



21.10.2019

((Assessment of the final exam for the first semester))

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Academic year 2018-2019

Q1. Consider a particle of a mass  $m$  subjected to the Hamiltonian

$$H = \begin{cases} \frac{p^2}{2m} + \frac{m\omega^2 r^2}{2} & : 0 \leq r \leq a \\ \frac{p^2}{2m} & : r > a \end{cases}$$

where  $r = \sqrt{x^2 + y^2}$ . Use the second-order perturbation to find the corrections to the ground state energy. (12M)

Q2. Consider a harmonic oscillator with a force constant  $k$  and a reduced mass  $m$ .

The small perturbation  $W = ax^3$  is applied to the oscillator. Compute the first order correction to the wave functions and first nonvanishing correction to the energies. (10M)

Q3. Obtain the matrix of Clebsch-Gordan coefficients when two angular momenta  $j_1 = 1/2$  and  $j_2 = 1/2$  are coupled. (12 M)

Q4. Consider a one-dimensional harmonic oscillator: (a) For the one-parameter family of wave functions  $\psi_\gamma(x) = \frac{1}{x^2 + \gamma}$ ; ( $\gamma > 0$ ), find a wave function that minimizes  $\langle H \rangle$ . What is the value of  $\langle H \rangle_{\min}$ . (b) Repeat the same procedure for  $\psi_\beta(x) = x \exp(-\beta x^2)$ . (12M)

Q5. Calculate the matrix representation of the angular momentum operators  $J_x, J_y, J_z$ , and  $J^2$ , for the values  $j = \frac{1}{2}, 1, \frac{3}{2}$ , respectively. (12M)

Q6. A. Show that for a harmonic oscillator, the matrix elements of operator  $p$  is given,

$$\langle n' | p | n \rangle = i \sqrt{\frac{m\hbar\omega}{2}} (\sqrt{n+1} \delta_{n',n+1} - \sqrt{n} \delta_{n',n-1})$$

B. Show that:  $[J_x, J_y] = \hbar J_z$

(12M)

Good Luck

Lecturer  
Dr. Hadey K. Mohamad

Head of Department



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Q1//

(14 Mark)

- (i) By Cayley-Hamilton theorem ,find  $A^{-1}$  of the matrix  $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$  ,then show that if  $0 < \theta < \pi$  ,then  $A^{-1}$  has no real Eigen values and consequently no Eigen vectors.
- (ii) Show that  $x^3 = \frac{2}{5}p_3(x) - \frac{3}{5}p_1(x)$  , where  $p_n(x)$  is Legendre polynomial .

Q2//

(12 Mark)

- (i) find the first three terms of Tayler series expansion of  $f(z) = \frac{1}{z^2+4}$  about  $z = -i$ , find the region of convergence.
- (ii) if  $D=P^{-1}AP$  then  $D^n=P^{-1}A^nP$  where  $A$  is a square matrix and  $P$  is a non-singular matrix .

Q3//

(14 Mark)

- (i) determine the poles of the following function and residue at each pole

$$\int \frac{(12z-7)dz}{(z-1)^2(2z+3)}$$
 where C is the circle: (a)  $|z| = 2$  (b)  $|z+i| = \sqrt{3}$

- (ii) if  $A^i_j$  and  $B^m_n$  are tensors then their sum and difference are tensors of the same rank and type.

Q4//

(15 Mark)

- (i) calculate  $\oint \frac{(2z^2+5)dz}{(z+2)^3(z^2+4)}$  , where C is the square with the vertices at (1+i),(2+i),(2+2i),(1+2i).
- (ii) if  $\phi = a_{jk} A^j A^k$  show that we can always write if  $\phi = b_{jk} A^j A^k$  where  $b_{jk}$  is symmetric.
- (iii) prove that  $x^2 J''_n(x) = (n^2 - n - x^2)J_n(x) + x J_{n+1}(x)$

Q5//

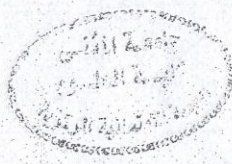
(15 mark)

- (i) An electrostatic field in the xy-plane is given by the potential function  $\zeta = x^2 - y^2$  . Find the stream function.
- (ii) solve  $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$  in series.

- (iii) find a matrix P which diagonalizes the matrix  $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$  verify  $P^{-1}AP = D$ .

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Subject: Adv. Classical  
Mechanics  
Stage: MSc.

Date: / / 2019  
Time : 3 hr.

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Q1// By using Definitions of D Alembert's find the relation for Lagrange equation?  
(14 marks)

Q2// A) Defined the constraints? What are the types of it and described it?  
What are the difficulties of constraints (8Marks)

B) Satisfying the relation  $\vec{N} = \dot{\vec{L}}$  where ,N, is torque and ,L, is angular momentum ?  
(6 Marks)

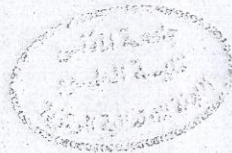
Q3// Use the (x,y) coordinate system of Fig(1) to find the kinetic energy, potential energy, and the lagrangian for a simple pendulum (length  $l$ , mass bob  $m$ ) moving in the x,y plane. Determine the transformation equations from the (x,y) rectangular system to the coordinate , Find the equation of motion.  
(14Marks)

Q4// // investigate the stability of circular orbits in a force field described by the Potential function:  
(14Marks)

$$U(r) = \frac{-k}{r^2} e^{-(2r/a)}$$

Q5// A particle of mass  $m$  starts at rest on top of a smooth fixed hemisphere of radius  $a$  (See Fig.2) . Find the force of constraint, and determine the angle at which the particle leaves the hemisphere?  
(14Marks)

Lecturer  
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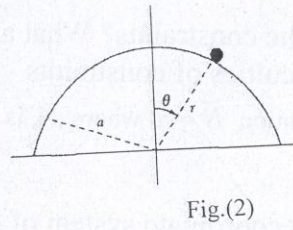
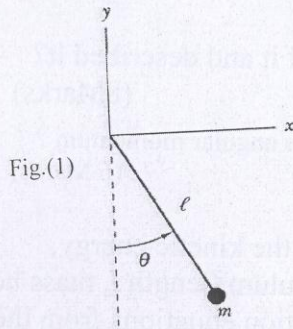
Head of Department  
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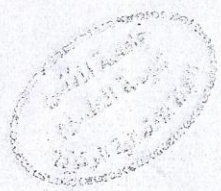
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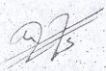
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