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**Muthanna University**
**College of Science**
**Department of Mathematics and Computer Application**

**Subject: Advanced Calculus II**
**Stage: Second**
**Date: / /2017**
**Time :Three Hours**

06. 06. 2017

45

**(( Final exam for the second semester))**
**Academic year 2016-2017**

Q1)) (a) Defined: sequence, convergence sequence, bounded sequence and monotonic sequence. Is every bounded sequence converge?

(b) Express the repeating decimal 0.123123123… as a rational number.

Q2)) (a) Given an example in which $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ both diverge but $\sum_{k=1}^{\infty}(a_k + b_k)$ is converges.

(b) Show that $\frac{1}{1+x} = \sum_{k=0}^{\infty}(-1)^k x^k$ ,for any real number $x$ with $|x| < 1$.

(c ) Prove $\lim_{n\to\infty}\left(1-\frac{x}{n}\right)^n = e^{-x}$

Q3))(a) Discuss the convergence of the following series: (1) $\frac{x}{1\times3} + \frac{x^2}{3\times5} + \frac{x^3}{5\times7} + \cdots \quad ,x > 0$

(2) $x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \frac{15}{17}x^4 + \cdots + \frac{n^2-1}{n^2+1}x^n + \cdots \quad ,x > 0$

(b) If $f(x) = (\frac{\pi-x}{2})^2$ in the interval $0 < x < 2\pi$ show that $f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\frac{\cos nx}{n^2}$

Q4))(a) Prove that the infinite series $\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + \cdots$, (I) converges to $\frac{a}{1-r}$ $if |r| < 1$

(II) diverges if $|r| \geq 1$

(b)Find the Fourier expression of $f(x) = x + x^2 \, , -\pi < x < \pi$ and prove $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

Q5))(a)Examine for convergence and absolute convergence of the following series:(1) $\sum_{k=1}^{\infty}\frac{(-1)^{k-1}k}{k^2+1}$

(2)$\sum_{k=1}^{\infty}\frac{(-1)^{k-1}\,2^k}{k^2}$

(b) Determine each of the following series if convergent or divergent by using root test:

(1)$\sum_{k=1}^{\infty}\frac{2^k}{k^2}$ (2)$\sum_{k=1}^{\infty}\frac{1}{k^k}$

(c)Find the Taylor series generated by the function:

$$f(x) = \frac{1}{5-x} \qquad at\ x = 2$$