 0 in the function.

Step2: form the AND of all the maxterms in step1.

Example: From the given truth table express F as a product of maxterms (PoS).

Given				Solution	
Inputs			Output	Step1	Step2
x	y	z	F	maxterms	Product of maxterms
0	0	0	**0**	(x+y+z)	F=(x+y+z) (x+y'+z) (x'+y+z) (x'+y+z') (x'+y'+z)
0	0	1	1		$F= M_0.M_2.M_4.\ M_5.M_6$
0	1	0	**0**	(x+y'+z)	$F(x,y,z)=\Pi(0,2,4,5,6)$
0	1	1	1		
1	0	0	**0**	(x'+y+z)	
1	0	1	**0**	(x'+y+z')	
1	1	0	**0**	(x'+y'+z)	
1	1	1	1		

Also, From the table F' can be expressed as follows:

Step1: forming a maxterm for each combination of the variables which produce 1 in the function.

Step2: form the AND of all the maxterms in step1.

Example: From the given truth table express F' as a product of maxterms

Given				Solution	
Inputs			Output	Step1	Step2
x	y	z	F	maxterms	Product of maxterms
0	0	0	0		F'= (x+y+z') (x+y'+z') (x'+y'+z')
0	0	1	**1**	(x+y+z')	$F'=M_1.M_3.M_7$
0	1	0	0		F'(x,y,z)= Π (1,3,7)
0	1	1	**1**	(x+y'+z')	
1	0	0	0		
1	0	1	0		
1	1	0	0		
1	1	1	**1**	(x'+y'+z')	

B. Standard Forms:

- Standard form is a simplified version of canonical form that represents Boolean outputs of digital circuits using Boolean Algebra.
- We discussed two canonical forms of representing the Boolean outputss. Similarly, there are two standard forms of representing the Boolean outputss. These are the simplified version of canonical forms:
  - Standard SoP form
  - Standard PoS form
- The main advantage of standard forms is that the number of inputs applied to logic gates can be minimized. Sometimes, there will be reduction in the total number of logic gates required.
- Sum terms: single variable or logical sum of several variables such as (A, B, (x+y), (A+C')).
- Product terms: single variable or logical product of several variables such as (x, y, AB', CD')
- Note: the expression x+y'z (not sum term nor product terms).

1- Standard Sum of Product (SoP)

Standard SoP is a Boolean expression containing AND terms, called product terms, of one or more literals each. The sum denotes the ORing of these terms. An example of a function expressed as a sum of product is

F1=y'+xy+x'yz'

The expression contains three product terms (y' one literal, xy two literals and x'yz' three literals). Their sum is in effect an OR operation. The logic diagram if a sum-of-product expression consist of a group of AND gates followed by a single OR gate.

2- Standard Product of Sum (PoS)

Standard PoS is a Boolean expression containing OR terms, called sum terms, of one or more literals each. The product denotes the ANDing of these terms. An example of a function expressed as a product of sum is

F2=x(y'+z)(x'+y+z)

The expression contains three sum terms (x one literal, y'+z two literals and x'+y+z three literals). The product is an AND operation. The logic diagram if a product-of-sum expression consist of a group of OR gates followed by a single AND gate.

Example: From the given truth table express F as a sum of product then simplify as a standard sum of product.

Given			
x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	**1**
0	1	1	**1**
1	0	0	0
1	0	1	0
1	1	0	**1**
1	1	1	0

Given				**Solution**	
Inputs			**Output**	**Step-1**	**Step-2**
X	**y**	**z**	**F**	**Minterms**	**SoP**
0	0	0	0		F= x'yz' + x'yz + xyz'
0	0	1	0		F= $m_2 + m_3 + m_6$
0	1	0	**1**	x'yz'	F(x,y,z)= Σ(2,3,6)
0	1	1	**1**	x'yz	
1	0	0	0		
1	0	1	0		
1	1	0	**1**	xyz'	
1	1	1	0		

Solution: the function equal to 1 in {(2,x'yz'),(3,x'yz),(6,xyz')}

So **F=x'yz'+x'yz+xyz'** (Sum of Minterms SoP)

Simplification of F

F=x'yz'+x'yz+xyz'	(distributive law)
=x'y(z'+z)+xyz'	(complement definition: A+A'=1)
=x'y.1+xyz'	(identity element: A.1=A)
=x'y+xyz'	(distributive law)
=y(x'+xz')	(distributive law)
=y(x'+x)(x'+z')	(complement definition: A+A'=1)
=y.1.(x'+z')	(identity element: A.1=A)
=y(x'+z')	(distributive law)
=x'y+yz'	

Example: From the given truth table express F as a product of maxterms PoS then simplify as a standard product of sum.

Given			
x	y	z	F
0	0	0	**0**
0	0	1	**0**
0	1	0	1
0	1	1	1
1	0	0	**0**
1	0	1	**0**
1	1	0	1
1	1	1	**0**

Given				**Solution**	
Inputs			**Output**	**Step-1**	**Step-2**
X	**y**	**z**	**F**	**Maxterms**	**PoS**
0	0	0	0	**(x+y+z)**	F= **(x+y+z)(x+y+z')(x'+y+z) (x'+y+z')(x'+y'+z')**
0	0	1	0	**(x+y+z')**	F= $M_0 . M_1 + M_4 + M_5 + M_7$
0	1	0	**1**		F(x,y,z)= ∏ (0,1,4,5,7)
0	1	1	**1**		
1	0	0	0	**(x'+y+z)**	
1	0	1	0	**(x'+y+z')**	
1	1	0	**1**		
1	1	1	0	**(x'+y'+z')**	

Solution: the function equal to 0 in {(0,x+y+z), (1,x+y+z'), (4,x'+y+z), (5,x'+y+z'), (7,x'+y'+z')}

So **F=(x+y+z)(x+y+z')(x'+y+z) (x'+y+z')(x'+y'+z')** product of maxterms

Simplification of F

F=(x+y+z) (x+y+z') (x'+y+z) (x'+y+z') (x'+y'+z')	(distributive law)
=[(x+y)+zz'] [(x'+y)+zz')] (x'+y'+z')	(complement definition: A.A'=0)
=[(x+y)+0] [(x'+y)+0] (x'+y'+z')	(identity element: A+0=A)
=(x+y) (x'+y) (x'+y'+z')	(distributive law)
=(y+xx') (x'+y'+z')	(complement definition: A.A'=0)
= (y+0) (x'+y'+z')	(identity element: A+0=A)
= y(x'+y'+z')	(distributive law)
= x'y+yy'+yz'	(complement definition: A.A'=0)
=x'y+0+yz'	(identity element: A+0=A)
=x'y+yz'	(distributive law) to convert the function from SOP to POS
=y(x'+z')	