Ministry of Higher Education
& Scientific Research
Al-Muthanna University
College of Science
Department of Mathematics
and Computer Applications

Subject: Complex analysis ~~II~~
Stage: 4th stage
Date: / / 2017
Time : 3 hours 06. 06. 2017

((Assessment of the final exam for the second semester))

Academic year 2016 -2017

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Q1/ State Cauchy theorem and its corollaries, and then find $\int_L \frac{dz}{z}$ *where L is the ellipse* $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

Q2/ State Cauchy integral's formula, and then evaluate $\int_C \frac{e^{z^2}}{z^2-6z} dz$; $c: |z-2| = 3$.

Q3/ Use Cauchy's residue theorem to evaluate $\oint_C \frac{2z+6}{z^2+4} dz$; $c: |z-i| = 2$.

Q4/ State the behavior theorem of integral as $R \to \infty$ *, and then evaluate the Cauchy principle value of* $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)(x^2+9)} dx$.

Q5/ A/ Compute $\int_C \frac{dz}{z^2+1}$; *where* $c: |z-1+2i| = \frac{1}{4}$.

B/ Show that : $\int_0^{\infty} \frac{x^{\frac{1}{3}}}{(x+1)^2} dx = \frac{2\pi}{3\sqrt{3}}$, $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta = \frac{\pi}{12}$.

Q6/ Define the Laurent series, then find the Laurent expansion of $f(z) = \frac{e^z}{z^{10}}$ *at* $z = 0$.

Q7/ State and prove the fundamental theorem of algebra, and give an example.

Q8/ Define the zeros of f(z), and prove that the polynomial $z^4 + z + 1$ *has one zero in each quadrant.*

Q9/ Define a conformal mapping, and then prove that: An analytic function f is conformal at every point z_0 *for which* $f'(z_0) \neq 0$.

Q10/ Find the bilinear transformation that maps the points $= -1, \infty, i$ *onto* $w = \infty, i, 1$.

note: 6 marks for each question

Good Luck