Arithmetic Operations on Binary numbers:

Binary arithmetic is used in digital systems mainly because the numbers (decimal and floating-point numbers) are stored in binary format in most computer systems. All arithmetic operations such as addition, subtraction, multiplication, and division are done in binary representation of numbers. It is necessary to understand the binary number representation to figure out binary arithmetic in digital computers.

1- Binary Addition Operation

The simplest arithmetic operation in binary is addition. It is a key for binary subtraction, multiplication, division.

- In addition: A + B = B + A
- There are five rules of binary addition.

0 + 0 = 0	
0 + 1 = 1	
1 + 0 = 1	
1 + 1 = 0	with carry 1
1 + 1 + 1 = 1	with carry 1

Ex1: add the following two binary numbers: $(1001)_2 + (1011)_2$

Carry:	1 1 1	
	1 0 0 1	9
	1 0 1 1	11
	1 0 1 0 0	20

HW: add the following two binary numbers: $(11010)_2 + (10100)_2$

2- Binary Subtraction Operation [Complement and Subtraction using 1's and 2's Complement]

A- Complement

Complements are used in digital computers for simplifying the subtraction operation and for logical manipulation. There are two types of complements for each base (r) system:

I. The r's complements (Radix Complement)

II. The (r-1)'s complements (Diminished Radix Complement)

Base	r's complements	(r-1)'s complements
10	10's complements	9's complements
2	2's complements	1's complements
8	8's complements	7's complements
16	16's complements	F's complements

1. The r’s complements (Radix Complement)

A positive number N in base r with an integer part of n digits, the r’s complement of N is defining as follows:

r^n-N	**for**	**$N \neq 0$**
0	**for**	**N=0**

Ex1: Find the r’s complement of $(7)_{10}$

Answer: N=7 , n=1, r=10

rn-N = 101-7 = 3

Ex2: Find the r’s complement of $(5690)_{10}$

Answer: N=5690 , n=4, r=10

rn-N = 104-5690 = 10000 – 5690 = 4310

Ex3: Find the 2’s complement of $(1101)_2$

Answer: N=1101 , n=4, r=2

rn-N = 24-1101 = (16)10 – (1101)2

= 16 – 13 = 3 = (0011)2

2. The (r-1)’s complements (Radix Complement)

A positive number N in base r with an integer part of n digits, the (r-1)’s complement of N is defining as follows:

$$\text{(r-1)’s complement} = r^n - N - 1$$

One’s complement (1’s):- can be found by changing all 1's to 0's and all 0's to 1's it used only with binary number.

Ex1: Find the 1’s complement of $(10101)_2$

Answer: Given Number: **$(10101)_2$**

1’s complement: **$(01010)_2$**

Ex2: Find the 1’s complement of $(1101)_2$

Answer: Given Number: **$(1101)_2$**

1’s complement: **$(0010)_2$**

Now we have:

r's complement = $r^n - N$
(r-1)'s complement = $\underline{r^n - N} - 1$
(r-1)'s complement = r's complement - 1
(r-1)'s complement + 1 = r's complement

This means, the 2's complement of binary number can be easily obtained by adding 1 to the Least Significant Bit (LSB) of 1's complement of the number.

Ex3: Find the 1's and 2's complement of $(1001)_2$

Answer: Given Number: $(1001)_2$

1's complement = $(0110)_2$

2's complement = 1's complement +1

$$= \begin{array}{r} 0\,1\,1\,0 \\ +\quad\quad 1 \\ \hline (\;0\,1\,1\,1\;)_2 \end{array}$$

B- Subtraction Operation

- In subtraction: $A + B \neq B + A$
- $A - B = A + (-B)$
- Flow the following steps to perform the subtraction operation (A - B):
  1- Take the 1's complement of the subtracted number (second number: B) to get $\overline{B}$. (1 → 0 , 0 → 1)
  2- Take the 2's complement of $\overline{B}$ and add it to the minuend number which remained unchanged.
  3- Check the result:
    1- If an end carry occurs discard it and the result is positive number.
    2- If an end carry does not occurs take the 2's complement to the result and put (-) sign (the result is negative number).

Ex1: Subtract the binary number $(1011011)_2$ from $(1101100)_2$

$$\begin{array}{rrr} & 1101100 & 108 \\ - & 1011011 & 91 \\ \hline \end{array}$$

1's complement of $(1011011)_2$ is: 0100100

2's complement of (0100100) is: 0100101

```
        1 1   1 1
          1 1 0 1 1 0 0
+         0 1 0 0 1 0 1
-----------------------------
        1 0 0 1 0 0 0 1
```

The final result: $(0010001)_2$ 17

An end carry occurs discard it

Ex2: Subtract the binary number $(1101100)_2$ from $(1011011)_2$.

```
     1011011        91
-    1101100        108
-----------------------------
```

1's complement of $(1101100)_2$ is: 0010011

2's complement of (0010011) is: 0010100

```
            1
          1 0 1 1 0 1 1
+         0 0 1 0 1 0 0
-----------------------------
          1 1 0 1 1 1 1
```

The final result: $-(0010001)_2$ -17

End carry does not occurs take the 2's complement to the result and put (-) sign

HW: Subtract the binary number $(10100101)_2$ from $(10101010)_2$.
HW: Subtract the binary number $(10101010)_2$ from $(10100101)_2$.

3- Binary Multiplication Operation

- There are four rules of binary multiplication.

0 * 0	= 0
0 * 1	= 0
1 * 0	= 0
1 * 1	= 1

Ex1: Multiply the following two binary numbers: $(1011)_2 * (111)_2$

	1 0 1 1	11
	1 1 1	7
carry	1 1 1 1 1	
	1 0 1 1	
	1 0 1 1	
+	1 0 1 1	
	1 0 0 1 1 0 1	77

Ex2: Multiply $(1001.11)_2$ by $(111.01)_2$

	1 0 0 1.1 1	9.75
	1 1 1.0 1	7.25
	1	
Carry	1 1 1 1 0 1 1	
	1 0 0 1 1 1	
	0 0 0 0 0 0	
	1 0 0 1 1 1	
	1 0 0 1 1 1	
+	1 0 0 1 1 1	
	1 0 0 0 1 1 0 1 0 1 1	70.6875

$(1\ \ 0\,0\,0\,1\,1\,0.1\,0\,1\,1)_2$

4- Binary Division Operation

Ex1: Divide $(1001110)_2$ by $(100)_2$

	10011.1	
100	1001110	78 / 4 = 19.5
	100	
	000111	
	100	
	0110	
	100	
	0100	Add 0 and put (.) on the result
	100	
	000	

The final result: $(10011.1)_2$

Ex2: Perform the following binary division:

1- $(11101)_2 / (10)_2$
2- $(11010)_2 / (10)_2$
3- $(10100)_2 / (111)_2$

Solve: $(11101)_2 / (10)_2$

```
           1110.1
10      |  11101          29 / 2 = 14.5
           10
           011
            10
            010
             10
             001
              00
              10          Add 0 and put (.) on the result
              10
              00
```

The final result: $(1110.1)_2$